DIFFERENTIATION — *Answers*

1 **a** $\frac{dy}{dx} = 3x^2$, grad = 27 **b** $\frac{dy}{dx} = 4 - 2x$, grad = –2 **c** $\frac{dy}{dx} = 4x - 8$, grad = 4 **d** $\frac{dy}{dx} = -3x^{-2}$, grad = $-\frac{1}{3}$

2 **a** $\frac{dy}{dx} = 6x + 1$
at (1, –1) grad = 7

b $\frac{dy}{dx} = 4x^3 + 6x^2$
at (–2, 0) grad = –8

c $y = 2x^2 - 3x$, $\frac{dy}{dx} = 4x - 3$
at (2, 2) grad = 5

d $\frac{dy}{dx} = 2x + 2x^{-2}$
at (2, 3) grad = $\frac{9}{2}$

e $\frac{dy}{dx} = 2x + 6$
at (–3, –1) grad = 0

f $\frac{dy}{dx} = 4 - 2x^{-3}$
at ($\frac{1}{2}$, 6) grad = –12

3 **a** $f(x) = x^2 + 2x + 1$
$f'(x) = 2x + 2$
$f'(4) = 10$

b $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$
$f'(4) = \frac{1}{4}$

c $f'(x) = 1 + 8x^{-3}$
$f'(4) = \frac{9}{8}$

d $f'(x) = -9x^{\frac{1}{2}}$
$f'(4) = -18$

4 **a** $x(x - 1)(x - 3) = 0$, $x = 0, 1, 3$
$\therefore$ (0, 0), (1, 0) and (3, 0)

b $\frac{dy}{dx} = 3x^2 - 8x + 3$
at (0, 0) grad = 3
at (1, 0) grad = –2
at (3, 0) grad = 6

5 **a** $\frac{dy}{dx} = 4x - 5$

b $4x - 5 = 7$
$x = 3$

6 $\frac{dy}{dx} = 3x^2 - 8$
$\therefore$ $3x^2 - 8 = 4$
$x^2 = 4$
$x = \pm 2$
$\therefore$ (–2, 8) and (2, –8)

7 **a** $\frac{dy}{dx} = 3x^2 + 2x - 4$
grad at $P = -3$

b grad at $Q = -3$
$\therefore$ $3x^2 + 2x - 4 = -3$
$3x^2 + 2x - 1 = 0$
$(3x - 1)(x + 1) = 0$
$x = -1$ (at P) or $\frac{1}{3}$
$\therefore$ $Q\left(\frac{1}{3}, -\frac{5}{27}\right)$

8 **a** $\frac{dy}{dx} = 2x$, grad = 4
$\therefore$ $y - 4 = 4(x - 2)$ [$y = 4x - 4$]

b $\frac{dy}{dx} = 2x + 3$, grad = 1
$\therefore$ $y - 2 = x + 1$ [$y = x + 3$]

c $\frac{dy}{dx} = 4x - 6$, grad = –2
$\therefore$ $y - 4 = -2(x - 1)$ [$y = -2x + 6$]

d $\frac{dy}{dx} = 3x^2 - 8x$, grad = 3
$\therefore$ $y + 7 = 3(x - 3)$ [$y = 3x - 16$]

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9 **a** $\frac{dy}{dx} = -2x$, grad = 6

