1. Find the coordinates of the points on the curve $y = \frac{1}{3}x^3 + \frac{9}{x}$ at which the tangent is parallel to the line $y = 8x + 3$.

[10]

2. Find $\frac{dy}{dx}$ in each of the following cases:

i. $y = \frac{(3x)^2 \times x^4}{x}$,

[3]

ii. $y = \sqrt[3]{x}$,

[3]

iii. $y = \frac{1}{2x^3}$.

[2]

3. It is given that $f(x) = \frac{6}{x^2} + 2x$.

i. Find $f'(x)$.

[3]

ii. Find $f''(x)$.

[2]

4. Given that $y = 6x^3 + \frac{4}{\sqrt{x}} + 5x$, find

(i) $\frac{dy}{dx}$,

[4]

(ii) $\frac{d^2y}{dx^2}$.

[2]

5. Given that $f(x) = 6x^3 - 5x$. Find

(a) $f'(x)$, [2]

(b) $f''(2)$. [2]

6. **In this question you must show detailed reasoning.**

Find the gradient of the curve $y = 3\cos 2x$ at the point where $x = \frac{1}{8}\pi$. [4]

7. It is given that $f(x) = (3 + x^2)(\sqrt{x} - 7x)$. Find $f'(x)$. [5]

8. **In this question you must show detailed reasoning.**

Find the values of x for which the gradient of the curve $y = \frac{2}{3}x^3 + \frac{5}{2}x^2 - 3x + 7$ is positive. Give your answer in set notation.

[5]

9. i. Solve the equation $x^2 - 6x - 2 = 0$, giving your answers in simplified surd form.

[3]

ii. Find the gradient of the curve $y = x^2 - 6x - 2$ at the point where $x = -5$.

[2]

10. a. Given that $f(x) = (x^2 + 3)(5 - x)$, find $f'(x)$.

[4]

b. Find the gradient of the curve $y = x^{-\frac{1}{3}}$ at the point where $x = -8$.

[4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			$\frac{dy}{dx} = x^2 - 9x^{-2}$	B1	x^2 from differentiating first term	
				M1	kx^{-2}	
				A1	$-9x^{-2}$ (no + c)	
			Gradient of line = 8	B1		
			$x^2 - 9x^{-2} = 8$	M1	Equate their $\frac{dy}{dx}$ to 8 (or their gradient of line, if clear)	**Note:** If equated to +/−1/8 then M0 but the next M1 and its dependencies are available
			$x^4 - 8x^2 - 9 = 0$ $k^2 - 8k - 9 = 0$	*M1	Use a correct substitution to obtain a 3 term quadratic or factorise into 2 brackets each containing x^2	If no substitution stated and treated as a quadratic (e.g. quadratic formula), no more marks until square rooting seen
			$(k - 9)(k + 1) = 0$	DM1	Correct method to solve 3 term quadratic – **dependent on previous M1**	**SC:** If spotted after first five marks– (3, 12) **B1**
			$k = 9$ (don't need $k = -1$)	A1	No extras	(−3, −12) **B1** Justifies exactly two solutions **B3**
			$x = 3, -3$	DM1	Attempt to find x by square rooting – accept one value	
					No extras	If curve equated to line and before differentiating:
			$y = 12, -12$	A1	<u>Examiner's Comments</u> Many candidates realised what needed to be done in this unstructured question and a large proportion secured the first five marks by correctly differentiating	First four marks **B1 M1 A1 B1** available as main scheme Then **M0** for equating as this not been explicitly done Allow the **M1** for the substitution **DM1** for quadratic as main scheme (dependent on a correct substitution)

					the equation of the curve and equating this to 8, the gradient of the line. A relatively common error at this stage was to equate to the negative reciprocal of the gradient, showing confusion regarding parallel and perpendicular gradients. The resulting disguised quadratic proved far more difficult than usual as many candidates did not recognise this out of context, as it is more usually seen as a question in its own right. Of the candidates who did realise the need to make a substitution, many did not multiply by x^2 and incorrectly substituted y for x^2 and y^2 for x^4; they secured no more marks. Those who proceeded correctly usually factorised the simple resulting quadratic and remembered to take the square root to find x, although it was quite common to omit the –3. Thereafter, the vast majority of successful candidates found the corresponding value(s) of y correctly, although a number erroneously substituted into the line rather than the curve. This question proved appropriately discriminating with less than a quarter of candidates scoring full marks.	A0 for the 9 (as follows wrong working) DM1 for square rooting (dependent on a correct substitution) A0 for the co-ordinates (as follows wrong working). Max mark 7/10
			Total	10		
2		i	$y = 9x^5$	M1	Obtain kx^5	If individual terms are differentiated then M0A0B0
		i		A1	Correct expression for y ($9x^5$) Follow through from their single kx^n, $n \neq 0$. Must be simplified.	
		i	$\frac{dy}{dx} = 45x^4$	B1 ft	Examiner's Comments Although a large number of candidates secured all three marks for this question, a lot of errors were seen in the initial stages. Most commonly, candidates forgot to square the 3 or divided both terms by x before simplifying. Candidates who obtained a single term	$\frac{3x^2 + x^4}{x}$ is not a misread M0A0B0

