1. Find $\frac{dy}{dx}$ when

i. $y = 2x^{-5}$,

[2]

ii. $y = \sqrt[3]{x}$.

[3]

2. The points P (2, 3.6) and Q (2.2, 2.4) lie on the curve $y = f(x)$. Use P and Q to estimate the gradient of the curve at the point where $x = 2$.

[2]

3. A and B are points on the curve $y = 4\sqrt{x}$. Point A has coordinates (9, 12) and point B has x-coordinate 9.5. Find the gradient of the chord AB.

The gradient of AB is an approximation to the gradient of the curve at A. State the x-coordinate of a point C on the curve such that the gradient of AC is a closer approximation.

[3]

4. Prove from first principles that the derivative of x^3 is $3x^2$. [5]

5.

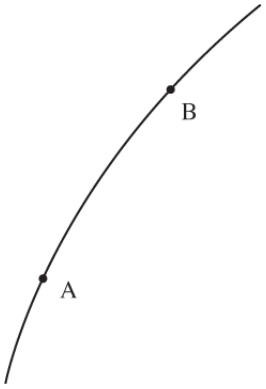

Fig. 3

Fig. 3 shows two points A and B on the curve $y = \log_{10} x$. At A, $x = 0.1$ and at B, $x = 0.2$.

(i) Calculate the gradient of the chord AB. [2]

(ii) The gradient of the chord AB gives an estimate for the gradient of the curve at A. On the copy of Fig. 3 below, mark a point C on the curve such that the gradient of the chord AC would give a better estimate. [1]

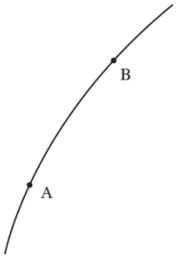

Fig. 3

6. Sanjiv is investigating points on the curve $y = f(x)$. P is the point $(x, f(x))$ and Q is the point $(x + h, f(x + h))$. He has produced the spreadsheet shown in Fig. 2.

	A	B	C	D	E	F	
1	x	f(x)	h	(x+h)	f(x+h)		
2	2	24	0.1	2.1	26.901	29.01	
3	2	24	0.01	2.01	24.281001	28.1001	
4	2	24	0.001	2.001	24.028010001	28.010001	
5	2	24	0.0001	2.0001	24.002800100001	28.00100001	
6							

Fig. 2

Column F shows the value of $\dfrac{f(x+h)-f(x)}{h}$ for different values of h.

(a) Explain what the values in column F represent. [1]

(b) Explain what the spreadsheet suggests about the curve at the point P. [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$-10x^{-6}$ isw	B1	for – 10 for x^{-6} ignore + *c* and *y* =	if **B0B0** then **SC1** for $-5 \times 2x^{-5-1}$ or better soi
		i		B1	**Examiner's Comments** The overwhelming majority of candidates scored full marks on this question. A few candidates omitted the minus sign, and others lost a mark because they calculated the power as – 5 – 1 = – 4. A small number of candidates integrated. Some of these did so incorrectly, obtaining the answer $\frac{-2x^{-6}}{-6}$.	
		ii	$y = x^{\frac{1}{3}}$ soi	B1	condone $y' = x^{\frac{1}{3}}$ f differentiation follows	
		ii	kx^{n-1}	M1	ft their fractional *n* ignore + *c* and *y* =	
		ii	$\frac{1}{3}x^{-\frac{2}{3}}$ isw	A1	**Examiner's Comments** Most candidates identified the correct power, and went on to differentiate correctly. However, a significant minority gave the new power as –⅓, and a few integrated instead of differentiating. In cases where candidates failed to identify ⅓ as the power, – 3 and $^3/_2$ were the most common errors.	allow 0.333 or better
			Total	**5**		

2			$\frac{2.4-3.6}{2.2-2}$ oe	M1		M1 may be embedded eg in equation of straight line
			– 6 cao	A1	Examiner's Comments Many candidates scored full marks here. A few switched the values in the numerator round to obtain + 6 and lost both marks. A small minority found the reciprocal of the gradient, which didn't score, and a tiny minority wrote down the correct calculation, but obtained an incorrect answer. Some candidates needlessly went on to obtain the equation of the chord, and then differentiated it just to convince themselves that the gradient really was – 6, and a few went straight to finding the equation of the chord, and then left the answer embedded, which cost the accuracy mark.	B2 if unsupported ignore subsequent work irrelevant to finding the gradient
			Total	**2**		
3			$\text{gradient} = \frac{4\sqrt{9.5}-12}{9.5-9}$	M1		$4\sqrt{38}-24$ $4\sqrt{38}-24$
			0.6577 to 0.66	A1	or 0.657656…isw	
			$9 < x_C < 9.5$	B1	Examiner's Comments This was done very well. Some candidates lost the second mark through premature rounding or simply giving the answer as 0.6. Only a few calculated the reciprocal of the gradient (which didn't score) and nearly all gave an appropriate value for x_c. A few candidates differentiated and substituted values in the derivative.	allow $8.53 \leq x_c < 9$
			Total	**3**		

4			Gradient of chord $= \frac{(x+h)^3 - x^3}{h}$ oe $= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$ Derivative is limit of gradient of chord as $h \to 0$ $3x^2 + 3xh + h^2 \to 3x^2$ **AG**	M1(AO 2.1) M1(AO 1.1a) A1(AO 1.1) E1(AO 2.4) B1(AO 2.2a) [5]	Expansion of cubic attempted, with at least some terms correct Completely correct expansion	Not just $x^3 + h^3$
			Total	5		
5	i		$\frac{\log_{10} 0.2 - \log_{10} 0.1}{0.2 - 0.1}$ or eg $\frac{-0.7 - -1}{0.2 - 0.1}$ seen 3.01 to 3.0103 isw or 10 $\log_{10} 2$ isw oe	M1 A1 [2]	**NB** $\frac{\log_{10} 2}{0.1}$ or $\frac{0.3}{0.1}$ allow –0.69 to –0.7 for $\log_{10} 2$ in gradient formula for **M1** **Examiner's Comments** Most knew what to do, but many slipped up by making a sign error in	condone omission of base 10; **B2** for 3.01…unsupported

					the numerator or by working with a rounded or truncated value of $\log_{10}$ 0.2, thus losing the accuracy mark.	
		ii	one point C marked on curve between A and B or before A	B1 [1]	<table><tr><td></td><td>condone omission of label of C</td></tr></table> **Examiner's Comments** Nearly all candidates correctly identified a suitable point on the curve. A few guessed wrongly and placed C to the right of B, and a very small number placed C off the curve altogether.	
			Total	**3**		
6		a	The formula gives the gradient of the chord PQ	B1 (AO 2.4) [1]	<table><tr><td></td><td></td></tr></table>	
		b	Column F suggests that the limit as $h \to 0$ is 28, and this is the gradient of (the tangent to) the curve at P	B1 (AO 2.4) [1]	<table><tr><td>Comment must include reference to gradient and the idea of a limit</td><td>Not essential to mention that the limit is f′(2)</td></tr></table>	
			Total	**2**		