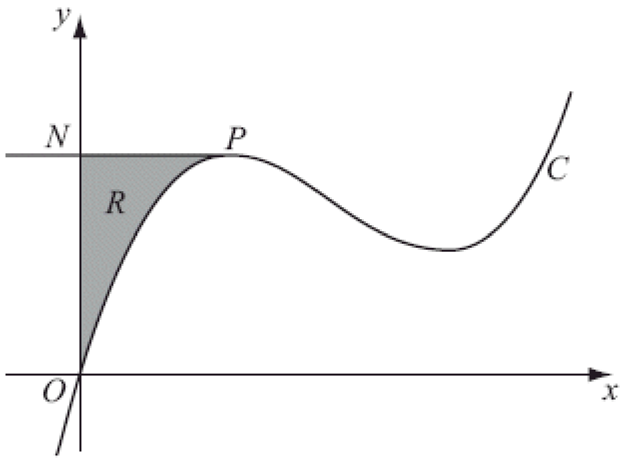
1.



The diagram above shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N. The region R is bounded by C, the y-axis and PN, as shown shaded in the diagram above.

(b) Use calculus to find the exact area of R.

(6)
(Total 9 marks)

2. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \; x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

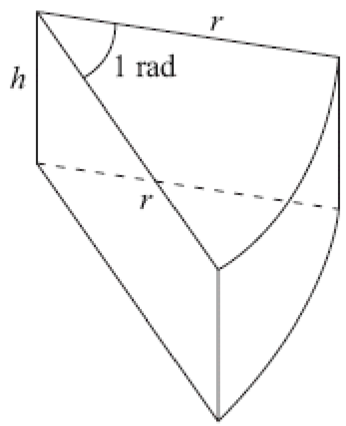
(b) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

(2)

(c) State the nature of the turning point.

(1)
(Total 10 marks)

3.



The diagram above shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(a) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}$$

(5)

(b) Use calculus to find the value of r for which S is stationary.

(4)

(c) Prove that this value of r gives a minimum value of S.

(2)

(d) Find, to the nearest cm^2, this minimum value of S.

(2)
(Total 13 marks)

4. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2.

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

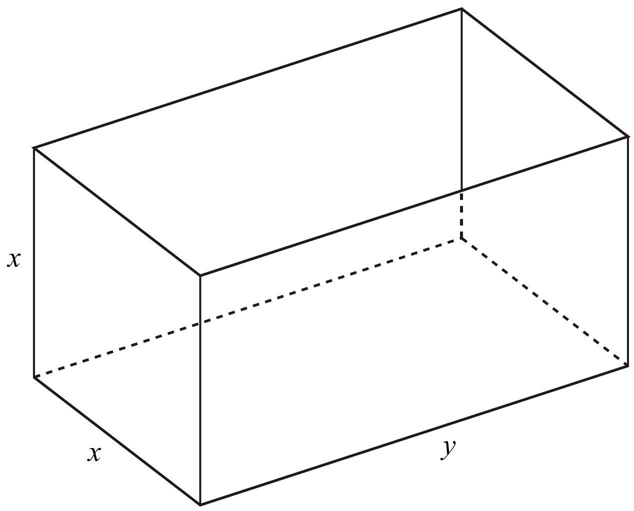
(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)
(Total 12 marks)

5.



The diagram above shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(a) Show that the area A m2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

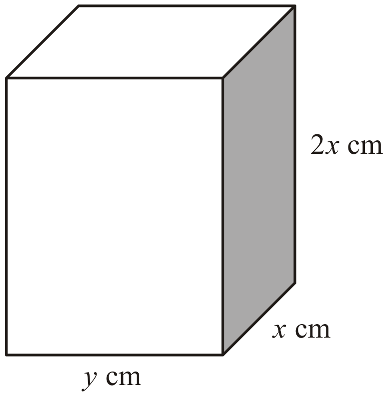
(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)
(Total 12 marks)

6.



The diagram above shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(c) Justify that the value of V you have found is a maximum.

(2)
(Total 11 marks)

7. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum.

(5)

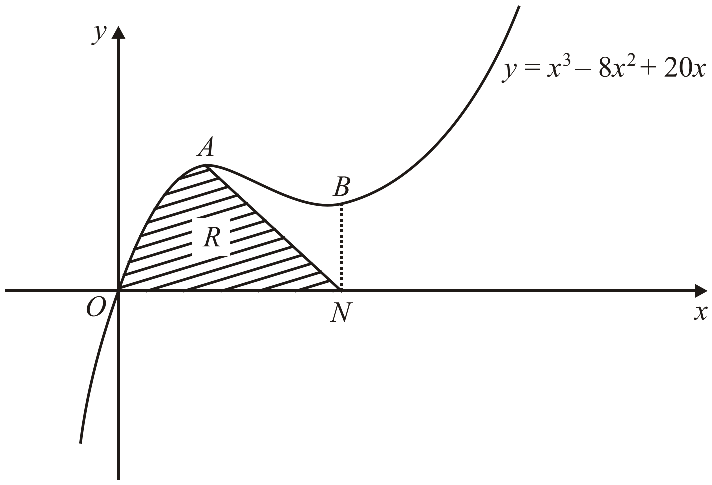
(b) Find $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ and hence verify that C is a minimum for this value of v.

(2)

(c) Calculate the minimum total cost of the journey.

(2)
(Total 9 marks)

8.



The figure above shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.
The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N.
The region R, shown shaded in the figure above, is bounded by the curve, the x-axis and the line from A to N.

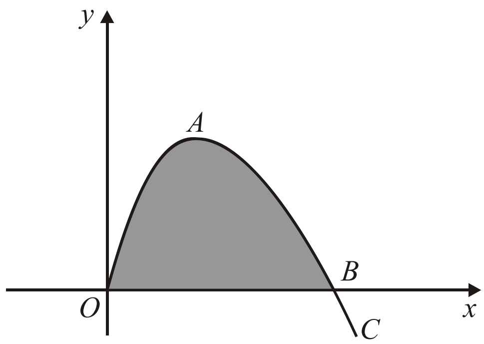
(c) Find $\int (x^3 - 8x^2 + 20x)\,dx$

(3)

(d) Hence calculate the exact area of R.

(5)
(Total 14 marks)

9.



The figure above shows part of the curve C with equation

$$y = 3x^{\frac{1}{2}} - x^{\frac{3}{2}},\ x \geq 0.$$

The point A on C is a stationary point and C cuts the x-axis at the point B.

(a) Show that the x-coordinate of B is 3.

(1)

(b) Find the coordinates of A.

(5)

(c) Find the exact area of the finite region enclosed by C and the x-axis, shown shaded in the figure above.

(5)
(Total 11 marks)

10. The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Using the result from part (a), find the coordinates of the turning points of C.

(4)

(c) Find $\frac{d^2y}{dx^2}$.

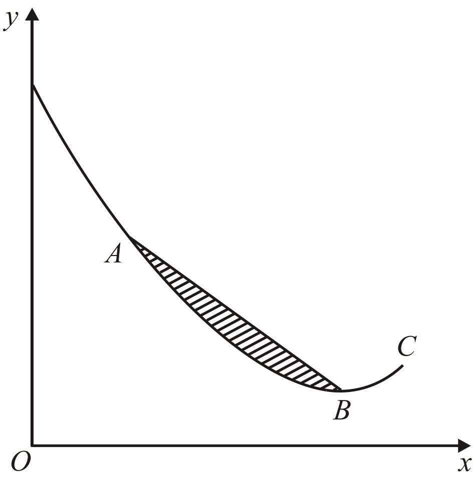
(2)

(d) Hence, or otherwise, determine the nature of the turning points of C.

(2)

(Total 10 marks)

11.



This diagram shows part of the curve C with equation

$$y = 2x^{\frac{3}{2}} - 6x + 10, \quad x \geq 0.$$

The curve C passes through the point $A(1, 6)$ and has a minimum turning point at B.

