Question	Scheme	Marks	AOs
1 (a) (i)	Uses $\frac{dy}{dx}=-3$ at $x=2 \Rightarrow 12a+60-39=-3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a+60-39=-3 \Rightarrow 12a=-24 \Rightarrow a=-2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $10=8a+60-78+b$	M1	3.1a
	Subs $a=-2$ into $10=8a+60-78+b \Rightarrow b=44$	A1	1.1b
		(4)	
(b)	$f(x)=-2x^3+15x^2-39x+44 \Rightarrow f'(x)=-6x^2+30x-39$	B1	1.1b
	Attempts to show that $-6x^2+30x-39$ has no roots Eg. calculates $b^2-4ac=30^2-4\times-6\times-39=-36$	M1	3.1a
	States that as $f'(x)\neq 0 \Rightarrow$ hence $f(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3+15x^2-39x+44 \equiv (x-4)(-2x^2+7x-11)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	
			(11 marks)

Notes

(a)(i)

M1: Attempts to use $\frac{dy}{dx}=-3$ at $x=2$ to form an equation in a. Condone slips but expect to see two of the powers reduced correctly

A1*: Correct differentiation with one correct intermediate step before $a=-2$

(a)(ii)

M1: Attempts to use the fact that $(2,10)$ lies on C by setting up an equation in a and b with $a=-2$ leading to $b=...$

A1: $b=44$

(b)

B1: $f'(x)=-6x^2+30x-39$ oe

M1: Correct attempt to show that "$-6x^2+30x-39$" has no roots.
This could involve an attempt at

- finding the numerical value of b^2-4ac
- finding the roots of $-6x^2+30x-39$ using the quadratic formula (or their calculator)
- completing the square for $-6x^2+30x-39$

A1*: A fully correct method with reason and conclusion. Eg as $b^2-4ac=-36<0, f'(x)\neq 0$ meaning that no stationary points exist

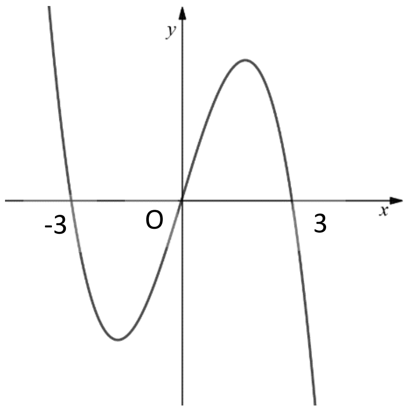
(c)

M1: For an attempt at division (seen or implied) Eg $-2x^3+15x^2-39x+b \equiv (x-4)\left(-2x^2 ... \pm \frac{b}{4}\right)$

A1: $(x-4)\left(-2x^2+7x-11\right)$ Sight of the quadratic with no incorrect working seen can score both marks.

(d)

See scheme. You can follow through on their value for b

Question	Scheme		Marks	AOs
2(a)	$9x-x^3=x\left(9-x^2\right)$		M1	1.1b
	$9x-x^3=x\left(3-x\right)\left(3+x\right)$ oe		A1	1.1b
			(2)	
(b)		A cubic with correct orientation	B1	1.1b
		Passes though origin, (3, 0) and (–3, 0)	B1	1.1b
			(2)	
(c)	$y=9x-x^3 \Rightarrow \frac{dy}{dx}=9-3x^2=0 \Rightarrow x=(\pm)\sqrt{3} \Rightarrow y=...$		M1	3.1a
	$y=(\pm)6\sqrt{3}$		A1	1.1b
	$\left\{k \in \mathbb{R} : -6\sqrt{3}<k<6\sqrt{3}\right\}$ oe		A1ft	2.5
			(3)	
				(7 marks)

Notes

(a)

M1: Takes out a factor of x or $-x$. Scored for $\pm x(\pm 9 \pm x^2)$ May be implied by the correct answer or $\pm x(\pm x \pm 3)(\pm x \pm 3)$.

Also allow if they attempt to take out a factor of $(\pm x \pm 3)$ so score for $(\pm x \pm 3)(\pm 3x \pm x^2)$

A1: Correct factorisation. $x(3-x)(3+x)$ on its own scores M1A1.

Allow eg $-x(x-3)(x+3)$, $x(x-3)(-x-3)$ or other equivalent expressions

Condone an = 0 appearing on the end and condone eg x written as $(x+0)$.

