Question	Scheme	Marks	AOs
1(a)	Attempts to find the value of $\frac{dy}{dx}$ at $x = 2$	M1	1.1b
	$\frac{dy}{dx} = 6x \Rightarrow$ gradient of tangent at P is 12	A1	1.1b
		(2)	
(b)	Gradient $PQ = \frac{3(2+h)^2 - 2 - 10}{(2+h)-2}$ oe	B1	1.1b
	$= \frac{3(2+h)^2 - 12}{(2+h)-2} = \frac{12h + 3h^2}{h}$	M1	1.1b
	$= 12 + 3h$	A1	2.1
		(3)	
(c)	Explains that as $h \to 0,\ 12 + 3h \to 12$ and states that the gradient of the chord tends to the gradient of (the tangent to) the curve	B1	2.4
		(1)	
			(6 marks)

Notes

(a)

M1: Attempts to differentiate, allow $3x^2 - 2 \to ...x$ and substitutes $x = 2$ into their answer

A1: cso $\frac{dy}{dx} = 6x \Rightarrow$ gradient of tangent at P is 12

(b)

B1: Correct expression for the gradient of the chord seen or implied.

M1: Attempts $\frac{\delta y}{\delta x}$, condoning slips, and attempts to simplify the numerator. The denominator must be h

A1: cso $12 + 3h$

(c)

B1: Explains that as $h \to 0,\ 12 + 3h \to 12$ and states that the gradient of the chord tends to the gradient of the curve

Question	Scheme	Marks	AOs
2 (a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=...$	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	

Notes:
(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PN is -3 with point $(7,4)$ to find the equation of line PN

So sight of $y-4=-3(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=...$ to score this mark.

A1: Achieves $y=-3x+25$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving their $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates.

A1: $P=\left(\frac{15}{2},\frac{5}{2}\right)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=\left(\frac{15}{2},\frac{5}{2}\right)$ and $(7,4)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: Full and careful work leading to a correct equation. Eg $(x-7)^2+(y-4)^2=\frac{5}{2}$ or its expanded form. Do not accept $(x-7)^2+(y-4)^2=\left(\sqrt{\frac{5}{2}}\right)^2$

(c)

M1: Attempts to find where $y=\frac{1}{3}x+k$ meets *C* using a vector approach

M1: For a full method leading to k . Scored for substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$

A1: $k=\frac{10}{3}$ only

Alternative I

M1: For solving $y=\frac{1}{3}x+k$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and x must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$ oe

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.
Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=\frac{10}{3}$ only

Alternative II

M1: For solving $y=-3x+25$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$, creating a 3TQ and solving.

M1: For substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ into $y=\frac{1}{3}x+k$ and finding k

A1: $k=\frac{10}{3}$ only

Question	Scheme	Marks	AOs
3(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \frac{dy}{dx} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5} = 3\times 5^2 - 20\times 5 + 27 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 5)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
		(4)	
(b)	Both *C* and *l* pass through (0, –23) and so *C* meets *l* again on the *y*-axis	B1	2.2a
		(1)	
(c)	$\pm\int\left(x^3 - 10x^2 + 27x - 23 - (2x - 23)\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right]_0^5$ $= \left(\frac{625}{4} - \frac{1250}{3} + \frac{625}{2}\right)(-0)$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative:		
	$\pm\int\left(x^3 - 10x^2 + 27x - 23\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right]_0^5 + \frac{1}{2}\times 5(23 + 13)$ $= -\frac{455}{12} + 90$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
			(9 marks)

Notes

(a)
B1: Correct derivative
M1: Substitutes $x = 5$ into their derivative. This may be implied by their value for $\frac{dy}{dx}$
M1: Fully correct straight line method using (5, – 13) and their $\frac{dy}{dx}$ at $x = 5$
A1: cao. Must see the full equation in the required form.
(b)
B1: Makes a suitable deduction.

Alternative via equating l and C and factorising e.g.

$$x^3 - 10x^2 + 27x - 23 = 2x - 23$$
$$x^3 - 10x^2 + 25x = 0$$
$$x(x^2 - 10x + 25) = 0 \Rightarrow x = 0$$

So they meet on the y-axis

(c)
M1: For an attempt to integrate $x^n \to x^{n+1}$ for ±"$C - l$"
A1ft: Correct integration in any form which may be simplified or unsimplified. (follow through their equation from (a))

If they attempt as 2 separate integrals e.g. $\int (x^3 - 10x^2 + 27x - 23)\,dx - \int (2x - 23)\,dx$ then award this mark for the correct integration of the curve as in the alternative.