					gained a follow-through mark for correct differentiation, but those who differentiated “term by term” received no credit.	Gradients and Differentiation of Standard Functions
		ii	$y = x^{\frac{1}{3}}$ $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$	B1 B1 B1	$\sqrt[3]{x} = x^{\frac{1}{3}}$ $kx^{-\frac{2}{3}}$ $\frac{1}{3}x^{-\frac{2}{3}}$. Allow $0.\dot{3}$ (not finite) Examiner's Comments Most candidates realised that $\sqrt[3]{x}$ s the same as $x^{\frac{1}{3}}$ and the large majority went on to differentiate correctly, although the resulting negative power caused issues for some candidates.	**SC** $\sqrt[3]{x} = x^{-\frac{1}{3}}$ differentiated to $-\frac{1}{3}x^{-\frac{4}{3}}$ **B1**
		iii	$y = \frac{1}{2}x^{-3}$ $\frac{dy}{dx} = -\frac{3}{2}x^{-4}$	M1 A1	kx^{-4} seen Examiner's Comments Whereas most candidates were able to get the power of x correct, rewriting the question as $y = 2x^3$ instead of $y = \frac{1}{2}x^{-3}$ was extremely common and as a result the modal mark for this question was 1 out of 2, achieved by almost half of candidates.	
			Total	8		
3		i	$f(x) = 6x^{2} + 2x$ $f'(x) = -12x^{-3} + 2$	M1	kx^{-3} obtained by differentiation	
		i		A1	$-12x^{-3}$	**ISW** incorrect simplification after correct expression

		i		B1	2x correctly differentiated to 2 **Examiner's Comments** This was very well done, with over 90% of candidates securing all three marks despite the added difficulty of negative powers of x. Even candidates whose overall total was very low recognised and performed the routine of differentiation efficiently. Where errors did occur, these were usually in converting the original expression.	Gradients and Differentiation of Standard Functions
		ii	$f''(x) = 36x^{-4}$	M1	Attempt to differentiate **their (i)** i.e. at least one term "correct"	Allow constant differentiated to zero
		ii		A1	Fully correct **cao** No follow through for **A** mark **Examiner's Comments** Again, this was very well done, with almost all candidates recognising the notation and differentiating again, usually successfully.	**ISW** incorrect simplification after correct expression
			Total	5		
4		i	$y = 6x^3 + 4x^{-\frac{1}{2}} + 5x$	B1	$\frac{4}{\sqrt{x}} = 4x^{-\frac{1}{2}}$ soi	
		i	$\frac{dy}{dx} = 18x^2 - 2x^{-\frac{3}{2}} + 5$	M1	Attempt to differentiate, any term correct	
		i		A1	Two correct terms	
		i		A1	Fully correct, no "+c" **Examiner's Comments**	

					This differentiation was extremely well done, with around four in five candidates securing all the available marks; the ability to recognise and deal with a fractional negative term was much better than in some previous sessions. This term remained the main cause of error, although some errors were made with the first term. The inclusion of a constant when differentiating is now very rare indeed.	Gradients and Differentiation of Standard Functions
		ii ii	$\frac{d^2y}{dx^2} = 36x + 3x^{-\frac{5}{2}}$	M1 A1	Attempt to differentiate their $\frac{dy}{dx}$ cao www in either part Examiner's Comments The need to differentiate again was apparent to most candidates, and again the standard of dealing with the fractional negative term was very high. Some candidates made arithmetical errors here and a few failed to simplify $\frac{6}{2}$.	Any term still involving x correct — follow through from their expression for the **M** mark only
			Total	6		
5		a	$18x^2$ … -5	B1 (AO1.1) B1 (AO1.1) [2]		
		b	$f''(x) = 36x$	M1 (AO1.1) A1FT (AO1.1)	FT their (a)	

			$f''(2) = 72$	[2]	FT their **(b)**	Gradients and Differentiation of Standard Functions
			Total	4		
6			DR $\frac{dy}{dx} = -6\sin 2x$ Substitute $x = \frac{1}{8}\pi$ n attempt at first derivative Obtain $-3\sqrt{2}$	M1(AO1.1) A1(AO1.1) M1(AO1.1) A1(AO1.1) [4]	For $k \sin 2x$ For completely correct derivative oe, e.g. $-\frac{6}{\sqrt{2}}$	
			Total	4		
7			seen or implied $3x^{\frac{1}{2}} - 21x + x^{\frac{5}{2}} - 7x^3$	B1 M1 A1	Attempts to expand brackets with 3/4 terms soi Alternative using product rule: **B1** as main scheme **M1*** Clear attempt at $uv' + vu'$	

