Question	Scheme	Marks	AOs
1 (a) (i)	Uses $\frac{dy}{dx} = -3$ at $x = 2 \Rightarrow 12a + 60 - 39 = -3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a + 60 - 39 = -3 \Rightarrow 12a = -24 \Rightarrow a = -2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $10 = 8a + 60 - 78 + b$	M1	3.1a
	Subs $a = -2$ into $10 = 8a + 60 - 78 + b \Rightarrow b = 44$	A1	1.1b
		(4)	
(b)	$f(x) = -2x^3 + 15x^2 - 39x + 44 \Rightarrow f'(x) = -6x^2 + 30x - 39$	B1	1.1b
	Attempts to show that $-6x^2 + 30x - 39$ has no roots Eg. calculates $b^2 - 4ac = 30^2 - 4 \times -6 \times -39 = -36$	M1	3.1a
	States that as $f'(x) \neq 0 \Rightarrow$ hence $f(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3 + 15x^2 - 39x + 44 \equiv (x-4)(-2x^2 + 7x - 11)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	
			(11 marks)

Notes

(a)(i)

M1: Attempts to use $\frac{dy}{dx} = -3$ at $x = 2$ to form an equation in a. Condone slips but expect to see two of the powers reduced correctly

A1*: Correct differentiation with one correct intermediate step before $a = -2$

(a)(ii)

M1: Attempts to use the fact that $(2,10)$ lies on C by setting up an equation in a and b with $a = -2$ leading to $b = ...$

A1: $b = 44$

(b)

B1: $f'(x) = -6x^2 + 30x - 39$ oe

M1: Correct attempt to show that "$-6x^2 + 30x - 39$" has no roots.
This could involve an attempt at

- finding the numerical value of $b^2 - 4ac$
- finding the roots of $-6x^2 + 30x - 39$ using the quadratic formula (or their calculator)
- completing the square for $-6x^2 + 30x - 39$

A1*: A fully correct method with reason and conclusion. Eg as $b^2 - 4ac = -36 < 0, f'(x) \neq 0$ meaning that no stationary points exist

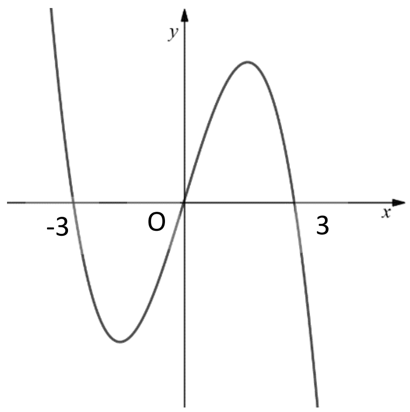
(c)

M1: For an attempt at division (seen or implied) Eg $-2x^3+15x^2-39x+b \equiv (x-4)\left(-2x^2 \ldots \pm \frac{b}{4}\right)$

A1: $(x-4)\left(-2x^2+7x-11\right)$ Sight of the quadratic with no incorrect working seen can score both marks.

(d)

See scheme. You can follow through on their value for *b*

Question	Scheme	Marks	AOs
2(a)	$9x - x^3 = x\left(9 - x^2\right)$	M1	1.1b
	$9x - x^3 = x\left(3 - x\right)\left(3 + x\right)$ oe	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	B1	1.1b
	Passes though origin, (3, 0) and (−3, 0)	B1	1.1b
		(2)	
(c)	$y = 9x - x^3 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = 9 - 3x^2 = 0 \Rightarrow x = (\pm)\sqrt{3} \Rightarrow y = ...$	M1	3.1a
	$y = (\pm)6\sqrt{3}$	A1	1.1b
	$\left\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\right\}$ oe	A1ft	2.5
		(3)	
			(7 marks)

Notes

(a)

M1: Takes out a factor of x or $-x$. Scored for $\pm x(\pm 9 \pm x^2)$ May be implied by the correct answer or $\pm x(\pm x \pm 3)(\pm x \pm 3)$.