If they combine the curve with the line first then the subsequent integration must be correct or a correct ft for their line and allow for ±"$C - l$"

dM1: Fully correct strategy for the area. Award for use of 5 as the limit and condone the omission of the "– 0". **Depends on the first method mark.**
A1: Correct exact value
Alternative:
M1: For an attempt to integrate $x^n \to x^{n+1}$ for $\pm C$
A1: Correct integration for $\pm C$
dM1: Fully correct strategy for the area e.g. correctly attempts the area of the trapezium and subtracts the area enclosed between the curve and the x-axis. Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the trapezium **and** the subtraction.

May see the trapezium area attempted as $\int (2x - 23)\,dx$ in which case the integration and use of the limits needs to be correct or correct follow through for their straight line equation.

Depends on the first method mark.

A1: Correct exact value

Note if they do $l - C$ rather than $C - l$ and the working is otherwise correct allow full marks if their final answer is given as a positive value. E.g. correct work with $l - C$ leading to $-\frac{625}{12}$ and then e.g. hence area is $\frac{625}{12}$ is acceptable for full marks.

If the answer is left as $-\frac{625}{12}$ then score A0

Question	Scheme	Marks	AOs
4(a)	$\frac{d}{dx}(3y^2)=6y\frac{dy}{dx}$ or $\frac{d}{dx}(qxy)=qx\frac{dy}{dx}+qy$	M1	2.1
	$3px^2+qx\frac{dy}{dx}+qy+6y\frac{dy}{dx}=0$	A1	1.1b
	$(qx+6y)\frac{dy}{dx}=-3px^2-qy \Rightarrow \frac{dy}{dx}=...$	dM1	2.1
	$\frac{dy}{dx}=\frac{-3px^2-qy}{qx+6y}$	A1	1.1b
		(4)	
(b)	$p(-1)^3+q(-1)(-4)+3(-4)^2=26$	M1	1.1b
	$19x+26y+123=0 \Rightarrow m=-\frac{19}{26}$	B1	2.2a
	$\frac{-3p(-1)^2-q(-4)}{q(-1)+6(-4)}=\frac{26}{19}$ or $\frac{q(-1)+6(-4)}{3p(-1)^2+q(-4)}=-\frac{19}{26}$	M1	3.1a
	$p-4q=22,\ 57p-102q=624 \Rightarrow p=...,q=...$	dM1	1.1b
	$p=2,\ q=-5$	A1	1.1b
		(5)	
		(9 marks)	

Notes

(a)

M1: For selecting the appropriate method of differentiating:

Allow this mark for either $3y^2 \to \alpha y\frac{dy}{dx}$ or $qxy \to \alpha x\frac{dy}{dx}+\beta y$

A1: Fully correct differentiation. Ignore any spurious $\frac{dy}{dx}=...$

dM1: A valid attempt to make $\frac{dy}{dx}$ the subject with 2 terms only in $\frac{dy}{dx}$ coming from qxy and $3y^2$

Depends on the first method mark.

A1: Fully correct expression

(b)

M1: Uses $x=-1$ and $y=-4$ in the equation of C to obtain an equation in p and q

B1: Deduces the correct gradient of the given normal.

This may be implied by e.g.

$19x+26y+123=0 \Rightarrow y=-\frac{19}{26}x+... \Rightarrow \text{Tangent equation is } y=\frac{26}{19}x+...$

M1: Fully correct strategy to establish an equation connecting p and q using $x=-1$ and $y=-4$ in their $\frac{dy}{dx}$ and the gradient of the normal. E.g. $(a)=-1\div\text{their}-\frac{19}{26}$ or $-1\div(a)=\text{their}-\frac{19}{26}$

dM1: Solves simultaneously to obtain values for p and q.

Depends on both previous method marks.