(a) Show that the x-coordinate of B is 4.

(4)

The finite region R, shown shaded in the diagram, is bounded by C and the straight line AB.

(b) Find the exact area of R.

(8)
(Total 12 marks)

12. Find the coordinates of the stationary point on the curve with equation $y = 2x^2 - 12x$.

(Total 4 marks)

13. The curve C has equation

$$y = 4x^2 + \frac{5x-1}{x}.$$

(a) Find $\frac{dy}{dx}$.

(3)

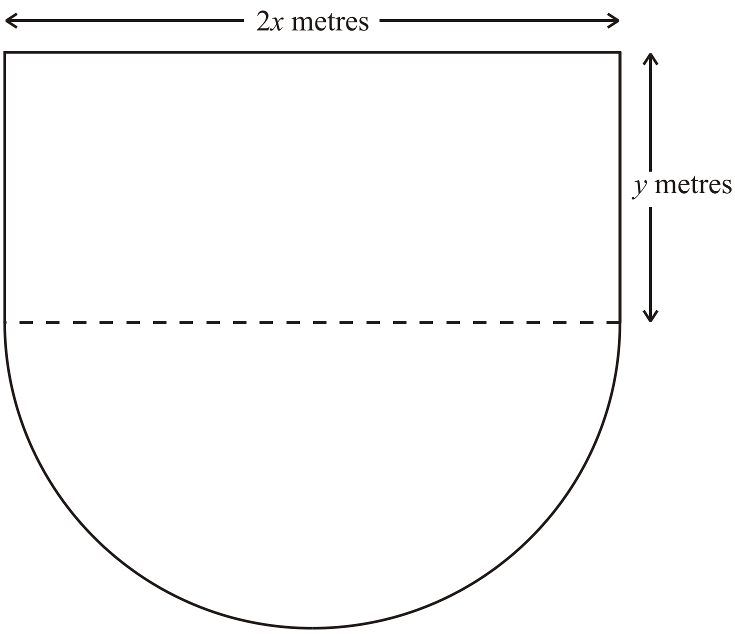
(b) Find the x-coordinate of the stationary point of C.

(3)

(c) Determine whether this stationary point is a maximum or a minimum.

(2)
(Total 8 marks)

14.



The diagram above shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(a) Show that the area, A m^2, of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2.$$

(4)

(b) Use calculus to find the value of x at which A has a stationary value.

(4)

(c) Prove that the value of x you found in part (b) gives the maximum value of A.

(2)

(d) Calculate, to the nearest m^2, the maximum area of the stage.

(2)
(Total 12 marks)

15. A manufacturing company produces closed cylindrical containers with base radius r cm and height h cm. The capacity of each container is 780 cm^3.

(a) Express h in terms of r.

(2)

(b) Show that the surface area, A cm^2, of a container is given by

$$A = \frac{1560}{r} + 2\pi r^2.$$

(2)

The surface area of a container is to be minimised.

(c) Use calculus to find the value of r for which A is a minimum.

(4)

(d) Prove that, for this value of r, A is a minimum.

(2)

(e) Calculate the minimum value of A.

(2)
(Total 12 marks)

16.

$$f(x) = \frac{(x^2 - 3)^2}{x^3}, \; x \neq 0.$$

(a) Show that $f(x) \equiv x - 6x^{-1} + 9x^{-3}$.

(2)

(b) Hence, or otherwise, differentiate $f(x)$ with respect to x.

(3)

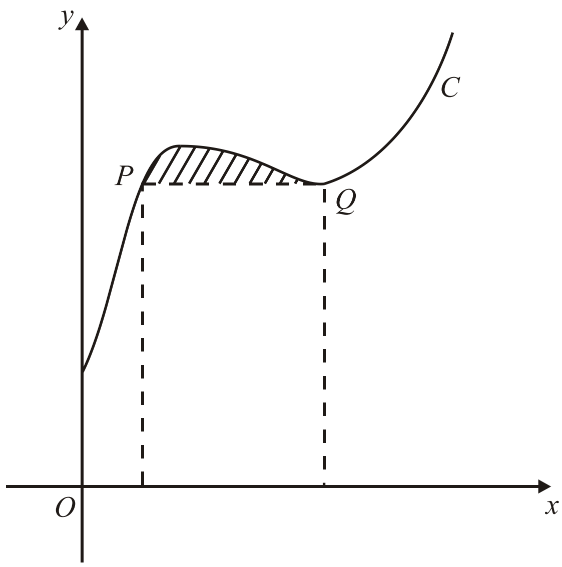
(c) Verify that the graph of $y = f(x)$ has stationary points at $x = \pm\sqrt{3}$.

(2)

(d) Determine whether the stationary value at $x = \sqrt{3}$ is a maximum or a minimum.

(3)
(Total 10 marks)

17.



The diagram above shows a sketch of part of the curve C with equation

$$y = x^3 - 7x^2 + 15x + 3, \qquad x \geq 0.$$

The point P, on C, has x-coordinate 1 and the point Q is the minimum turning point of C.

(a) Find $\frac{dy}{dx}$.

(2)

(b) Find the coordinates of Q.

(4)

(c) Show that PQ is parallel to the x-axis.

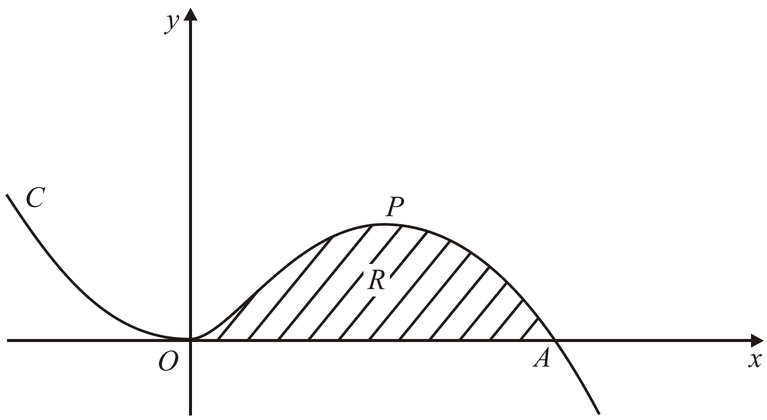
(2)

(d) Calculate the area, shown shaded in the diagram above, bounded by C and the line PQ.

(6)

(Total 14 marks)

18.



The diagram above shows part of the curve C with equation

$$y = \tfrac{3}{2}x^2 - \tfrac{1}{4}x^3.$$

The curve C touches the x-axis at the origin and passes through the point $A(p, 0)$.

(a) Show that $p = 6$.

(1)

(b) Find an equation of the tangent to C at A.

(4)

The curve C has a maximum at the point P.

(c) Find the x-coordinate of P.

(2)

The shaded region R, in the diagram above, is bounded by C and the x-axis.

(d) Find the area of R.

(4)
(Total 11 marks)

19. A container made from thin metal is in the shape of a right circular cylinder with height h cm and base radius r cm. The container has no lid. When full of water, the container holds 500 cm^3 of water.

(a) Show that the exterior surface area, A cm^2, of the container is given by

$$A = \pi r^2 + \frac{1000}{r}.$$

(4)

(b) Find the value of r for which A is a minimum.

(4)

(c) Prove that this value of r gives a minimum value of A.

(2)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)
(Total 12 marks)

20. The curve with equation $y = (2x + 1)(x^2 - k)$, where k is a constant, has a stationary point where $x = 1$.

(a) Determine the value of k.

(4)

(b) Find the coordinates of the stationary points and determine the nature of each.

(8)

(c) Sketch the curve and mark on your sketch the coordinates of the points where the curve crosses the coordinate axes.

(3)
(Total 15 marks)

21. A pencil holder is in the shape of an open circular cylinder of radius r cm and height h cm. The surface area of the cylinder (including the base) is 250 cm^2.

(a) Show that the volume, V cm^3, of the cylinder is given by $V = 125r - \dfrac{\pi r^3}{2}$.

