1. Differentiate $2x^3 + 9x^2 - 24x$. Hence find the set of values of x for which the function $f(x) = 2x^3 + 9x^2 - 24x$ is increasing.

[4]

2.

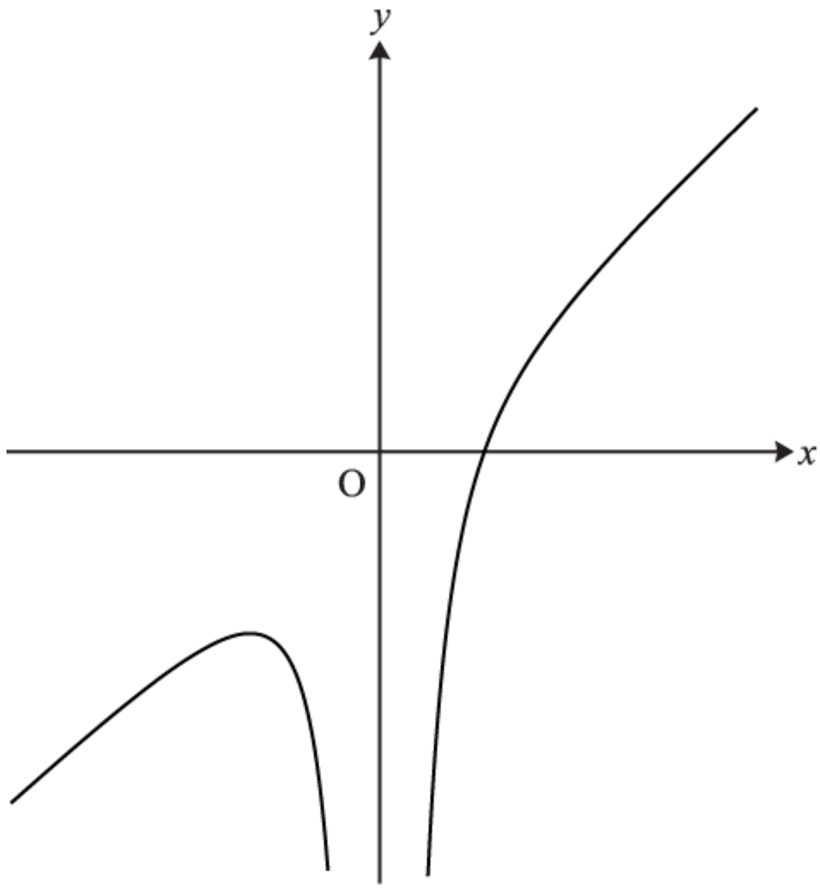

Fig. 11

Fig. 11 shows a sketch of the curve with equation $y = x - \frac{4}{x^2}$.

i. Find $\frac{dy}{dx}$ and show that $\frac{d^2y}{dx^2} = -\frac{24}{x^4}$.

[3]

ii. Hence find the coordinates of the stationary point on the curve. Verify that the stationary point is a maximum.

[5]

iii. Find the equation of the normal to the curve when $x = -1$. Give your answer in the form $ax + by + c = 0$.

[5]

3. Use calculus to find the set of values of x for which $x^3 - 6x$ is an increasing function. [5]

4. i. Calculate the gradient of the chord of the curve $y = x^2 - 2x$ joining the points at which the values of x are 5 and 5.1. [2]

ii. Given that $f(x) = x^2 - 2x$, find and simplify $\dfrac{f(5+h) - f(5)}{h}$. [4]

iii. Use your result in part (ii) to find the gradient of the curve $y = x^2 - 2x$ at the point where $x = 5$, showing your reasoning. [2]

iv. Find the equation of the tangent to the curve $y = x^2 - 2x$ at the point where $x = 5$. Find the area of the triangle formed by this tangent and the coordinate axes. [5]

5. Prove that $f(x) = x^3 - 3x^2 + 6x + 5$ is an increasing function for all real values of x. [5]

6. Find the equation of the normal to the curve $y = 2x^3$ at the point on the curve where $x = 2$. Give your answer in the form $ax + by = c$. [5]

7. The standard formulae for the volume V and total surface area A of a solid cylinder of radius r and height h are

$$V = \pi r^2 h \text{ and } A = 2\pi r^2 + 2\pi r h.$$

You are given that $V = 400$.