A1: Correct values

Alternative for (b):

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-3p+4q}{-q-24} \Rightarrow y+4 = \frac{q+24}{4q-3p}(x+1)$$

M1A1

$$\Rightarrow y(4q-3p)+4(4q-3p) = (q+24)x+q+24$$

M1

$$19x+26y+123=0 \Rightarrow q+24=19 \Rightarrow q=-5$$

$$3p-4q=26 \Rightarrow 3p+20=26 \Rightarrow p=2$$

M1A1

M1: Uses (–1, –4) in the tangent gradient and attempts to form normal equation
A1: Correct equation for normal
M1: Multiplies up so that coefficients can be compared
dM1: Full method comparing coefficients to find values for *p* and *q*
A1: Correct values

Question	Scheme	Marks	AOs
5(a)	$y=\text{cosec}^3\theta \Rightarrow \frac{dy}{d\theta}=-3\text{cosec}^2\theta\text{cosec}\,\theta\cot\theta$	B1	1.1b
	$\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$	M1	1.1b
	$\frac{dy}{dx}=\frac{-3\text{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y=8\Rightarrow \text{cosec}^3\theta=8\Rightarrow \sin^3\theta=\frac{1}{8}\Rightarrow \sin\theta=\frac{1}{2}$	M1	3.1a
	$\theta=\frac{\pi}{6}\Rightarrow\frac{dy}{dx}=\frac{-3\text{cosec}^3\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)}{2\cos\left(\frac{2\pi}{6}\right)}=...$ **or** $\sin\theta=\frac{1}{2}\Rightarrow\frac{dy}{dx}=\frac{\frac{-3}{\sin^3\theta}\times\frac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)}=\frac{-3\times8\times\frac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\frac{1}{4}\right)}$	M1	2.1
	$=-24\sqrt{3}$	A1	2.2a
		(3)	
		(6 marks)	

Notes

(a)

B1: Correct expression for $\frac{dy}{d\theta}$ seen or implied in any form e.g. $\frac{-3\cos\theta}{\sin^4\theta}$

M1: Obtains $\frac{dx}{d\theta}=k\cos 2\theta$ or $\alpha\cos^2\theta+\beta\sin^2\theta$ (from product rule on $\sin\theta\cos\theta$)

and attempts $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$

A1: Correct expression in any form.

May see e.g. $\frac{-3\cos\theta}{2\sin^4\theta\cos 2\theta}$, $-\frac{3}{4\sin^4\theta\cos\theta-2\sin^3\theta\tan\theta}$

(b)

M1: Recognises the need to find the value of $\sin\theta$ or θ when y = 8 and uses the y parameter to establish its value. This should be correct work leading to $\sin\theta=\frac{1}{2}$ or e.g. $\theta=\frac{\pi}{6}$ or 30°.

M1: Uses their value of $\sin\theta$ or θ in their $\frac{dy}{dx}$ from part (a) (working in exact form) in an attempt to obtain an exact value for $\frac{dy}{dx}$. May be implied by a correct exact answer.

If no working is shown but an exact answer is given you may need to check that this follows their $\frac{dy}{dx}$.

A1: Deduces the correct gradient

Question	Scheme	Marks	AOs
6a	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)$	M1	1.2
	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)=0\Rightarrow 2\alpha+\frac{1}{2}-\frac{\alpha^2}{4}=0 \quad \Rightarrow \alpha=...$	dM1	1.1b
	$\alpha=-0.243$ (3dp) only	A1	2.3
		(3)	
b	$f'(0)=\frac{1}{2}\cos 0\Rightarrow ...\Rightarrow y=...x+3$	M1	1.1b
	$y=\frac{1}{2}x+3$	A1	1.1b
		(2)	
		(5 marks)	

Notes

(a) **Note on EPEN this is M1A1A1 but we are marking this as M1dM1A1**

Accept to be in terms of α or another variable e.g. x

Note: -0.243 **with no working is 0 marks**

M1: Fully substitutes $\cos x=1-\frac{x^2}{2}$ into the derivative.

dM1: Attempts to multiply out to achieve a 3TQ (= 0) **and** attempts to find a value for α. Condone slips. Allow solving the quadratic via any method (usual rules apply).

If they use a calculator then you may need to check this.

A1: $(\alpha=)-0.243$ only cao Can only be scored provided a correct 3TQ is seen. If both roots found then the other one must be rejected (or a choice made of -0.243 e.g. underlining it or a tick)

Condone $x=-0.243$

(b)

M1: Attempts to find the gradient of the curve when $x=0$ and achieves an equation of the form $y=\text{"}f'(0)\text{"}x+3$.

$x=0$ must be fully substituted in and a value must be found for the gradient. Do not allow this mark if they attempt to use a changed gradient e.g. the gradient of the normal.