Also allow if they attempt to take out a factor of $(\pm x \pm 3)$ so score for $(\pm x \pm 3)(\pm 3x \pm x^2)$

A1: Correct factorisation. $x(3 - x)(3 + x)$ on its own scores M1A1.

Allow eg $-x(x - 3)(x + 3)$, $x(x - 3)(-x - 3)$ or other equivalent expressions

Condone an = 0 appearing on the end and condone eg x written as $(x + 0)$.

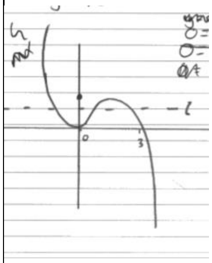
(b)

B1: Correct shape (negative cubic) appearing anywhere on a set of axes. It must have a minimum to the left and maximum to the right. Be tolerant of pen slips. Judge the intent of the shape. (see examples)

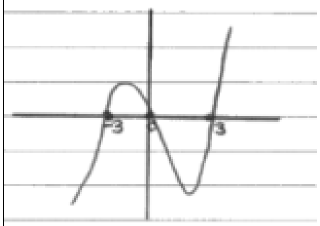
B1: Passes **through** each of the origin, (3, 0) and (−3, 0) and no other points on the x axis. (The graph should not turn on any of these points).
The points may be indicated as just 3 and −3 on the axes. Condone x and y to be the wrong way round eg $(0, -3)$ for (−3, 0) as long as it is on the correct axis but do not allow (−3, 0) to be labelled as (3, 0).

Examples

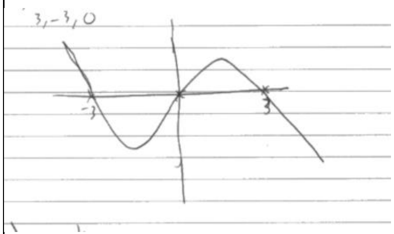
B1B0

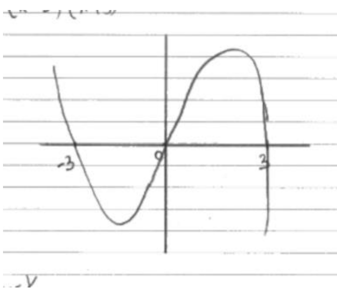


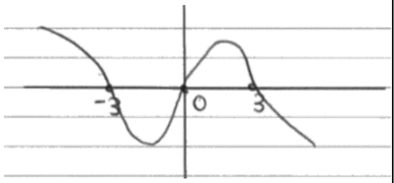
B0B1



B1B1







(c) ***Be aware the value of y can be solved directly using a calculator which is not acceptable***

M1: Uses a correct strategy for the y value of either the maximum or minimum. E.g. differentiates to achieve a quadratic, solves $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and uses their x to find y

A1: Either or both of the values $(\pm)6\sqrt{3}$.

Cannot be scored for an answer without any working seen.

A1ft: Correct answer in any acceptable set notation following through their $6\sqrt{3}$.

Condone $\left\{"-6\sqrt{3}" < k < "6\sqrt{3}"\right\}$ or $\left\{"-6\sqrt{3}" < k\right\} \cap \left\{k < "6\sqrt{3}"\right\}$ but not $\left\{"-6\sqrt{3}" < k\right\} \cup \left\{k < "6\sqrt{3}"\right\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Must be in terms of k

Question	Scheme	Marks	AOs
3	Sets $f'(4)=0 \Rightarrow 16+2a+b=0$	M1	2.1
	Integrates $f'(x)=4x+a\sqrt{x}+b \Rightarrow \{f(x)=\}2x^2+\frac{2}{3}ax^{\frac{3}{2}}+bx \ \{+c\}$	M1 A1ft	1.1b 1.1b
	Deduces that $c=-5$	B1	2.2a
	Full and complete method using the given information $f'(4)=0$ and $f(4)=3$ in order to find values for *a* and *b* Note: $a=-15$ and $b=14$	ddM1	3.1a
	$\{f(x)=\}2x^2-10x^{\frac{3}{2}}+14x-5$	A1	1.1b
		(6)	
		(6 marks)	

Notes:

M1: For the key step in setting $f'(4)=0 \Rightarrow 16+2a+b=0$ to set up an equation in *a* and *b*. Condone slips.