(i) Show that $A = 2\pi r^2 + \dfrac{800}{r}$. [2]

(ii) Find $\dfrac{dA}{dr}$ and $\dfrac{d^2A}{dr^2}$. [4]

(iii) Hence find the value of r which gives the minimum surface area. Find also the value of the surface area in this case. [4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			$6x^2 + 18x - 24$	B1		
			their $6x^2 + 18x - 24 = 0$ or >0 or $\leqslant 0$	M1		or sketch of $y = 6x^2 + 18x - 24$ with attempt to find x-intercepts
			-4 and $+1$ identified oe	A1		
			$x < -4$ and $x > 1$ cao	A1	or $x \leqslant -4$ and $x \leq 1$ **Examiner's Comments** Most candidates differentiated correctly and identified the correct values of x. The final mark was often lost, either due to a misunderstanding of what had been found — answer given as –4 < x < 1 or poor notation — answer given as –4 > x > 1. Those who used a graphical approach with the derivative generally scored full marks. A few candidates missed the last term out, converted the first plus sign to a minus sign or failed to multiply 2 by 3 correctly, and lost the first mark.	if **B0M0** then **SC2** for fully correct answer
			Total	4		
2		i	$y' = 1 + 8x^{-3}$	M2	M1 for just $8x^{-3}$ or $1 - 8x^{-3}$	
		i	$y'' = -24x^{-4}$ oe	A1	**Examiner's Comments** Most knew what to do here, but $8x^{-3}$ and $1 - 8x^{-3}$ were often seen. Only a few candidates failed to show sufficient detail of their working to earn the third mark following a fully correct $\frac{dy}{dx}$	but not just $\frac{-24}{x^4}$ as AG
		ii	their $y' = 0$ soi	M1		
		ii	$x = -2$	A1	A0 if more than one x-value	$x = -2$ must have been correctly obtained for all marks after first M1

		ii	$y = -3$	A1	A0 if more than one y-value	
		ii	substitution of $x = -1$ in their y'	M1	or considering signs of gradient either side of −2 with negative x-values signs for gradients identified to verify maximum	condone any bracket error
		ii	< 0 or = − 1.5 oe correctly obtained isw	A1	**Examiner's Comments** There were many correct solutions, although even some of the better candidates neglected to find the corresponding value of y, or evaluated the second derivative as + 1.5, and concluded the stationary value must be a local maximum. A few obtained $x = 2$ following correct differentiation, but never bothered to look at the graph to realise that this must be wrong. It was surprising just how many candidates solved $8x^{-3} = 0$ to obtain $x = 2$, without realising that something must have gone wrong.	must follow from M1 A1 A0 M1 or better
		iii	$y = -5$ soi	B1		
		iii	substitution of $x = -1$ in their y'	M1	may be implied by − 7	
		iii	grad normal = ${}^{-1}/_{\text{their} -7}$	M1*	may be implied by e.g. ${}^{1}/_{7}$	
		iii	$y -$ their $(-5) =$ their ${}^{1}/_{7}(x - -1)$	M1dep*	or their $(-5) =$ their ${}^{1}/_{7} \times (-1) + c$	
		iii	$-x + 7y + 34 = 0$ oe	A1	allow e.g. $y - \frac{1}{7}x + \frac{34}{7} = 0$ **Examiner's Comments** This was generally well done. Only a small minority of candidates did not understand how to obtain the gradient of the normal, and many obtained follow through marks, at least. Some candidates slipped up finding the value of y, and a few made sign errors when finishing off.	must see = 0 do not allow e.g. $y = \frac{x}{7} - \frac{34}{7}$
			Total	**13**		
3			$3x^2 - 6$ seen	B1		
			their $y' = 0$ or $y' > 0$ or $y' \geq 0$	M1	must be quadratic with at least one of only two terms correct	