(4)

(b) Use calculus to find the value of r for which V has a stationary value.

(3)

(c) Prove that the value of r you found in part (b) gives a maximum value for V.

(2)

(d) Calculate, to the nearest cm^3, the maximum volume of the pencil holder.

(2)

(Total 11 marks)

1. (a) $\dfrac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required) A1

At $x = 2$, $\dfrac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $k = 28$ (*) A1 cso

N.B. The '= 0' must be seen at some stage to score the final mark.

Alternatively: (using $k = 28$)

$\dfrac{dy}{dx} = 3x^2 - 20x + 28$ (A1)

'Assuming' $k = 28$ only scores the final cso mark if there is justification that $\dfrac{dy}{dx} = 0$ at $x = 2$ represents the maximum turning point. 3

Note

M: $x^n \to cx^{n-1}$ (c constant, $c \neq 0$) for one term, seen in part (a).

(b) $\int (x^3 - 10x^2 + 28x)\,dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}$ Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$ A1

$\left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2\right]_0^2 = ...$ $\left(= 4 - \frac{80}{3} + 56 = \frac{100}{3}\right)$

(With limits 0 to 2, substitute the limit 2 into a 'changed function')

y-coordinate of $P = 8 - 40 + 56 = 24$ Allow if seen in part (a) B1

(The B1 for 24 may be scored by implication from later working)

Area of rectangle = $2 \times (\text{their } y\text{-coordinate of } P)$

Area of R = $(\text{their } 48) - \left(\text{their } \frac{100}{3}\right) = \frac{44}{3}$ $\left(14\frac{2}{3} \text{ or } 14.\dot{6}\right)$ A1

If the subtraction is the 'wrong way round', the final A mark is lost. 6

Note

1st M: $x^n \to cx^{n+1}$ (c constant, $c \neq 0$) for one term.
Integrating the gradient function loses this M mark.

2ndM: Requires use of limits 0 and 2, with 2 substituted into a 'changed function'. (It may, for example, have been differentiated).

Final M: Subtract their values either way round. This mark is dependent on the use of calculus and a correct method attempt for the area of the rectangle.

A1: Must be exact, not 14.67 or similar, but isw after seeing, say, $\frac{44}{3}$.

Alternative: (effectively finding area of rectangle by integration)

$$\int \{24 - (x^3 - 10x^2 + 28x)\}\,dx = 24x - \left(\frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}\right), \text{ etc.}$$

This can be marked equivalently, with the 1st A being for integrating the same 3 terms correctly. The 3rd M (for subtraction) will be scored at the same stage as the 2nd M. If the subtraction is the 'wrong way round', the final A mark is lost.

[9]

2. (a) $\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10 \right]$

$[y' =]$ $6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$ A1

Puts their $\frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}} = 0$

So $x =$ $, \frac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1) A1

$x = 4, \Rightarrow y = 12\times2 - 4^{\frac{3}{2}} - 10,$ so $y = 6$ dM1 A1 7

Note

1st for an attempt to differentiate a fractional power $x^n \rightarrow x^{n-1}$

A1 a.e.f – can be unsimplified

2nd for forming a suitable equation using their y'=0

3rd for correct processing of fractional powers leading to $x = \ldots$ (Can be implied by $x = 4$)

A1 is for $x = 4$ only. If $x = 0$ also seen and not discarded they lose this mark only.

4th for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but $y = 6$ can imply M1A1

(b) $y'' = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$ M1A1 2

Note

for differentiating their y' again

A1 should be simplified

(c) [Since $x > 0$] It is a maximum B1 1

Note

B1 Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier).

(Treat parts (a),(b) and (c) together for award of marks)

[10]

3. (a) (Arc length =) $r\theta = r \times 1 = r$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$). B1

(Sector area =) $\frac{1}{2}r^2\,\theta = \frac{1}{2}r^2 \times 1 = \frac{r^2}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$). B1

Surface area = 2 sectors + 2 rectangles + curved face

$(= r^2 + 3rh)$ (See notes below for what is allowed here)

Volume = 300 = $\frac{1}{2}r^2h$ B1

Sub for h: $S = r^2 + 3 \times 3 \times \frac{600}{r} = r^2 + \frac{1800}{r}$ (*) A1cso 5

Note

for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an r^2 (or $r^2\,\theta$) term and an rh (or $rh\theta$) term.

(b) $\frac{dS}{dr} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$ M1A1

$\frac{dS}{dr} = 0 \Rightarrow r^3 = ...,\ r = \sqrt[3]{900}$ or AWRT 9.7 (NOT – 9.7 or ± 9.7) A1 4

Note

In parts (b), (c) and (d), ignore labelling of parts

1st for attempt at differentiation (one term is sufficient) $r^n \to kr^{n-1}$

2nd for setting their derivative (a 'changed function') = 0 and solving as far as r^3 = ... (depending upon their 'changed function', this could be r = ... or r^2 = ...,etc., but the algebra must deal with a negative power of r and should be sound apart from possible sign errors, so that r^n = ... is consistent with their derivative).

(c) $\frac{d^2S}{dr^2} = ...$ and consider sign, $\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum A1ft 2

Note

for attempting second derivative (one term is sufficient) $r^n \to kr^{n-1}$, and cosnsidering its sign. Substitution of a value of r is not required. (Equating it to zero is M0).

A1ft for a correct second derivative (or correct ft from their first derivative) and a valid reason (e.g. > 0), and conclusion. The actual value of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum.

Alternative:

Find value of $\frac{dS}{dr}$ on each side of their value of r and consider sign.

A1ft: Indicate sign change of negative to positive for $\frac{dS}{dr}$, and conclude minimum.

Alternative:

Find value of S on each side of their value of r and compare with their 279.65.

A1ft: Indicate that both values are more than 279.65, and conclude minimum.

(d) $S_{\min} = (9.65...)^2 + \frac{1800}{9.65...}$

(Using their value of r, however found, in the given S formula)

= 279.65… (AWRT: 280) (Dependent on full marks in part (b)) A1 2

[13]

4. (a) $2\pi rh + 2\pi r^2 = 800$ B1

$h = \dfrac{400 - \pi r^2}{\pi r}$, $V = \pi r^2 \left(\dfrac{400 - \pi r^2}{\pi r}\right) = 400r - \pi r^3$ (*) A1 4

Alternative

$A = 2\pi r^2 + 2\pi rh$, $\frac{A}{2} \times r = \pi r^3 + \pi r^2 h$ is Equate to $400r$ **B1**

Then $V = 400r - \pi r^3$ is **A1**

Notes

B1: For any correct form of this equation (may be unsimplified, may be implied by 1st

: Making h the subject of their three or four term formula

Substituting expression for h into $\pi r^2 h$ (independent mark) Must now be expression in r only.

A1: cso

(b) $\dfrac{dV}{dr} = 400 - 3\pi r^2$ A1

$400 - 3\pi r^2 = 0$ $r^2 = ...,$ $r = \sqrt{\dfrac{400}{3\pi}}$ (= 6.5 (2 s.f.)) A1

$V = 400r - \pi r^3 = 1737 = \dfrac{800}{3}\sqrt{\dfrac{400}{3\pi}}(\text{cm}^3)$ A1 6

(accept awrt 1737 or exact answer)

Notes

At least one power of r decreased by 1 **A1**: cao

Setting $\dfrac{dV}{dr} = 0$ and finding a value for correct power of r for candidate

A1: This mark may be credited if the value of V is correct. Otherwise answers should round to 6.5 (allow ± 6.5) or be exact answer

Substitute a positive value of r to give V **A1**: 1737 or 1737.25 ….. or exact answer

(c) $\dfrac{d^2V}{dr^2} = -6\pi r$, Negative, $\therefore$ maximum A1 2

(Parts (b) and (c) should be considered together when marking)

Other methods for part (c):

Either:M: Find value of $\frac{\mathrm{d}V}{\mathrm{d}r}$ on each side of " $r=\sqrt{\frac{400}{3\pi}}$ "and consider sign.