M1: For attempting to integrate $f'(x)$. Award for $x^n \to x^{n+1}$ or $b \to bx$
This may come after finding values for *a* or *b* or both.

A1ft: $\{f(x)=\}2x^2+\frac{2}{3}ax^{\frac{3}{2}}+bx \ \{+c\}$ or, e.g., $\{f(x)=\}2x^2+\frac{2}{3}ax^{\frac{3}{2}}+(-16-2a)x \ \{+c\}$
Allow ft on their *b* in terms of *a* if they substituted in from their $f'(4)=0 \Rightarrow 16+2a+b=0$
Do not ft if they have a value(s) for *a* or *b*
This may be left unsimplified but the indices must be processed.
isw once the mark is awarded. Condone the omission of the + *c*
This accuracy mark requires only the previous M mark to be scored.

B1: Deduces that the constant term in $f(x)$ is -5.
Note that deducing $b=-5$ is B0. It must be the constant in a changed function.

ddM1: For a complete strategy to find values for both *a* and *b*.
Do not be concerned about the logistics of how they solve the simultaneous equations – this may be done on a calculator.
Note: $a=-15$ and $b=14$
This is dependent on **both** previous method marks and so must include use of both

- $f'(4)=0$ (their $16+2a+b=0$ o.e.)
- $f(4)=3$ (their $32+\frac{16}{3}a+4b-5=3$ o.e.)

A1: $\{f(x)=\}2x^2-10x^{\frac{3}{2}}+14x-5$ or exact simplified equivalent, e.g., use of $x\sqrt{x}$ in place of $x^{\frac{3}{2}}$
Apply isw once a correct expression is seen.

Question	Scheme	Marks	AOs
4(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	
(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = ...$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = ...$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0,$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = ...$ $\left(\frac{dy}{dx}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0,$ $\left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
			(7 marks)

Notes

(a)(i)

M1: $x^n \rightarrow x^{n-1}$ for at least one power of x

A1: $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(b)(i)

M1: Substitutes $x = 1$ into their $\frac{dy}{dx}$

A1: Obtains $\frac{dy}{dx} = 0$ following a correct derivative and makes a conclusion which can be minimal e.g. tick, QED etc. which may be in a preamble e.g. stationary point when $\frac{dy}{dx} = 0$ and then shows $\frac{dy}{dx} = 0$

Alternative:

M1: Attempts to solve $\frac{dy}{dx} = 0$ by factorisation. This may be by using the factor of $(x - 1)$ or possibly using a calculator to find the roots and showing the factorisation. Note that they may divide by 4 before factorising which is acceptable. Need to either see either $4(x-1)^2(5x-8)$ or $(x-1)^2(5x-8)$ for the factorisation or $x = \frac{8}{5}$ and $x = 1$ seen as the roots.

A1: Obtains $x = 1$ and makes a conclusion as above

(b)(ii)

M1: Considers the value of the second derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

(NB $\frac{d^2y}{dx^2} = (x-1)(60x-84)$ so may use this factorised form when considering $x < 1$, $x > 1$ for sign change of second derivative)

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that the stationary point is a point of inflection. Sufficient reason is e.g. "sign change"/ "> 0, < 0". If values are given they should be correct (but be generous with accuracy) but also just allow "> 0" and "< 0" provided they are correctly paired. The interval must be where $x < 1.4$

Alternative 1 for (b)(ii)

