DIFFERENTIATION

1 Find the gradient at the point with x-coordinate 3 on each of the following curves.

a $y = x^3$ **b** $y = 4x - x^2$ **c** $y = 2x^2 - 8x + 3$ **d** $y = \frac{3}{x} + 2$

2 Find the gradient of each curve at the given point.

a $y = 3x^2 + x - 5$ $(1, -1)$ **b** $y = x^4 + 2x^3$ $(-2, 0)$

c $y = x(2x - 3)$ $(2, 2)$ **d** $y = x^2 - 2x^{-1}$ $(2, 3)$

e $y = x^2 + 6x + 8$ $(-3, -1)$ **f** $y = 4x + x^{-2}$ $(\frac{1}{2}, 6)$

3 Evaluate $f'(4)$ when

a $f(x) = (x + 1)^2$ **b** $f(x) = x^{\frac{1}{2}}$ **c** $f(x) = x - 4x^{-2}$ **d** $f(x) = 5 - 6x^{\frac{3}{2}}$

4 The curve with equation $y = x^3 - 4x^2 + 3x$ crosses the x-axis at the points A, B and C.

a Find the coordinates of the points A, B and C.

b Find the gradient of the curve at each of the points A, B and C.

5 For the curve with equation $y = 2x^2 - 5x + 1$,

a find $\frac{dy}{dx}$,

b find the value of x for which $\frac{dy}{dx} = 7$.

6 Find the coordinates of the points on the curve with the equation $y = x^3 - 8x$ at which the gradient of the curve is 4.

7 A curve has the equation $y = x^3 + x^2 - 4x + 1$.

a Find the gradient of the curve at the point P $(-1, 5)$.

Given that the gradient at the point Q on the curve is the same as the gradient at the point P,

b find, as exact fractions, the coordinates of the point Q.

8 Find an equation of the tangent to each curve at the given point.

a $y = x^2$ $(2, 4)$ **b** $y = x^2 + 3x + 4$ $(-1, 2)$

c $y = 2x^2 - 6x + 8$ $(1, 4)$ **d** $y = x^3 - 4x^2 + 2$ $(3, -7)$

9 Find an equation of the tangent to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = 3 - x^2$ $(-3, -6)$ **b** $y = \frac{2}{x}$ $(2, 1)$

c $y = 2x^2 + 5x - 1$ $(\frac{1}{2}, 2)$ **d** $y = x - 3\sqrt{x}$ $(4, -2)$

10 Find an equation of the normal to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = x^2 - 4$ $(1, -3)$ **b** $y = 3x^2 + 7x + 7$ $(-2, 5)$

c $y = x^3 - 8x + 4$ $(2, -4)$ **d** $y = x - \frac{6}{x}$ $(3, 1)$

11 Find, in the form $y = mx + c$, an equation of

a the tangent to the curve $y = 3x^2 - 5x + 2$ at the point on the curve with x-coordinate 2,

b the normal to the curve $y = x^3 + 5x^2 - 12$ at the point on the curve with x-coordinate -3.

12 A curve has the equation $y = x^3 + 3x^2 - 16x + 2$.

a Find an equation of the tangent to the curve at the point P (2, −10).

The tangent to the curve at the point Q is parallel to the tangent at the point P.

b Find the coordinates of the point Q.

13 A curve has the equation $y = x^2 - 3x + 4$.

a Find an equation of the normal to the curve at the point A (2, 2).

The normal to the curve at A intersects the curve again at the point B.

b Find the coordinates of the point B.

14 $$\text{f}(x) \equiv x^3 + 4x^2 - 18.$$

a Find $\text{f}'(x)$.

b Show that the tangent to the curve $y = \text{f}(x)$ at the point on the curve with x-coordinate -3 passes through the origin.

15 The curve C has the equation $y = 6 + x - x^2$.

a Find the coordinates of the point P, where C crosses the positive x-axis, and the point Q, where C crosses the y-axis.

b Find an equation of the tangent to C at P.

c Find the coordinates of the point where the tangent to C at P meets the tangent to C at Q.

16 The straight line l is a tangent to the curve $y = x^2 - 5x + 3$ at the point A on the curve.

Given that l is parallel to the line $3x + y = 0$,

a find the coordinates of the point A,

b find the equation of the line l in the form $y = mx + c$.

17 The line with equation $y = 2x + k$ is a normal to the curve with equation $y = \frac{16}{x^2}$.

Find the value of the constant k.

18 A ball is thrown vertically downwards from the top of a cliff. The distance, s metres, of the ball from the top of the cliff after t seconds is given by $s = 3t + 5t^2$.

Find the rate at which the distance the ball has travelled is increasing when

a $t = 0.6$,

b $s = 54$.

19 Water is poured into a vase such that the depth, h cm, of the water in the vase after t seconds is given by $h = kt^{\frac{1}{3}}$, where k is a constant. Given that when $t = 1$, the depth of the water in the vase is increasing at the rate of 3 cm per second,

a find the value of k,

b find the rate at which h is increasing when $t = 8$.