A: Indicate sign change of positive to negative for $\frac{\mathrm{d}V}{\mathrm{d}r}$ and conclude max.

Or: M: Find value of V on each side of " $r=\sqrt{\frac{400}{3\pi}}$ "and compare with "1737".

A: Indicate that both values are less than 1737 or 1737.25, and conclude max.

Notes

needs complete method **e.g.**attempts differentiation (power reduced) of their first derivative and considers its sign
A1(first method) should be $-6\pi r$ (do not need to substitute r and can condone wrong r if found in (b))
Need to conclude maximum or indicate by a tick that it is maximum.
Throughout allow confused notation such as dy/dx for dV/dr

[12]

5. (a) (Total area) = $3xy + 2x^2$ B1

(Vol:) $x^2y = 100$ ($y = \frac{100}{x^2}$, $xy = \frac{100}{x}$) B1

Deriving expression for area in terms of x only
(Substitution, or clear use of, y or xy into expression for area)

(Area =) $\frac{300}{x} + 2x^2$ **AG** A1cso 4

First B1: Earned for correct unsimplified expression, isw.

(b) $\frac{\mathrm{d}A}{\mathrm{d}x} = -\frac{300}{x^2} + 4x$ M1A1

Setting $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ and finding a value for correct power of x, for cand.

[$x^3 = 75$]

$x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$) A1 4

First At least one power of × decreased by 1, and no "c" term.

(c) $\dfrac{d^2A}{dx^2} = \dfrac{600}{x^3} + 4$ = positive, > 0; therefore minimum A1 2

For Find $\dfrac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or "positive"

A1: Candidate's $\dfrac{d^2A}{dx^2}$ must be correct for their $\dfrac{dA}{dx}$, sign must be + ve and conclusion "so minimum", (allow QED, ft). (may be wrong x, or even no value of x found)

Alternative:

Find value of $\dfrac{dA}{dx}$ on either side of " $x = \sqrt[3]{75}$ " and consider sign

A1: Indicate sign change of negative to positive for $\dfrac{dA}{dx}$, and conclude minimum.

OR Consider values of A on either side of " $x = \sqrt[3]{75}$ " and compare with"107"

A1: Both values greater than "x = 107" and conclude minimum.
Allow marks for (c) and (d) where seen; even if part labelling confused.
Throughout , allow confused notation, such as dy/dx for dA/dx.

(d) Substituting found value of x into (a)
(Or finding y for found × and substituting both in $3xy + 2x^2$)
$[y = \dfrac{100}{4.2172^2} = 5.6228]$
Area = 106.707 awrt 107 A1 2

[12]

6. (a) $4x^2 + 6xy = 600$ M1A1

$V = 2x^2y = 2x^2\left(\dfrac{600-4x^2}{6x}\right)$ $V = 200x - \dfrac{4x^3}{3}$ (*) M1A1cso 4

1st M: Attempting an expression in terms of x and y for the total surface area (the expression should be dimensionally correct).

1st A: Correct expression (not necessarily simplified), equated to 600.

2nd M: Substituting their y into $2x^2y$ to form an expression in terms of x only,
(Or substituting y from $2x^2y$ into their area equation).

(b) $\frac{\mathrm{d}V}{\mathrm{d}x} = 200 - 4x^2$ B1

Equate their $\frac{\mathrm{d}V}{\mathrm{d}x}$ to 0 and solve for x^2 or x : $x^2 = 50$ or $x = \sqrt{50}$ (7.07...) M1A1

Evaluate V: $V = 200(\sqrt{50}) - \frac{4}{3}(50\sqrt{50}) = 943$ cm^3 Allow awrt M1A1 5

1st A: Ignore $x = -\sqrt{50}$, if seen.

The 2nd M mark (for substituting their x value into the given expression for V) is dependent on the 1st M.

Final A: Allow also exact value $\frac{400\sqrt{50}}{3}$ or $\frac{2000\sqrt{2}}{3}$ or equiv. single term.

(c) $\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = -8x$ Negative. $\therefore$ Maximum A1ft 2

Allow marks if the work for (c) is seen in (b) (or vice-versa).

M: Find second derivative and consider its sign.

A: Second derivative following through correctly from their $\frac{\mathrm{d}V}{\mathrm{d}x}$, and correct reason/conclusion (it must be a maximum, not a minimum).
An actual value of x does not have to be used... this mark can still be awarded if not x value has been found or if a wrong x value is used.

Alternative:

M: Find value of $\frac{\mathrm{d}V}{\mathrm{d}x}$ on each side of " $x = \sqrt{50}$" and consider sign.

A: Indicate sign change of positive to negative for $\frac{\mathrm{d}V}{\mathrm{d}x}$, and conclude max.

Alternative:
M: Find value of V on each side of "$x = \sqrt{50}$" and compare with "943".
A: Indicate that both values are less than 943, and conclude max.

[11]

7.	(a)	$\frac{dC}{dv} = -1400v^{-2} + \frac{2}{7}$	A1	
		$-1400v^{-2} + \frac{2}{7} = 0$		
		$v^2 = 4900$	**d**M1	
		$v = 70$	A1cso	5

Attempt to differentiate $v^n \to v^{n-1}$. Must be seen and marked in part (a) not part (b).
Must be differentiating a function of the form $av^{-1} + bv$.

o.e.

($-1400v^{-2} + \frac{2}{7} + c$ is A0) A1

Their $\frac{dC}{dv} = 0$. Can be implied by their $\frac{dC}{dv} = P + Q \to P = \pm Q$.

Dependent on both of the previous Ms.

Attempt to rearrange their $\frac{dC}{dv}$ into the form v^n = number or v^n – number = 0, $n \neq 0$. dM1

$v = 70$ cso but allow $v = \pm 70$. $v = 70$ km per h also acceptable. A1cso

Answer only is 0 out of 5.

Method of completing the square: send to review.

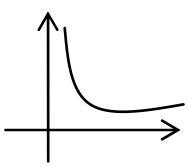
Trial and improvement $f(v) = \frac{1400}{v} + \frac{2v}{7}$

Attempts to evaluate $f(v)$ for 3 values a, b, c where (i) $a < 70$, $b = 70$ and $c > 70$ or (ii) a, $b < 70$ and $c > 70$ or (iii) $a < 70$ and b, $c > 70$.

All 3 correct and states $v = 70$ (exact) A1

Then 2nd M0, 3rd M0, 2nd A0.

Graph



Correct shape (ignore anything drawn for $v < 0$).

$v = 70$ (exact) A1

Then 2nd M0, 3rd M0, 2nd A0.

(b) $\dfrac{d^2C}{dv^2} = 2800v^{-3}$

$v = 70$, $\dfrac{d^2C}{dv^2} > 0$ {$\Rightarrow$ minimum}

or $v = 70$, $\dfrac{d^2C}{dv^2} = 2800 \times 70^{-3}$ {$= \dfrac{2}{245} = 0.00816...$}{$\Rightarrow$ minimum} A1ft 2

Attempt to differentiate their $\dfrac{dC}{dv}$; $v^n \to v^{n-1}$ (including $v^0 \to 0$).

