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⬡ **DIFFERENTIATION**   *Answers*
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1   **a**   volume $= 2x^2h = 4000$

$\therefore\ h = \dfrac{2000}{x^2}$

b $A\ = 2x^2 + 2(2xh) + 2(xh)$

$= 2x^2 + 6xh$

$= 2x^2 + (6x \times \dfrac{2000}{x^2})$

$= 2x^2 + \dfrac{12\,000}{x}$

c $\dfrac{\mathrm{d}A}{\mathrm{d}x} = 4x - 12\,000x^{-2}$

SP: $4x - 12\,000x^{-2} = 0$

$x^3 = 3000$

$x = \sqrt[3]{3000} = 14.4$ (3sf)

d   min $A = 1250$ (3sf)

e $\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2} = 4 + 24\,000x^{-3}$

when $x = \sqrt[3]{3000}$, $\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2} = 12$

$\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2} > 0\ \therefore$ minimum

2   **a**   S.A. $= 2\pi r^2 + 2\pi rh = 30\,000$

$\therefore\ \pi rh = 15\,000 - \pi r^2$

$h = \dfrac{15\,000}{\pi r} - r$

$V\ = \pi r^2 h$

$= \pi r^2 (\dfrac{15\,000}{\pi r} - r)$

$= 15\,000r - \pi r^3$

b $\dfrac{\mathrm{d}V}{\mathrm{d}r} = 15\,000 - 3\pi r^2$

SP: $15\,000 - 3\pi r^2 = 0$

$r^2 = \dfrac{5000}{\pi}$

$r = \sqrt{\dfrac{5000}{\pi}}$ [$= 39.9$ (3sf)]

max volume $= 399\,000$ cm^3 (3sf)

$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} = -6\pi r$

when $r = \sqrt{\dfrac{5000}{\pi}}$, $\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} = -752$

$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} < 0\ \therefore$ maximum

3   **a**   S.A. $= 2x^2 + 4xl = k$

$\therefore\ 4xl = k - 2x^2$

$l = \dfrac{k - 2x^2}{4x}$

b $V\ = x^2 l$

$= x^2 \times \dfrac{k - 2x^2}{4x}$

$= \tfrac{1}{4}kx - \tfrac{1}{2}x^3$

$\dfrac{\mathrm{d}V}{\mathrm{d}x} = \tfrac{1}{4}k - \tfrac{3}{2}x^2$

SP: $\tfrac{1}{4}k - \tfrac{3}{2}x^2 = 0$

$x^2 = \tfrac{1}{6}k$

$x = \sqrt{\dfrac{k}{6}}$

$\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2} = -3x$

when $x = \sqrt{\dfrac{k}{6}}$, $\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2} < 0\ \therefore$ maximum

$l = \dfrac{k - \frac{1}{3}k}{4\sqrt{\frac{k}{6}}} = \tfrac{2}{3}k \times \tfrac{1}{4} \times \sqrt{\dfrac{6}{k}}$

$= \dfrac{k}{6} \times \sqrt{\dfrac{6}{k}} = \sqrt{\dfrac{k}{6}}$

$\therefore$ maximum V when $l = x\ \therefore$ prism is a cube