M1: Shows that second derivative at $x = 1$ is zero and **then finds the third derivative at $x = 1$**

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is "$\neq 0$" but must follow a correct third derivative and a correct value if evaluated. For reference $\left(\frac{d^3y}{dx^3}\right)_{x=1} = -24$

Alternative 2 for (b)(ii)

M1: Considers the value of the first derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

A1: Fully correct work with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is e.g. "same sign"/"both negative"/"< 0, < 0". If values are given they should be correct (but be generous with accuracy). The interval must be where $x < 1.4$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$f'(x)$	-32	-24.3	-17.92	-12.74	-8.64	-5.5	-3.2	-1.62	-0.64	-0.14	0
$f''(x)$	84	70.2	57.6	46.2	36	27	19.2	12.6	7.2	3	0

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7
$f'(x)$	-0.1	-0.32	-0.54	-0.64	-0.5	0	0.98
$f''(x)$	-1.8	-2.4	-1.8	0	3	7.2	12.6

Question	Scheme	Marks	AOs
5 (a)	$2 < x < 6$	B1	1.1b
		(1)	
(b)	States either $k > 8$ or $k < 0$	M1	3.1a
	States e.g. $\{k : k > 8\} \cup \{k : k < 0\}$	A1	2.5
		(2)	
(c)	**Please see notes for alternatives**		
	States $y = ax(x-6)^2$ or $f(x) = ax(x-6)^2$	M1	1.1b
	Substitutes $(2,8)$ into $y = ax(x-6)^2$ and attempts to find a	dM1	3.1a
	$y = \frac{1}{4}x(x-6)^2$ or $f(x) = \frac{1}{4}x(x-6)^2$ o.e	A1	2.1
		(3)	
			(6 marks)
Notes: Watch for answers written by the question. If they are beside the question and in the answer space, the one in the answer space takes precedence			

(a)

B1: Deduces $2 < x < 6$ o.e. such as $x > 2, x < 6$ $\quad x > 2$ and $x < 6$ $\quad \{x : x > 2\} \cap \{x : x < 6\}$ $\quad x \in (2,6)$

Condone attempts in which set notation is incorrectly attempted but correct values can be seen or implied E.g. $\{x > 2\} \cap \{x < 6\}$ $\{x > 2, x < 6\}$. Allow just the open interval $(2,6)$

Do not allow for incorrect inequalities such as e.g. $x > 2$ or $x < 6$, $\{x : x > 2\} \cup \{x : x < 6\}$ $x \in [2,6]$

(b)

M1: Establishes a correct method by finding one of the (correct) inequalities

States either $k > 8$ (condone $k \geqslant 8$) or $k < 0$ (condone $k \leqslant 0$)

Condone for this mark $y \leftrightarrow k$ or $f(x) \leftrightarrow k$ and $8 < k < 0$

A1: Fully correct solution in the form $\{k : k > 8\} \cup \{k : k < 0\}$ or $\{k | k > 8\} \cup \{k | k < 0\}$ either way around but condone $\{k < 0\} \cup \{k > 8\}$, $\{k : k < 0 \cup k > 8\}$, $\{k < 0 \cup k > 8\}$. It is not necessary to mention $\mathbb{R}$, e.g. $\{k : k \in \mathbb{R}, k > 8\} \cup \{k : k \in \mathbb{R}, k < 0\}$ Look for $\{ \; \}$ and $\cup$

Do not allow solutions not in set notation such as $k < 0$ or $k > 8$.

(c)

M1: Realises that the equation of C is of the form $y = ax(x-6)^2$. Condone with $a = 1$ for this mark.

So award for sight of $ax(x-6)^2$ even with $a = 1$

dM1: Substitutes $(2,8)$ into the form $y = ax(x-6)^2$ and attempts to find the value for a.

It is dependent upon having an equation, which the (y =..) may be implied, of the correct form.