$\therefore\ y + 6 = 6(x + 3)$

$y + 6 = 6x + 18$

$6x - y + 12 = 0$

b $\frac{dy}{dx} = -2x^{-2}$, grad = $-\frac{1}{2}$

$\therefore\ y - 1 = -\frac{1}{2}(x - 2)$

$2y - 2 = -x + 2$

$x + 2y - 4 = 0$

c $\frac{dy}{dx} = 4x + 5$, grad = 7

$\therefore\ y - 2 = 7(x - \frac{1}{2})$

$2y - 4 = 14x - 7$

$14x - 2y - 3 = 0$

d $\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$, grad = $\frac{1}{4}$

$\therefore\ y + 2 = \frac{1}{4}(x - 4)$

$4y + 8 = x - 4$

$x - 4y - 12 = 0$

10 **a** $\frac{dy}{dx} = 2x$, grad = 2

$\therefore$ grad of normal = $-\frac{1}{2}$

$\therefore\ y + 3 = -\frac{1}{2}(x - 1)$

$2y + 6 = -x + 1$

$x + 2y + 5 = 0$

b $\frac{dy}{dx} = 6x + 7$, grad = –5

$\therefore$ grad of normal = $\frac{1}{5}$

$\therefore\ y - 5 = \frac{1}{5}(x + 2)$

$5y - 25 = x + 2$

$x - 5y + 27 = 0$

c $\frac{dy}{dx} = 3x^2 - 8$, grad = 4

$\therefore$ grad of normal = $-\frac{1}{4}$

$\therefore\ y + 4 = -\frac{1}{4}(x - 2)$

$4y + 16 = -x + 2$

$x + 4y + 14 = 0$

d $\frac{dy}{dx} = 1 + 6x^{-2}$, grad = $\frac{5}{3}$

$\therefore$ grad of normal = $-\frac{3}{5}$

$\therefore\ y - 1 = -\frac{3}{5}(x - 3)$

$5y - 5 = -3x + 9$

$3x + 5y - 14 = 0$

11 **a** $x = 2\ \therefore\ y = 4$

$\frac{dy}{dx} = 6x - 5$, grad = 7

$\therefore\ y - 4 = 7(x - 2)$

$y = 7x - 10$

b $x = -3\ \therefore\ y = 6$

$\frac{dy}{dx} = 3x^2 + 10x$, grad = –3

$\therefore$ grad of normal = $\frac{1}{3}$

$\therefore\ y - 6 = \frac{1}{3}(x + 3)$

$y = \frac{1}{3}x + 7$

12 **a** $\frac{dy}{dx} = 3x^2 + 6x - 16$, grad = 8

$\therefore\ y + 10 = 8(x - 2)$ $[y = 8x - 26]$

b $3x^2 + 6x - 16 = 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = 2$ (at P) or -4

$\therefore\ Q\ (-4, 50)$

13 **a** $\frac{dy}{dx} = 2x - 3$, grad = 1

$\therefore$ grad of normal = –1

$\therefore\ y - 2 = -(x - 2)$ $[y = 4 - x]$

b $x^2 - 3x + 4 = 4 - x$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 2$ (at A) or 0

$\therefore\ B\ (0, 4)$

14 **a** $f'(x) = 3x^2 + 8x$

b $x = -3\ \therefore\ y = -9$

grad = 3

$\therefore\ y + 9 = 3(x + 3)$

$y = 3x$ which passes through (0, 0)

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PMT

15 **a** $y = 0 \Rightarrow 6 + x - x^2 = 0$

$(2 + x)(3 - x) = 0$

$x = -2, 3$

+ve x-axis $\therefore P\ (3, 0)$

$x = 0 \Rightarrow y = 6 \therefore Q\ (0, 6)$

b $\frac{dy}{dx} = 1 - 2x$

grad at $P = -5$

$y = -5(x - 3)$ $[y = 15 - 5x]$

c grad at $Q = 1$

tangent at Q: $y = x + 6$

$\therefore 15 - 5x = x + 6$

$x = \frac{3}{2}$

$\therefore (\frac{3}{2}, \frac{15}{2})$

16 **a** grad of $l = -3$

for curve, $\frac{dy}{dx} = 2x - 5$

$\therefore$ at A, $2x - 5 = -3$

$x = 1$

$\therefore A\ (1, -1)$

b $y + 1 = -3(x - 1)$

$y = -3x + 2$

17 grad of normal = 2

$\therefore$ grad of curve $= -\frac{1}{2}$

for curve, $\frac{dy}{dx} = -32x^{-3}$

$\therefore -\frac{32}{x^3} = -\frac{1}{2}$

$x^3 = 64$

$x = 4 \therefore (4, 1)$

sub. $1 = 8 + k$

$k = -7$

18 **a** $\frac{ds}{dt} = 3 + 10t$

$t = 0.6 \Rightarrow \frac{ds}{dt} =$ 9 metres per second

b $54 = 3t + 5t^2$

$5t^2 + 3t - 54 = 0$

$(5t + 18)(t - 3) = 0$

$t > 0 \therefore t = 3$

$\therefore \frac{ds}{dt} =$ 33 metres per second

19 **a** $\frac{dh}{dt} = \frac{1}{3}kt^{-\frac{2}{3}}$

when $t = 1$, $\frac{dh}{dt} = 3$

$\therefore \frac{1}{3}k = 3$

$k = 9$

b $\frac{dh}{dt} = 3 \times 8^{-\frac{2}{3}} = 0.75$ cm per second

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