			$\sqrt{2}$ and $-\sqrt{2}$ dentified?	A1	may be implied by use with inequalities or by ± 1.41[4213562] to 3 sf or more	$\lvert x\rvert=\sqrt{2}$ implies **A1** NB **just** $-\sqrt{2}>x>\sqrt{2}$ or $\sqrt{2}<x<-\sqrt{2}$ or $x>\pm\sqrt{2}$ mplies the first **A1** then **A0A0**
			$x<-\sqrt{2}$ or $x\le-\sqrt{2}$ sw	A1	if **A1A0A0**, allow **SC1** forfully correct answer in decimal form to 3 sf or more or **A2** for $\lvert x\rvert>\sqrt{2}$ or $\lvert x\rvert\ge\sqrt{2}$	
			$x>\sqrt{2}$ or $x\ge\sqrt{2}$	A1	**Examiner's Comments** The majority of candidates differentiated successfully and went on to identify $\pm\sqrt{2}$ correctly. A few neglected the negative root, losing an easy mark. Thereafter candidates went astray in a variety of ways. Many candidates used incorrect forms when writing their inequalities. $x>\pm\sqrt{2}$ was seen frequently and many candidates combined their separate inequalities in illegal ways such as $\sqrt{2}<x<-\sqrt{2}$. These candidates were penalised if the correct inequalities were not seen first. Candidates should realise that it is good practise to write the two inequalities separately first, before any attempt is made to combine them. Some candidates decimalised $\pm\sqrt{2}$ were penalised for having a slight inaccuracy in their answer. A few candidates didn't differentiate at all, thereby ignoring the instruction to use calculus and so made no progress.	
			Total	5		
4		i	$\dfrac{(5.1^2-10.2)-(5^2-10)}{5.1-5}$ oe	M1	condone omission of brackets	0 for 8.1 unsupported
		i	8.1	A1	**Examiner's Comments** The majority of candidates gained full marks on this question. A significant minority differentiated and substituted in the midpoint, or the endpoints of the chord and found the mean. Whilst these approaches do achieve the correct numerical answer, they nevertheless went unrewarded.	

		ii	$\dfrac{(5+h)^2-2(5+h)-\text{ their }15}{h}$ oe	M1	condone omission of brackets	
		ii	$25 + 10h + h^2 - 10 - 2h$ oe seen	M1	allow one sign error	
		ii	numerator is $8h + h^2$	A1		
		ii	$8 + h$ isw	A1	**Examiner's Comments** Many candidates clearly didn't understand the notation, and either produced expressions involving x and h, or "expanded brackets" and worked with $5f + fh$. A good number of candidates did understand what this question was about, and successfully substituted to obtain correct expressions. Some made sign errors or slips in arithmetic: $h + 12$ was a common wrong answer, and a few knew what the answer was supposed to be and "back-engineered" their incorrect work accordingly.	
		iii	$h \to 0$	M1	may be embedded; allow eg "tends to 0"	**M0** for differentiation of $x^2 - 2x$ **M0** for following from part (i) **M0** for $h = 0$
		iii	their 8	A1	**FT** their $k + h$ from part (ii) **Examiner's Comments** Only a few candidates used the correct terminology or notation here. Some worked with $h = 0$ and a good number ignored part (ii) and differentiated. Neither approach scored.	
		iv	$y = 8x - 25$ isw	B1	or $y - 15 = 8(x - 5)$ isw or $y = 8x + c$ and $c = -25$ stated isw	
		iv	non-zero numerical value for x-intercept on their straight line found	M1		
		iv	[x =] 3.125 oe	A1	may be embedded in calculation for area	

		iv	$\frac{1}{2} \times$ their non-zero y-intercept × their $\frac{25}{8}$	M1	condone arithmetic slips in finding values of intercepts	or integration and evaluation of their $\int_0^{25/8} (8x-25)\mathrm{d}x$; lower limit must be 0
		iv	$\frac{625}{16}$ or $39\frac{1}{16}$ or 39.0625 isw	A1	accept rounded to 1 dp or better for **A1**; but **A0** if **final** answer negative **Examiner's Comments** Many candidates found the correct equation and went on to achieve full marks. Some didn't read the question carefully and used (5, 15) with (3.125, 0). A small number of candidates found the equation of the normal and were thus only able to access two method marks.	
			Total	13		
5			$3x^2 - 6x + 6$ $6x - 6 = 0$ $x = 1$ gradient has a minimum value of 3 at x =1. This is a minimum value because the gradient function is parabolic and the coefficient of x^2 is positive. As the gradient is always positive the function is always increasing oe	M1(AO2.1) A1(AO1.1) M1(AO2.1) A1(AO1.1) E1(AO2.2a) [5]	Differentiation All correct Differentiates again or completes the square or uses the discriminant *alternatively* gradient is never zero since discriminant is negative / can't solve from completing square, and must therefore be always positiyve since term in x^2 is positive oe NB $3(x-1)^2 + 3$ NB – 36	