$\dfrac{d^2C}{dv^2}$ must be correct. Ft only from their value of v and provided their value of v is +ve.
Must be some (minimal) indication that their value of v is being used.
Statement: "When v = their value of v, $\dfrac{d^2C}{dv^2} > 0$" is sufficient provided $2800v^{-3} > 0$ for their value of v.
If substitution of their v seen: correct substitution of their v into $2800v^{-3}$, but, provided evaluation is +ve, ignore incorrect evaluation.
N.B. Parts in mark scheme in { } do not need to be seen. A1ft

<u>Examples</u>

$\dfrac{d^2C}{dv^2} = 2800v^{-3}$

$v = 70$, $\dfrac{d^2C}{dv^2} > 0$ A1

$\dfrac{d^2C}{dv^2} = 2800v^{-3}$

> 0 A0 (no indication that a value of v is being used)

Answer from (a): $v = 30$

$\frac{d^2C}{dv^2} = 2800v^{-3}$

$v = 30,\ \frac{d^2C}{dv^2} > 0$ A1ft

$\frac{d^2C}{dv^2} = 2800v^{-3}$

$v = 70,\ \frac{d^2C}{dv^2} = 2800 \times 70^{-3}$

$= 8.16$ A1 (correct substitution of 70 seen, evaluation wrong but positive)

$\frac{d^2C}{dv^2} = 2800v^{-3}$

$v = 70,\ \frac{d^2C}{dv^2} = 0.00408$ A0 (correct substitution of 70 not seen)

(c) $v = 70,\ C = \frac{1400}{70} + \frac{2 \times 70}{7}$

$C = 40$ A1 2

Substitute their value of v that they think will give C_{min} (independent of the method of obtaining this value of v and independent of which part of the question it comes from).

40 or £40
Must have part (a) completely correct (i.e. all 5 marks) to gain this A1. A1

Answer only gains M1A1 provided part (a) is completely correct..

[9]

8. (a) $\frac{dy}{dx} = 3x^2 - 16x + 20$ A1

$3x^2 - 16x + 20 = 0$ $(3x - 10)(x - 2) = 0$ $x = ...,\ \frac{10}{3}$ and 2 dM1 A1 4

The second M is dependent on the first, and requires an attempt to solve a 3 term quadratic.

(b) $\frac{d^2y}{dx^2} = 6x - 16$ At $x = 2$, $\frac{d^2y}{dx^2} = ...$

-4 (or < 0, or both), therefore maximum A1ft 2

Attempt second differentiation and substitution of one of the x values.

A1ft: Requires correct second derivative and negative value of the second derivative, but ft from their x value.

(c) $\int (x^3 - 8x^2 + 20x)dx = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{20x^2}{2} (+C)$ A1 A1 3

All 3 terms correct: A1 A1,

Two terms correct: A1 A0,

One power correct: A0 A0.

(d) $4 - \frac{64}{3} + 40$ $\left(= \frac{68}{3}\right)$

A: $x = 2$: $y = 8 - 32 + 40 = 16$ (May be scored elsewhere) B1

Area of Δ = $\frac{1}{2}\left(\frac{10}{3} - 2\right) \times 16$ $\left(\frac{1}{2}(x_B - x_A) \times y_A\right)$ $\left(= \frac{32}{3}\right)$

Shaded area = $\frac{68}{3} + \frac{32}{3} = \frac{100}{3}$ $\left(= 33\frac{1}{3}\right)$ A1 5

Limits Substituting their lower x value into a 'changed' expression.

Area of triangle Fully correct method.

Alternative for the triangle (finding an equation for the straight line then integrating) requires a fully correct method to score the M mark.

Final Fully correct method (beware valid alternatives!)

[14]

9. (a) $y = 0 \Rightarrow x^{\frac{1}{2}}(3 - x) = 0 \Rightarrow x = 3$ B1 1

or $3\sqrt{3} - 3^{\frac{3}{2}} = 3\sqrt{3} - 3\sqrt{3} = 0$

(b) $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$ $x^n \mapsto x^{n-1}$ A1

$\frac{dy}{dx} = 0 \Rightarrow x^{\frac{1}{2}} = x^{-\frac{1}{2}}$ Use of $\frac{dy}{dx} = 0$

$\Rightarrow x = 1$ A1

A: (1, 2) A1 5

(c) $\int\left(3x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx = 2x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}$ $x^n \mapsto x^{n+1}$ A1+A1

Accept unsimplified expressions for As

Area = $[...]_0^3$ = $2 \times 3\sqrt{3} - \frac{2}{5} \times 9\sqrt{3}$ Use of correct limits

Area is $\frac{12}{5}\sqrt{3}$ (units^2) A1 5

For final A1, terms must be collected together but accept exact equivalents, e.g. $\frac{4}{5}\sqrt{27}$

[11]

10. (a) $\frac{dy}{dx} = 6x^2 - 10x - 4$ A1 2

(b) $6x^2 - 10x - 4 = 0$

$2(3x + 1)(x - 2)$ [= 0]

$x = 2$ or $-\frac{1}{3}$ (both x values) A1

Points are (2, <u>– 10</u>) and ($-\frac{1}{3}$, $2\frac{19}{27}$ or $\frac{73}{27}$ or 2.70 or better)

(both y values) A1 4

(c) $\frac{d^2y}{dx^2} = 12x - 10$ A1 2

(d) $x - 2 \Rightarrow \dfrac{d^2y}{dx^2}(= 14) \geq 0 \therefore [(2, -10)]$ is a <u>Min</u>

$x = -\frac{1}{3} \Rightarrow \dfrac{d^2y}{dx^2}(= 14) \geq 0 \therefore [(-\frac{1}{3}, \frac{73}{27})]$ is a <u>Max</u> A1 2

[10]

(a) for some correct attempt to differentiate $x^n \to x^{n-1}$

(b) 1st for setting their $\dfrac{dy}{dx} = 0$

2nd for attempting to solve 3TQ but it must be based on their $\dfrac{dy}{dx}$.

NO marks for answers only in part (b)

(c) for attempting to differentiate their $\dfrac{dy}{dx}$

(d) for one correct use of their second derivative or a full method to determine the nature of one of their stationary points

A1 both correct (= 14 and = – 14) are not required

11. (a) $\dfrac{dy}{dx} = 3x^{\frac{1}{2}} - 6$ A1

$3x^{\frac{1}{2}} - 6 = 0,\ x^{\frac{1}{2}} = 2 \qquad x = 4$ (*) A1 4

First for decrease of 1 in power of x of at least one term (disappearance of "10" sufficient)

Second for putting $\dfrac{dy}{dx} = 0$ *and finding x = ...*

(b) $\int\left(2x^{3/2} - 6x + 10\right)\, dx = \left[\frac{4x^{5/2}}{5} - 3x^2 + 10x\right]$ A1 A1

$\left[\frac{4x^{5/2}}{5} - 3x^2 + 10x\right]_1^4 = \left(\frac{4\times 4^{5/2}}{5} - (3\times 16) + 40\right) - \left(\frac{4}{5} - 3 + 10\right)$ A1ft

(= 17.6 – 7.8 = 9.8)

Finding area of trapezium = $\frac{1}{2}(6 + 2) \times 3$ (=12) A1

[A = (1, 6), B = (4, 2)]

Or by integration: $\left[\frac{22x - 2x^2}{3}\right]_1^4$

Area of R =12 – 9.8 = 2.2 A1 8

First Power of at least one term increased by 1

First A1: For $\frac{4x^{5/2}}{5}$

Second A1: For $-3x^2 + 10x$

Second for limits requires $|[\ ]^4 - [\ ]_1|$ *(allow candidate's "4")*

and some processing of "integral", $[y]_1^4$ *is M0*

A1ft requires ***1*** *and* ***4*** *substituted in candidate's* ***3-termed integrand*** *(unsimplified)*

Area of trapezium: attempt at ½ $(y_A + y_B)(x_B - 1)$

or $\int \frac{22 - 4x}{3} dx$ *(A1 correct unsimplified)*

EXTRA

Attempting integral |(equation of line – equation of curve)! Third

$= \int |\,(-\frac{8}{3} + \frac{14}{3}x - 2x^{\frac{3}{2}})\,|\, dx$ Fourth A1

Performing integration: First

$\left[(-\frac{8}{3}x + \frac{7}{3}x^2) - (\frac{4}{5}x^{\frac{5}{2}})\right]$ $\left[(\frac{4}{5}x^{\frac{5}{2}})\right]$ First A1

$\left[|\,(-\frac{8}{3}x + \frac{7}{3}x^2)\,|\right]$ allow as follow through in this case. Second A1

Limits M1A1 $\sqrt{\ }$ Second

Third A1

Answer A1 Fifth A1

[12]

12. $\frac{dy}{dx} = 4x - 12$ B1

$4x - 12 = 0$ $\quad x = 3$ A1ft

$y = -18$ A1 4

equate $\frac{dy}{dx}$ *(not just y) to zero and proceed to x = …*

A1ft follow through only from a linear equation in x.