A1: Uses all of the information to form a correct **equation** for C $\quad y = \frac{1}{4}x(x-6)^2$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{1}{4}x(x-6)^2$ but not $C = \frac{1}{4}x(x-6)^2$.

Allow this to be written down for all 3 marks

Examples of alternative methods

....

Alternative I part (c):
Using the form $y = ax^3 + bx^2 + cx$ and setting up then solving simultaneous equations.
There are various versions of this but can be marked similarly

M1: Realises that the equation of *C* is of the form $y = ax^3 + bx^2 + cx$ and forms two equations in *a*, *b* and *c*. Condone with *a* =1 for this mark.
Note that the form $y = ax^3 + bx^2 + cx + d$ is M0 until *d* is set equal to 0.
There are four equations that could be formed, only two are necessary for this mark.
Condone slips
Using $(6, 0)$ $\Rightarrow 216a+36b+6c=0$
Using $(2, 8)$ $\Rightarrow 8a+4b+2c=8$
Using $\frac{dy}{dx}=0$ at $x = 2$ $\Rightarrow 12a+4b+c=0$
Using $\frac{dy}{dx}=0$ at $x = 6$ $\Rightarrow 108a+12b+c=0$

dM1: Forms and solves three different equations, one of which must be using (2, 8) to find values for *a*, *b* and *c*. A calculator can be used to solve the equations

A1: Uses all of the information to form a correct equation for *C* $y = \frac{1}{4}x^3 - 3x^2 + 9x$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{1}{4}x^3 - 3x^2 + 9x$

....

Alternative II part (c)
Using the gradient and integrating

M1: Realises that the gradient of *C* is zero at 2 and 6 so sets $f'(x) = k(x-2)(x-6)$ oe **and** attempts to integrate. Condone with *k* = 1

dM1: Substitutes $x = 2, y = 8$ into $f(x) = k(...x^3 + ...x + ...)$ and finds a value for *k*

A1: Uses all of the information to form a correct equation for *C* $y = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$ o.e.

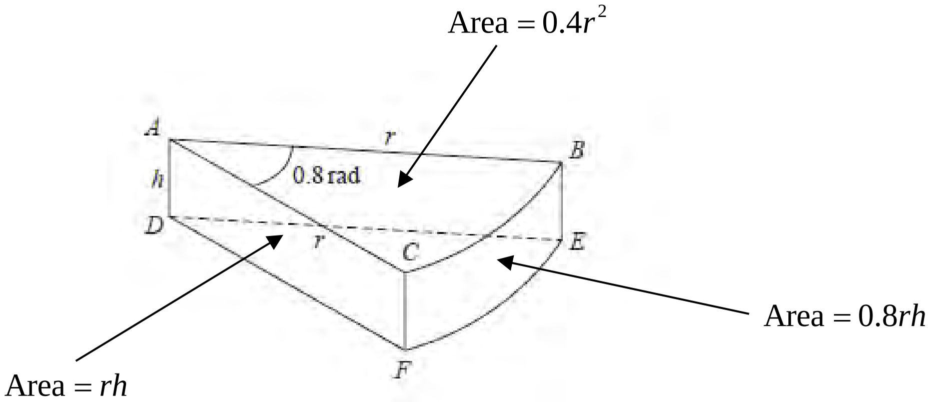
ISW after a correct answer. Condone $f(x) = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$

....