			Total	5		
6			$\left[\dfrac{dy}{dx}=\right] kx^2$ soi When $x = 2$, $\left[\dfrac{dy}{dx}=\right]$ 24 $-\dfrac{1}{their\ 24}$ $x = 2$, $y = 16$ $x + 24y = 386$ oe	M1 A1 M1 B1 A1 [5]	$k > 0$ their 24 must come from evaluating their derivative **NB** $y-16=-\frac{1}{24}(x-2)$ coefficients in any exact form eg $\dfrac{1}{24}x+y=\dfrac{193}{12}$ but not rounded or truncated decimals **Examiner's Comments** Most candidates were familiar with this sort of question and obtained the first four marks without difficulty. A few slipped up with the arithmetic, and a similar number found the equation of the tangent. A very small number of candidates integrated or went straight to working with $y = mx + c$	**NB** $6x^2$ **M0** if their 24 from elsewhere eg integration
			Total	5		

7		i	correct rearrangement of 400 = $\pi r^2 h$ seen, where h is not in the denominator substitution seen to obtain given answer $A = 2\pi r^2 + \frac{800}{r}$ not from wrong working	B1 B1 [2]	eg $h = \frac{400}{\pi r^2}, rh = \frac{400}{\pi r}, \pi rh = \frac{400}{r}$ or $2\pi rh = \frac{2 \times 400}{r}$ — allow embedded versions of these if **B0B0 allow SC2** for eg 400 $\pi r^2 h$ used $\frac{800}{r} = \frac{2 \times 400}{r}$ $\left(\text{or } \frac{2V}{r}\right) = \frac{2 \times \pi r^2 h}{r}$ used to obtain $A\ 2\pi r^2 + 2\pi rh$ — must see all the steps if starting from $A = 2\pi r^2 + \frac{800}{r}$ **Examiner's Comments** Most candidates scored full marks here, but poor algebra let some candidates down. A wide variety of solutions were seen, some of which very elaborate.	
		ii	$\left(\frac{dA}{dr} =\right) 4\pi r - \frac{800}{r^2}$ oe $\left(\frac{d^2A}{dr^2} =\right) 4\pi + \frac{1600}{r^3}$ oe	B1 B1 B1 B1	for first term for second term **FT** to give non-zero first term **FT** negative power of r to — A maximum of **B1B0B1B0** is available if 2[nd] term left in terms of h	

				[4]	<table><tr><td>give non-zero second term</td><td></td></tr></table> Examiner's Comments In spite of the correct expression being given in part (i), some candidates worked with an expression involving *h*, which inhibited much further progress. Some candidates worked with 800^{-r} and some disregarded π or treated it as a variable. The majority, however, differentiated successfully to obtain full marks.	
		iii	their $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ seen $r = \sqrt[3]{\frac{200}{\pi}}$ or 3.99… isw $\frac{\mathrm{d}^2 A}{\mathrm{d}r^2} > 0$ justified so minimum oe or check gradient either side of *their* positive *r* A = 300 to 301	M1 A1 B1 A1 [4]	<table><tr><td>**A0** for two or more values eg $r = 0$, 3.99 or ± 3.99 eg $4\pi > 0$ and $\frac{1600}{r^3} > 0$ **NB** 12π or 37.699… to 38 **NB** 300.530027931</td><td>**NB** 3.99294542466 simply stating that second derivative is positive is insufficient ignore units</td></tr></table> Examiner's Comments A sizeable minority of candidates failed to score any marks in this part, beginning with an inequality in the second derivative. A good number of candidates started on the right track by setting the first derivative to zero, but then failed to make progress. Only rarely did	

					candidates successfully find *r* and *A* and then use the second derivative correctly to establish that they had indeed found the minimum surface area.	
			Total	10		