DIFFERENTIATION

1 In each case, find any values of x for which $\frac{dy}{dx} = 0$.

a $y = x^2 + 6x$ **b** $y = 4x^2 + 2x + 1$ **c** $y = x^3 - 12x$ **d** $y = 4 + 9x^2 - x^3$

e $y = x^3 - 5x^2 + 3x$ **f** $y = x + \frac{9}{x}$ **g** $y = (x^2 + 3)(x - 3)$ **h** $y = x^{\frac{1}{2}} - 2x$

2 Find the set of values of x for which f(x) is increasing when

a $f(x) \equiv 2x^2 + 2x + 1$ **b** $f(x) \equiv 3x^2 - 2x^3$ **c** $f(x) \equiv 3x^3 - x - 7$

d $f(x) \equiv x^3 + 6x^2 - 15x + 8$ **e** $f(x) \equiv x(x - 6)^2$ **f** $f(x) \equiv 2x + \frac{8}{x}$

3 Find the set of values of x for which f(x) is decreasing when

a $f(x) \equiv x^3 + 2x^2 + 1$ **b** $f(x) \equiv 5 + 27x - x^3$ **c** $f(x) \equiv (x^2 - 2)(2x - 1)$

4 $$f(x) \equiv x^3 + kx^2 + 3.$$

Given that $(x + 1)$ is a factor of f(x),

a find the value of the constant k,

b find the set of values of x for which f(x) is increasing.

5 Find the coordinates of any stationary points on each curve.

a $y = x^2 + 2x$ **b** $y = 5x^2 - 4x + 1$ **c** $y = x^3 - 3x + 4$

d $y = 4x^3 + 3x^2 + 2$ **e** $y = 2x + 3 + \frac{8}{x}$ **f** $y = x^3 - 9x^2 - 21x + 11$

g $y = \frac{1}{x} - 4x^2$ **h** $y = 2x^{\frac{3}{2}} - 6x$ **i** $y = 9x^{\frac{2}{3}} - 2x + 5$

6 Find the coordinates of any stationary points on each curve. By evaluating $\frac{d^2y}{dx^2}$ at each stationary point, determine whether it is a maximum or minimum point.

a $y = 5 + 4x - x^2$ **b** $y = x^3 - 3x$ **c** $y = x^3 + 9x^2 - 8$

d $y = x^3 - 6x^2 - 36x + 15$ **e** $y = x^4 - 8x^2 - 2$ **f** $y = 9x + \frac{4}{x}$

g $y = x - 6x^{\frac{1}{2}}$ **h** $y = 3 - 8x + 7x^2 - 2x^3$ **i** $y = \frac{x^4 + 16}{2x^2}$

7 Find the coordinates of any stationary points on each curve and determine whether each stationary point is a maximum, minimum or point of inflexion.

a $y = x^2 - x^3$ **b** $y = x^3 + 3x^2 + 3x$ **c** $y = x^4 - 2$

d $y = 4 - 12x + 6x^2 - x^3$ **e** $y = x^2 + \frac{16}{x}$ **f** $y = x^4 + 4x^3 - 1$

8 Sketch each of the following curves showing the coordinates of any turning points.

a $y = x^3 + 3x^2$ **b** $y = x + \frac{1}{x}$ **c** $y = x^3 - 3x^2 + 3x - 1$

d $y = 3x - 4x^{\frac{1}{2}}$ **e** $y = x^3 + 4x^2 - 3x - 5$ **f** $y = (x^2 - 2)(x^2 - 6)$