Question	Scheme	Marks	AOs
6 (a)	Sets up an allowable equation using volume $=240$ E.g. $\frac{1}{2}r^2\times0.8h=240\Rightarrow h=\frac{600}{r^2}$ o.e.	M1 A1	3.4 1.1b
	Attempts to substitute their $h=\frac{600}{r^2}$ into $(S=)\frac{1}{2}r^2\times0.8+\frac{1}{2}r^2\times0.8+2rh+0.8rh$	dM1	3.4
	$S=0.8r^2+2.8rh=0.8r^2+2.8\times\frac{600}{r}=0.8r^2+\frac{1680}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dS}{dr}\right)=1.6r-\frac{1680}{r^2}$	M1 A1	3.1a 1.1b
	Sets $\frac{dS}{dr}=0\Rightarrow r^3=1050$ $r=\text{awrt }10.2$	dM1 A1	2.1 1.1b
		(4)	
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right)=1.6+\frac{3360}{r^3}$ and considers its value or sign	M1	1.1b
	E.g. Correct $\frac{d^2S}{dr^2}=1.6+\frac{3360}{r^3}$ with $\left.\frac{d^2S}{dr^2}\right\|_{r=10.2}=5>0$ proving a minimum value of S	A1	1.1b
		(2)	
			(10 marks)

Notes:

Volume $=0.4r^2h$



Total surface area = $2rh+0.8r^2+0.8rh$

(a)

M1: Attempts to use the fact that the volume of the toy is 240 cm^3

Sight of $\frac{1}{2}r^2 \times 0.8 \times h = 240$ leading to $h = ...$ or $rh = ...$ scores this mark

But condone an equation of the correct form so allow for $kr^2h = 240 \Rightarrow h = ...$ or $rh = ...$

A1: A correct expression for $h = \frac{600}{r^2}$ or $rh = \frac{600}{r}$ which may be left unsimplified.

This may be implied when you see an expression for S or part of S E.g $2rh = 2r \times \frac{600}{r^2}$

dM1: Attempts to substitute their $h = \frac{a}{r^2}$ o.e. such as $hr = \frac{a}{r}$ into a **correct** expression for S

Sight of $\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + rh + rh + 0.8rh$ with an appropriate substitution

Simplified versions such as $0.8r^2 + 2rh + 0.8rh$ used with an appropriate substitution is fine.

A1*: Correct work leading to the given result.

$S =$, $SA =$ or surface area = must be seen at least once in the correct place

The method must be made clear so expect to see evidence. For example

$S = 0.8r^2 + 2rh + 0.8rh \Rightarrow S = 0.8r^2 + 2r \times \frac{600}{r^2} + 0.8r \times \frac{600}{r^2} \Rightarrow S = 0.8r^2 + \frac{1680}{r}$ would be fine.

(b) There is no requirement to see $\frac{dS}{dr}$ in part (b). It may even be called $\frac{dy}{dx}$.

M1: Achieves a derivative of the form $pr \pm \frac{q}{r^2}$ where p and q are non- zero constants

A1: Achieves $\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$

dM1: Sets or implies that their $\frac{dS}{dr} = 0$ and proceeds to $mr^3 = n,\quad m \times n > 0$. It is dependent upon a correct attempt at differentiation. This mark may be implied by a correct answer to their $pr - \frac{q}{r^2} = 0$

A1: $r =$ awrt 10.2 or $\sqrt[3]{1050}$

(c)

M1: Attempts to substitute their positive r (found in (b)) into $\left(\frac{d^2S}{dr^2} =\right) e \pm \frac{f}{r^3}$ where e and f are non zero and finds its value or sign.

Alternatively considers the sign of $\left(\frac{d^2S}{dr^2} =\right) e \pm \frac{f}{r^3}$ (at their positive r found in (b))

Condone the $\frac{d^2S}{dr^2}$ to be $\frac{d^2y}{dx^2}$ or being absent, but only for this mark.

A1: States that $\frac{d^2S}{dr^2}$ or $S'' = 1.6 + \frac{3360}{r^3} =$ awrt $5 > 0$ proving a minimum value of S

This is dependent upon having achieved $r =$ awrt 10 and a correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$

It can be argued without finding the value of $\frac{d^2S}{dr^2}$. E.g. $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3} > 0$ as $\underline{r > 0}$, so minimum value of S. For consistency it is also dependent upon having achieved $r =$ awrt 10

Do **NOT** allow $\frac{d^2y}{dx^2}$ for this mark