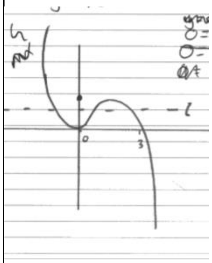
(b)

B1: Correct shape (negative cubic) appearing anywhere on a set of axes. It must have a minimum to the left and maximum to the right. Be tolerant of pen slips. Judge the intent of the shape. (see examples)

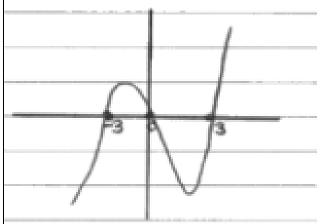
B1: Passes **through** each of the origin, (3, 0) and (–3, 0) and no other points on the x axis. (The graph should not turn on any of these points).
The points may be indicated as just 3 and -3 on the axes. Condone x and y to be the wrong way round eg $(0,-3)$ for (–3, 0) as long as it is on the correct axis but do not allow (–3, 0) to be labelled as (3, 0).

Examples

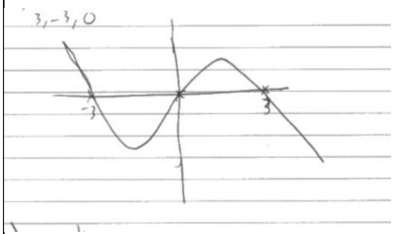
B1B0

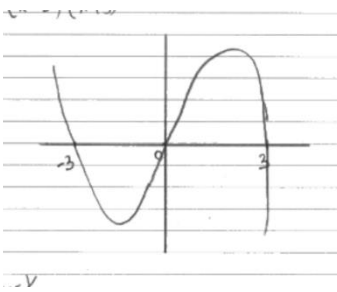


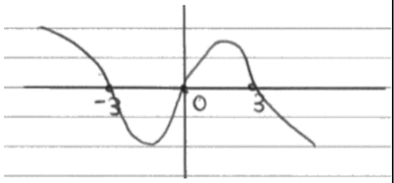
B0B1



B1B1







(c) ***Be aware the value of *y* can be solved directly using a calculator which is not acceptable***

M1: Uses a correct strategy for the y value of either the maximum or minimum. E.g. differentiates to achieve a quadratic, solves $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and uses their x to find y

A1: Either or both of the values $(\pm)6\sqrt{3}$.

Cannot be scored for an answer without any working seen.

A1ft: Correct answer in any acceptable set notation following through their $6\sqrt{3}$.

Condone $\left\{"-6\sqrt{3}" < k < "6\sqrt{3}"\right\}$ or $\left\{"-6\sqrt{3}" < k\right\} \cap \left\{k < "6\sqrt{3}"\right\}$ but not $\left\{"-6\sqrt{3}" < k\right\} \cup \left\{k < "6\sqrt{3}"\right\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Must be in terms of k

Question	Scheme	Marks	AOs
3	$\dfrac{2(x+h)^2-2x^2}{h}=\ldots$	M1	2.1
	$\dfrac{2(x+h)^2-2x^2}{h}=\dfrac{4xh+2h^2}{h}$	A1	1.1b
	$\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}(4x+2h)=4x$*	A1*	2.5
		(3)	
		(3 marks)	

Notes:

Throughout the question allow the use of δx for h or any other letter e.g. α if used consistently.
If δx is used then you can condone e.g. $\delta^2 x$ for δx^2 as well as condoning e.g. poorly formed δ's

M1: Begins the process by writing down the gradient of the chord and attempts to expand the correct bracket – you can condone "poor" squaring e.g. $(x+h)^2=x^2+h^2$.

Note that $\dfrac{2(x-h)^2-2x^2}{-h}=\ldots$ is also a possible approach.

A1: Reaches a correct fraction oe with the x^2 terms cancelled out.

E.g. $\dfrac{4xh+2h^2}{h}$, $\dfrac{\cancel{2x^2}+4xh+2h^2-\cancel{2x^2}}{h}$, $4x+2h$

A1*: Completes the process by applying a limiting argument and deduces that $\dfrac{dy}{dx}=4x$ with no errors seen. The "$\dfrac{dy}{dx}=$" doesn't have to appear but there must be something equivalent e.g. "$f'(x)=$" or "Gradient =" which can appear anywhere in their working. If $f'(x)$ is used then there is no requirement to see f (x) defined first. Condone e.g. $\dfrac{dy}{dx}\to 4x$ or $f'(x)\to 4x$.