			$\frac{3}{2}x^{-\frac{1}{2}}-21+\frac{5}{2}x^{\frac{3}{2}}-21x^2$	M1 A1 [5]	<table><tr><td>Correct expression for *f*(*x*) in index form Attempt to differentiate their expression with at least one non-zero term correct Correct expression for *f'*(*x*) **cao ISW** any attempts to put back into root form.</td><td>**A1** All terms fully correct **M1*dep** Attempt to expand brackets with at least two terms simplified correctly **A1** Correct expression for *f'*(*x*)</td></tr><tr><td></td><td></td></tr></table> **Examiner's Comments** This question tested both index notation and simple differentiation, with errors more common in the former process. Although almost all recognised that $\sqrt{x}=x^{\frac{1}{2}}$, a significant number incorrectly processed $x^2\times\sqrt{x}$ as x or $x^{\frac{3}{2}}$. The method mark was then still available for differentiating their expression, and was almost always earned. A small number of candidates used the product rule, which created a lot more work but was generally efficiently applied.	Gradients and Differentiation of Standard Functions

			Total	5	Gradients and Differentiation of Standard Functions
8			DR $\frac{dy}{dx} = 2x^2 + 5x - 3$ $2x^2 + 5x - 3 > 0 \Rightarrow (2x - 1)(x + 3) > 0$ $x < -3$ or $x > \frac{1}{2}$ $\{x : x < -3\} \cup \{x : x > \frac{1}{2}\}$	M1 (AO 1.1a) A1 (AO 1.1) M1 (AO 1.1) M1 (AO 1.1) A1 (AO 2.5) [5]	Attempt to differentiate (all powers reduced by 1) Correct differentiation of all terms Attempt to find critical values by any appropriate method (e.g. factorising, completing the square, quadratic formula) Choose 'outside region' for their critical values
			Total	5	

						Gradients and Differentiation of Standard Functions
9		i	$\dfrac{6 \pm \sqrt{(-6)^2 - 4\times1\times-2}}{2\times1}$	M1	Valid attempt to use quadratic formula	**No marks for attempting to factorise**
		i	$= \dfrac{6 \pm \sqrt{44}}{2}$	A1		
		i	$= 3 \pm \sqrt{11}$	A1	Both roots correct and simplified	
			OR:			
		i	$(x - 3)^2 - 9 - 2 = 0$ $x - 3 = \pm\sqrt{11}$	M1 A1	Correct method to complete square Rearranged to correct form **cao**	Must get to $(x - 3)$ and ± stage for the M mark, constants combined correctly gets A1
		i	$x = 3 \pm \sqrt{11}$	A1	<u>Examiner's Comments</u> Almost all candidates recognised from the wording of the question that factorisation was not appropriate and most opted to use the quadratic formula. Most were successful in the initial substitution but a significant number failed to deal accurately with the negative value of *c*. Many had difficulty simplifying the resulting surd. Some only divided the first term by 2; others erroneously divided √44 by 2 to get √22. Most of the candidates who attempted to complete the square were successful, although a number failed to find two roots. Overall, about 70% of candidates secured full marks.	
9		ii	$\dfrac{dy}{dx} = 2x - 6$	B1		
		ii	$= -16$	B1	www	

					Examiner's Comments This was generally more successful than part (i). Almost all candidates correctly differentiated the expression and most accurately substituted the given value of x to get –16; 85% of candidates gained both marks. The most common error was to use 5 instead of –5.	Gradients and Differentiation of Standard Functions
			Total	5		
10			$\frac{dy}{dx} = -\frac{1}{3}x^{-\frac{4}{3}}$ When $x = -8$ $\frac{dy}{dx} = -\frac{1}{3} \times (-8)^{-\frac{4}{3}}$ $\frac{dy}{dx} = -\frac{1}{3} \times \frac{1}{16} = -\frac{1}{48}$	M1 A1 B1 A1	Attempt to differentiate i.e. $-\frac{1}{3}x^{-\frac{k}{3}}$ soi for positive integer k Fully correct $(-8)^{-\frac{4}{3}} = \frac{1}{16}$ www Must use –8 Final answer Examiner's Comments Although the differentiation required here was more demanding than that of part (a), many candidates were able to secure the first two marks. Evaluation of	$x^{-\frac{1}{3}}$ misread as $x^{\frac{1}{3}}$ earns max 2/4: $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ **M1 A0 MR** $(-8)^{-\frac{2}{3}} = \frac{1}{4}$ **B1** Final answer $\frac{1}{12}$ **A0 MR**

					$(-8)^{-\frac{4}{3}}$ proved very challenging, with many ignoring one or both minus signs, not understanding the index or making calculation errors. Even those who were successful often then made errors in finding the product of two unit fractions.	Gradients and Differentiation of Standard Functions
			Total	4		