[4]

Alternative:

$y = 2x(x - 6) \Rightarrow$ Cirve crosses x-axis at 0 and 6 B1

(By symmetry) $x = 3$ A1ft

$y = -18$ A1

Alternative:

$(x - 3)^2$ B1

for $(x - 3)^2$ *seen somewhere*

$y = 2(x^2 - 6x) = 2\{(x - 3)^2 - 9]$ $\quad x = 3$

for attempt to complete square and deduce x = …

A1ft $[(x - a)^2 \Rightarrow x = a]$

$Y = -18$ A1

13. (a) Correct strategy for differentiation e.g. $y = 4x^2 + (5x - 1)x^{-1}$ multiplied out with correct differentiation method, or product or quotient rules applied correctly to $\frac{5x-1}{x}$.

$\frac{dy}{dx} = 8x, +\frac{1}{x^2}$ B1 for $8x$ seen anywhere. B1, A1 3

(b) Putting $\frac{dy}{dx} = 0$

So $8x^3 + 1 = 0 \Rightarrow x = -\frac{1}{2}$. A1 3

requires multiplication by denominator and use of a root in the solution

(c) Complete method:

Either $\frac{d^2y}{dx^2} = 8 - \frac{2}{x^3}$, with x value substituted,

or gradient either side checked

Completely correct argument, either > 0 with no error seen,

(24)or –ve to +ve gradient, then **minimum** stated A1 2

[8]

14. (a) Perimeter $\Rightarrow 2x + 2y + \pi x = 80$ B1

Area $\rightarrow \quad A = 2xy + \frac{1}{2}\pi x^2$ B1

$y = \frac{80 - 2x - \pi x}{2}$ and sub in to A

$\Rightarrow A = 80x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2$

i.e. $A = 80x - (2 + \frac{\pi}{2})x^2$ (*) A1 c.s.o 4

(b) $\frac{dA}{dx} = 80 - 2(2 + \frac{\pi}{2})x$ or $80 - 4x - \pi x$ (or equiv.) A1

$\frac{dA}{dx} = 0 \Rightarrow 40 = (2 + \frac{\pi}{2})x$ so $x = , \frac{40}{2 + \frac{\pi}{2}}$ or $\frac{80}{4 + \pi}$ or Awrt 11.2M1, A1 4

(c) $\frac{d^2A}{dx^2} = -4 - \pi$

$< 0 \quad \therefore A$ is Max A1 2

(d) Max Area = $80(b) - (2 + \frac{\pi}{2})(b)^2$

$= \underline{448(m^2)}$ (448 only for A1) A1 cao 2

[12]

15. (a) $\pi r^2 h = 780,\ h = \frac{780}{\pi r^2}$ A1 2

(b) $A = 2\pi r^2 + 2\pi rh$ and substitute for h.

$A = 2\pi r^2 + \frac{1560}{r}$ (*) A1 2

(c) $\frac{dA}{dr} = 4\pi r - 1560r^{-2}$ A1

Equate to zero and proceed to $r^3 = \ldots$ or $r = \ldots$, coping with indices.

$r = \sqrt[3]{\frac{1560}{4\pi}} \quad \left(= \sqrt[3]{\frac{390}{\pi}} \approx 4.99 \approx 5.0\right)$ A1 4

(d) Attempt second derivative and consider its sign/value.

$\frac{d^2A}{dr^2} = 4\pi + 3120r^{-3}$ Correct second derivative, > 0, $\therefore$ minimum. A1 2

(e) Substitute value of r (or values of r and h) into their A formula.

469 (or a.w.r.t.) or 470 (2 s.f.) A1cso 2

[12]

16. (a) $(x^4 - 6x^2 + 9)$

$(x^4 - 6x^2 + 9) \div x^3 = x - 6x^{-1} + 9x^{-3}$ (*) A1 2

(b) $f'(x) = 1 + 6x^{-2} - 27x^{-4}$ A1 A1 3

First A1: 2 terms correct (unsimplified)

Second A1: all 3 correct (simplified)

(c) When $x = \pm\sqrt{3}$, $f'(x) = 1 + \frac{6}{(\sqrt{3})^2} - \frac{27}{(\sqrt{3})^4}$

$\left(= 1 + \frac{6}{3} - \frac{27}{9}\right) = 0$, $\therefore$ Stationary A1 2

(d) $f''(x) = -12x^{-3} + 108x^{-5}$

M: Attempt to diff. f′(x), not g(x) f′(x).

$f''(\sqrt{3}) = -\frac{12}{(\sqrt{3})^3} + \frac{108}{(\sqrt{3})^5}$ $(\approx -2.309 + 6.928 = 4.619)$ $\left(= \frac{8}{\sqrt{3}}\right)$ A1

> 0, $\therefore$ Minimum (not dependent on a numerical version of $f''(x)$) A1ft 3

[10]

17. (a) $\frac{dy}{dx} = 3x^2 - 14x + 15$ A1 2

(b) $3x^2 - 14x + 15 = 0$
$(3x - 5)(x - 3) = 0 \quad x = \ldots,$ A1
(A1 requires <u>correct</u> quadratic factors).
$y = 12$ A1 4
(Following from x = 3)

(c) P: $x = 1 \quad y = 12$ B1
Same y-coord. as Q (or "zero gradient"), so PQ is parallel to the x-axis B1 2

(d) $\int (x^3 - 7x^2 + 15x + 3)\,dx = \frac{x^4}{4} - \frac{7x^3}{3} + \frac{15x^2}{2} + 3x$ A1 A1
(First A1: 3 terms correct, Second A1: all correct)
$\left[\frac{x^4}{4} - \frac{7x^3}{3} + \frac{15x^2}{2} + 3x\right]_1^3 = \left(\frac{81}{4} - 63 + \frac{135}{2} + 9\right) - \left(\frac{1}{4} - \frac{7}{3} + \frac{15}{2} + 3\right)$
$\left(33\frac{3}{4} - 8\frac{5}{12}\right) - 24 = 25\frac{1}{3} - (2 \times 12) = 1\frac{1}{3}$ A1 6
(or equiv. or 3 s.f or better)

[14]

18. (a) Solve $\frac{3}{2}x^2 - \frac{1}{4}x^3 = 0$ to find $p = 6$, or verify: $\frac{3}{2} \times 6^2 - \frac{1}{4} \times 6^3 = 0$ (*) B1 1

(b) $\frac{dy}{dx} = 3x - \frac{3x^2}{4}$ A1
$m = -9$, $y - 0 = -9(x - 6)$ (Any correct form) A1 4

(c) $3x - \frac{3x^2}{4} = 0$, $x = 4$ A1ft 2

(d) $\int \left(\frac{3x^2}{2} - \frac{x^3}{4}\right)dx = \frac{x^3}{2} - \frac{x^4}{16}$ (Allow unsimplified versions) A1
$[\ldots\ldots]_0^6 = \frac{6^3}{2} - \frac{6^4}{16} = 27$ M: Need 6 and 0 as limits. A1 4

[11]

19. (a) $\pi r^2 h = 500$, $2\pi rh + \pi r^2$ (could be implied at next step) B1, B1

$A = 2\pi r\left(\frac{500}{\pi r^2}\right) + \pi r^2 = \pi r^2 + \frac{1000}{r}$ (*) A1 cso 4

(b) $\frac{dA}{dr} = 2\pi r - 1000r^{-2}$ A1

$2\pi r - 1000r^{-2} = 0 \quad r = \sqrt[3]{\frac{500}{\pi}}$ (≈ 5.42) A1 4

(c) $\frac{d^2A}{dr^2} = 2\pi + 2000r^{-3}, > 0$ therefore minimum. A1 2

Follow through from their first derivative and their r value.
A1 requires conclusions, but not evaluation of second derivative.