Condone missing brackets so allow e.g. $\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}4x+2h=4x$

Do not allow h = 0 if there is never a reference to h→0

e.g. $\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}4x+2(0)=4x$ is acceptable

but e.g. $\dfrac{dy}{dx}=\dfrac{4xh+2h^2}{h}=4x+2h=4x+2(0)=4x$ is not if there is no h→0 seen.

The h→0 does not need to be present throughout the proof e.g. on every line.

They must reach $4x+2h$ at the end and not $\dfrac{4xh+2h^2}{h}$ (without the h's cancelled) to complete the limiting argument.

Question	Scheme	Marks	AOs
4	$\dfrac{\sin(x+h)-\sin x}{h}$	B1	2.1
	$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$	M1	1.1b
		A1	1.1b
	(As $h \to 0$), $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right) \to 0\times\sin x+1\times\cos x$	dM1	2.1
	so $\dfrac{dy}{dx}=\cos x$ *	A1*	2.5
			(5 marks)

Notes

Throughout the question allow the use of $h=\delta x$ if used consistently

There is no requirement to see "gradient of chord" written down.

B1: Gives the correct fraction such as $\dfrac{\sin(x+h)-\sin x}{x+h-x}$ or $\dfrac{\sin x-\sin(x+h)}{-h}$ or $\dfrac{\sin(x+h)-\sin(x-h)}{2h}$ or $\dfrac{\sin(x-h)-\sin x}{x-h-x}$. Condone invisible brackets. May be implied by $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

M1: Uses the compound angle formula for $\sin(x\pm h)$ to give $\sin x\cos h\pm\cos x\sin h$

A1: Achieves $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$ or equivalent (may be implied by further work).
Allow invisible brackets to be recovered.

dM1: **It is dependent on both the B and the M marks being awarded.**
Complete attempt to apply the given limits to the gradient of their chord. They must isolate $\left(\dfrac{\cos h-1}{h}\right)$ and replace with 0 and isolate $\left(\dfrac{\sin h}{h}\right)$ and replace with 1.

e.g. $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\sin x\times 0+\cos x\times 1$

Accept as a minimum $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\cos x$ (implying the application of the limits)

If they do not fully show $\left(\dfrac{\cos h-1}{h}\right)$ and $\left(\dfrac{\sin h}{h}\right)$ being isolated but proceed from

e.g. $\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}$ to $0\times\sin x+\cos x$ (or e.g. $0+\cos x$) then this can be implied and score dM1

$\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}=\cos x$ is dM0

Condone if limit notation remains within their expression after the limits have been applied.
e.g. $\lim\limits_{h\to 0}(\sin x\times 0+\cos x\times 1)$

Alternatively, condone use of the small angle approximations such that

$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h} \to \dfrac{-\dfrac{h^2}{2}\sin x+h\cos x}{h}=-\dfrac{h}{2}\sin x+\cos x$ and replaces $\dfrac{h}{2}$ with 0

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x$ with no errors seen. (cso)

We need to see $h \to 0$ **at some point in their solution and linking** $\frac{\mathrm{d}y}{\mathrm{d}x}$ **with** $\cos x$ e.g.

- $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \lim_{h \to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right) = \cos x$
- $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \lim_{h \to 0}\left(-\frac{h}{2}\sin x + \cos x\right) = 0 \times \sin x + \cos x = \cos x$ (using small angle approximations)
- $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} = \sin x \times 0 + 1 \times \cos x = \cos x$ as $h \to 0$

Condone $\mathrm{f}'(x)$ or y' in place of $\frac{\mathrm{d}y}{\mathrm{d}x}$

Give final A0 for no evidence of limiting arguments**:**

e.g. when $h = 0$ $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right) = \sin x \times 0 + \cos x \times 1 = \cos x$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h} = 1$ and $\frac{\cos h - 1}{h} = 0$ and attempting to apply these (without seeing e.g. $h \to 0$ at some point in their solution)

If they work in another variable (e.g. θ) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin x(\cos h - 1)$ e.g. $\frac{\sin x \cos h - 1 + \cos x \sin h}{h}$ but accept terms written as e.g. $\sin x \frac{\cos h - 1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.