Also allow attempts using the small angle approximation:

$$f'(x)\approx 2x+\frac{1}{2}\left(1-\frac{x^2}{2}\right) \text{ when } x=0,\ f'(0)=\text{"}\frac{1}{2}\text{"}\Rightarrow y=\text{"}f'(0)\text{"}x+3$$

A1: $y=\frac{1}{2}x+3$ or equivalent in the form $y=mx+c$ isw Stating just the values $m=0.5,\ c=3$ without the correct equation is A0

Question	Scheme	Marks	AOs
7(a)	$x^3 \to ...x^2$ and $3y^2 \to ...y\frac{dy}{dx}$	M1	1.1b
	$2xy \to 2y + 2x\frac{dy}{dx}$	B1	1.1b
	$3x^2 + 2x\frac{dy}{dx} + 2y + 6y\frac{dy}{dx} = ... \Rightarrow \frac{dy}{dx} = ...$	M1	2.1
	$\frac{dy}{dx} = -\frac{2y+3x^2}{2x+6y}$	A1	1.1b
		(4)	
(b)	$\frac{dy}{dx} = -\frac{2(5)+3(-2)^2}{2(-2)+6(5)}$ or e.g. $3(-2)^2 + 2(-2)\frac{dy}{dx} + 2\times5 + 6\times5\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = ... \left(-\frac{11}{13}\right)$	M1	1.1b
	$y - 5 = "\frac{13}{11}"(x+2)$	dM1	1.1b
	$13x - 11y + 81 = 0$	A1	2.2a
		(3)	
		(7 marks)	

Notes

(a) Allow equivalent notation for the $\frac{dy}{dx}$ e.g. y'

M1: Attempts to differentiate $x^3 \to ...x^2$ **and** $3y^2 \to ...y\frac{dy}{dx}$ where … are constants

B1: Correct application of the product rule on $2xy$: $2xy \to 2x\frac{dy}{dx} + 2y$

Note that some candidates have a spurious $\frac{dy}{dx} = ...$ at the start (as their intention to differentiate) and this can be ignored for the first 2 marks

M1: For a valid attempt to make $\frac{dy}{dx}$ the subject, with exactly 2 different terms in $\frac{dy}{dx}$ coming from $3y^2$ and $2xy$. Look for $(... \pm ...)\frac{dy}{dx} = ... \Rightarrow \frac{dy}{dx} = ...$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\frac{dy}{dx} = ...$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

A1: $\frac{dy}{dx} = -\frac{2y+3x^2}{2x+6y}$ oe e.g. $\frac{dy}{dx} = \frac{-2y-3x^2}{2x+6y}, \frac{2y+3x^2}{-2x-6y}$ Isw once a correct expression is seen.

Note that it is sometimes unclear if the minus sign(s) is/are correctly placed and you may have to use your judgement. Evidence may be available in part (b) to help you decide if they have the correct expression.

(b)

M1: Substitutes x = -2 and y = 5 into $\frac{dy}{dx} = "-\frac{2y+3x^2}{2x+6y}"$

They must have x's and y's in their $\frac{dy}{dx}$ but condone slips in substitution provided the intention is clear.

As a minimum look for at least one x and at least one y substituted correctly.

Note that this mark may be implied by their value for $\frac{dy}{dx}$ and may be implied if, for example, they find the negative reciprocal or the reciprocal of $"-\frac{2y+3x^2}{2x+6y}"$ and then substitute x = -2 and y = 5

Alternatively, substitutes x = -2 and y = 5 into their attempt to differentiate and then rearranges to find a value or numerical expression for $\frac{dy}{dx}$

dM1: Attempts to find the equation of the normal using their gradient of the tangent and x = -2 and y = 5 correctly placed. Score for an expression of the form $(y-5) = "\frac{13}{11}"(x+2)$ or if they use $y = mx + c$ they must proceed as far as c =… Must be using the **negative reciprocal** of the tangent gradient.

Note that $y - 5 = \frac{2x+6y}{2y+3x^2}(x+2)$ is not a correct method unless the gradient is evaluated first *before* expanding.

A1: $13x - 11y + 81 = 0$ or any integer multiple of this equation including the "= 0", not just a, b, c given.

e.g., $26x - 22y + 162 = 0$ is likely if they don't cancel down their gradient.