Other methods:

for evaluating $\frac{dA}{dr}$ *(or A) either side of their r value,*

A1 for correct reasoning and conclusion.

(d) $A = \pi r^2 + \frac{1000}{r} = 277$ (nearest integer) A1 2

(Allow the M mark even if r is negative)

[12]

20. (a) $y = (x^2 - k)(2x + 1) = 2x^3 + x^2 - 2kx - k$

$\frac{dy}{dx} = 6x^2 + 2x - 2k$ A1

$x = 1, \ \frac{dy}{dx} = 0 \ \Rightarrow \ k = 4$ A1 4

(b) Stationary points occur when $6x^2 + 2x - 8 = 0$

$(x - 1)(3x + 4) = 0$

$x = -\frac{4}{3}$ (and $x = 1$) A1

$x = 1, y = -9$ A1

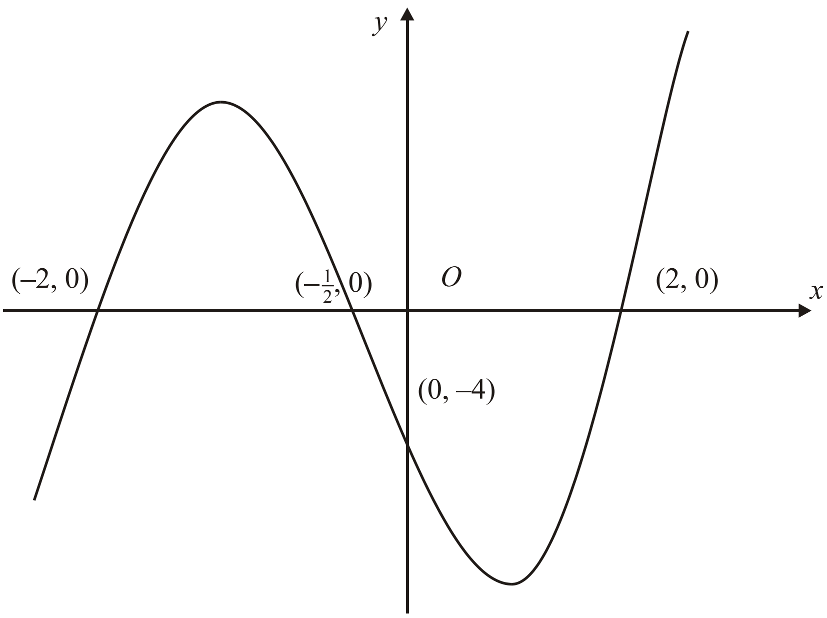
$x = -\frac{4}{3}, y = \left(\frac{16}{9} - 4\right)\left(-\frac{8}{3} + 1\right) = \frac{100}{27}$ A1

$\frac{d^2y}{dx^2} = 12x + 2$

At $x = 1$, $\frac{d^2y}{dx^2} = 14 > 0$, $\therefore$ $y = (1, -9)$ is a minimum point A1

At $x = -\frac{4}{3}$, $\frac{d^2y}{dx^2} = -14 < 0$, $\therefore$ $y = (-\frac{4}{3}, \frac{100}{27})$ is a maximum point A1 8

(c)



Shape	G1	
3 x-coordinates	G1 ft	
y coordinate	G1 ft	3

[15]

21. (a) Surface Area = $2\pi rh + \pi r^2$ B1

$h = \dfrac{250 - \pi r^2}{2\pi r}$

Attempt h =

$V = \pi r^2 h = \pi r^2 \times \dfrac{(250 - \pi r^2)}{2\pi r}$

V = and sub for h

$V = 125r - \dfrac{\pi r^3}{2}$ **(*)** A1 c.s.o. 4

(b) $\dfrac{dV}{dr} = 125 - \dfrac{3\pi}{2}r^2$

$\dfrac{dV}{dr} = 0 \Rightarrow r^2 = \dfrac{250}{3\pi}$ so $r = \sqrt{\dfrac{250}{3\pi}} = 5.15...$ A1 3

(c) $\dfrac{d^2V}{dr^2} = -3\pi r$

When $r = 5.15...$ this is < 0, therefore a maximum A1 2

(d) Max V is $125(5.15..) - \dfrac{\pi(5.15..)^3}{2}$

i.e. Maximum volume is 429.19… = 429 (cm^3) A1 2

[11]

1. To establish the x-coordinate of the maximum turning point in part (a), it was necessary to differentiate and to use $\dfrac{dy}{dx} = 0$. Most candidates realised the need to differentiate, but the use of the zero was not always clearly shown.

Methods for finding the area in part (b) were often fully correct, although numerical slips were common. Weaker candidates often managed to integrate and to use the limits 0 and 2, but were then uncertain what else (if anything) to do. There were some attempts using y coordinates as limits. While the most popular method was to simply subtract the area under the curve from the area of the appropriate rectangle, integrating $24 - (x^3 - 10x^2 + 28x)$ between 0 and 2 was also frequently seen. Occasional slips included confusing 24 (the y-coordinate of P) with 28, subtracting 'the wrong way round' and failing to give the final answer as an exact number.

2. (a) A pleasing majority of the candidates were able to differentiate these fractional powers correctly, but a sizeable group left the constant term on the end. They then put the derivative equal to zero. Solving the equation which resulted caused more problems as the equation contained various fractional powers. Some tried squaring to clear away the fractional powers, but often did not deal well with the square roots afterwards. There were many who expressed $6x^{-1/2} = 1/(6x^{1/2})$ and tended to get in a muddle after that. Those who took out a factor $x^{1/2}$ usually ended with $x = 0$ as well as $x = 4$ and if it was not discounted, they lost an accuracy mark. Those who obtained the solution $x = 4$ sometimes neglected to complete their solution by finding the corresponding y value. Some weaker candidates did not differentiate at all in part (a), with some integrating, and others substituting various values into y.

(b) The second derivative was usually correct and those who had made a slip earlier by failing to differentiate 10, usually differentiated it correctly this time!

(c) Candidates needed to have the correct second derivative to gain this mark. As the derivative was clearly negative this mark was for just stating that the turning point was a maximum.

3. Many candidates had difficulty in their attempts to establish the given result for the surface area in part (a) of this question. Solutions often consisted of a confused mass of formulae, lacking explanation of whether expressions represented length, area or volume. Formulae for arc length and sector area usually appeared at some stage, but it was often unclear how they were being used and at which point the substitution $\theta = 1$ was being made. It was, however, encouraging to see well-explained, clearly structured solutions from good candidates.

Having struggled with part (a), some candidates disappointingly gave up. The methods required for the remainder of the question were, of course, more standard and should have been familiar to most candidates.

In part (b), most candidates successfully differentiated the given expression then formed an equation in r using $\frac{dS}{dr} = 0$. While many solved $2r - \frac{1800}{r^2} = 0$ successfully, weaker candidates were sometimes let down by their algebraic skills and could not cope correctly with the negative power of r. A common slip was to proceed from $r^3 = 900$ to $r = 30$.

In part (c), the majority of candidates correctly considered the sign of the second derivative to establish that the value of S was a minimum, although occasionally the second derivative was equated to zero.

Those who proceeded as far as part (d) were usually able to score at least the method mark, except when the value of r they substituted was completely inappropriate, such as the value of the second derivative.

4. Part (a) required a proof. Common mistakes in the formula for the surface areas were to omit either one or both ends. Algebraic mistakes caused problems with rearranging to make h the subject and some candidates did not know the volume formula. This part was often not attempted or aborted at an early stage.

Parts (b) and (c) were answered well. Most candidates knew that they should differentiate and equate to zero although many could not manage to correctly evaluate r (poor calculator work) and it was common to forgot to evaluate V. Part (c) was often incorporated in (b) (and vice versa!), but generally contained all the elements necessary to score both marks. Most solutions used the second derivative here and there were relatively few of the alternative methods of determining a maximum point. Only a few candidates were unsure of the procedures for establishing the nature of stationary points.

5. For the better candidates this was a very good source of marks, but it proved quite taxing for many of the candidates who were able to spend time on the question. In part (a) the $2x^2$ term in the given answer was usually produced $\frac{300}{x}$ but the work to produce was often unconvincing, and it was clear that the given answer, which was an aid for subsequent parts, enabled many candidates to gain marks that otherwise would have been lost. It was common to see steps retraced to correct an initial wrong statement, such as $A = 2x^2 + 4xy$, but sometimes the resulting presentation was not very satisfactory and often incomplete, and the ability to translate "the capacity of the tank is 100m^3" into an algebraic equation was quite often lacking.

In part (b) the two most common errors were in differentiating $\frac{300}{x}$, often seen as 300 or –300, and in solving the correct equation $-\frac{300}{x^2} + 4x = 0$. It was surprising, too, to see so many candidates who, having successfully reached the stage $4x^3 = 300$, gave the answer $x = 8.66$, i.e. $\sqrt{75}$

In part (c) the most common approach, by far, was to consider $\frac{d^2A}{dx^2}$, and although the mark scheme was kind in some respects, it was expected that the sign, rather than just the value, of $\frac{d^2A}{dx^2}$ was commented upon.

The method mark in the final part was usually gained although there was a significant minority of candidates who substituted their value of $\frac{d^2A}{dx^2}$, rather than their answer to part (b), into the expression for A.

6. Responses to this question that were blank or lacking in substance suggested that some candidates were short of time at the end of the examination. Although many good solutions were seen, it was common for part (b) to be incomplete.
The algebra in part (a) was challenging for many candidates, some of whom had difficulty in writing down an expression for the total surface area of the brick and others who were unable to combine this appropriately with the volume formula. It was common to see several attempts at part (a) with much algebraic confusion.
Working with the given formula, most candidates were able to score the first three marks in part (b), but surprisingly many, having found $x \approx 7.1$, seemed to think that this represented the maximum value of V. Failing to substitute the value of x back into the volume formula lost them two marks.
Almost all candidates used the second derivative method, usually successfully, to justify the maximum value in part (c), but conclusions with a valid reason were sometimes lacking.

7. Some candidates did not understand the need to differentiate in part (a) and put $C = 0$. This should have resulted in an equation with no real values of v. However, these candidates often employed some creative algebra to obtain an answer of $v = 70$. Most candidates attempted to solve $\frac{\mathrm{d}C}{\mathrm{d}v} = 0$ but a few had difficulty rearranging their equation in a form from which they could find a value of v. Most of the errors seen came from incorrect differentiation of $\frac{2v}{7}$. A few candidates attempted a solution by trial and improvement for which only two out of five marks were available. Most candidates used the correct method to find the differential in (b); the most common error was to give $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ as $\pm 1400v^{-3}$. Some candidates lost the accuracy mark in (b) because they neither substituted their value of v from (a) nor gave any other convincing indication as to why $\frac{\mathrm{d}^2C}{\mathrm{d}v^2} > 0$ (e.g. $v > 0$ as speed). Part (c) was usually done well.

8. In general, candidates scored well on parts (a) and (c) of this question, usually managed part (b), but struggled with part (d). Most knew the method for part (a), and were able to differentiate correctly and solve the appropriate quadratic equation. Although part (b) asked for the value of the second derivative at A, some candidates equated the derivative $6x - 16$ to zero, solved this equation and then tried to use this result to justify the maximum. The vast majority of candidates were successful in part (c), performing the indefinite integration. Many marks were lost, however, in part (e), where candidates often had little idea how to calculate the required area. A common approach was to use limits 0 to $\frac{10}{3}$ (rather than 0 to 2), and those who continued often seemed confused as to which area they should subtract. Some supported their arguments with reference to the diagram, but more often than not triangles (or trapezia) being used were not clearly identified. For some, working was further complicated by their decision to find the area of a triangle by using the equation of a straight line, and integrating. A few produced very clear, concise and accurate methods, which were a pleasure to mark amidst the convoluted efforts of the majority.

9. The first part of this question gave difficulty to many. There are a number of possible approaches but many just wrote down $3x^{\frac{1}{2}} - x^{\frac{3}{2}} = 3\sqrt{3} - 3^{\frac{3}{2}} = 0$ and this was thought inadequate unless they could show that $3^{\frac{3}{2}}$ or $\sqrt{27}$ was $3\sqrt{3}.3^{\frac{3}{2}} = 3 \times 3^{\frac{1}{2}}$ would have been sufficient demonstration of this. In part (b), not all could solve $\frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$ and a few found the second derivative and equated that to zero. In part (c), most could gain the first four of the five marks available but cleaning up the final answer to the single surd, $\frac{12}{5}\sqrt{3}$ or its equivalent, proved difficult and many had recourse to their calculators, which did not fulfil the condition of the question that an exact answer is to be given.

10. The differentiation in this question was answered very well in deed with over 95% scoring full marks in parts (a) and (d). In part (b) most candidates set their answer from part (a) equal to zero and proceeded to solve. Some tried to use the quadratic formula and occasionally ran into difficulties with the double minus signs but most found correct values for *x*. The *y* coordinates caused problems for some. There were a few problems with the arithmetic but others simply substituted back in their expression for $\frac{dy}{dx}$ and (usually), of course, obtained 0. Most knew how to use their second derivative to determine the nature of their turning points, there was some poor arithmetic but the method was usually demonstrated clearly. A common error in part (d) was to put their second derivative equal to zero and solve for *x*. A few candidates seemed not to know, or chose not to use, the conventional terms "maximum" and "minimum" and unsatisfactory phrases such as "turning up" or "it's got a hill" were seen instead.

11. This was often a very good source of marks for candidates, with many candidates scoring full marks. In part (a) the majority of candidates realised that differentiation was required and were able to complete the solution, although it was clear that some candidates were helped by the answer being given. However, solutions such as $x^{\frac{1}{2}} = 2 \Rightarrow x = \frac{2}{\frac{1}{2}} = 4$ were seen.

The mark scheme was quite generous in part (b) for finding the area under the curve, but in general the integration was performed well. This helped candidates score well here, even if they did not have a complete method to find the required area, or made mistakes in finding the area of the trapezium.

Candidates who found the equation of the line *AB* (some did as a matter of course but then did not use it) in order to find the area under the line, or to find the required area using a single integral, clearly made the question harder, more time consuming and open to more errors, in this case. Errors in finding the equation of the line were quite common; usually these occurred in finding the gradient or in manipulating the algebra, but it was not uncommon to see a gradient of –3 used from $\frac{dy}{dx}$ in part (a) with $x = 1$ substituted.

12. There were many completely correct answers to this question. Most candidates used the expected method of equating $\frac{dy}{dx}$ to zero, but occasional solutions based on the symmetry of the quadratic curve were seen. Weaker candidates sometimes found the points of intersection of the curve with the *x*-axis but could not proceed any further.

13. (a) B1 seemed to be the most common mark for this part for the 8x. Many had no idea how to deal with the other part – some tried to multiply out but got confused by the indices. Others did successfully use the quotient rule, and some were able to deal with the differentiation correctly.

(b) The first was given often, and quite a few gained the second as well. The most common mistakes here were to have +½ as an answer, or $^3\sqrt{5/8}$ from incorrect working in (a)

(c) The majority of candidates used the second differential method rather than considering the sign of dy/dx either side of the turning point. Some sadly didn't have a value for x to substitute, and others didn't have an x term in their expression for $\frac{d^2y}{dx^2}$. (The most common wrong answer was just $\frac{d^2y}{dx^2}$=8). If the candidate had parts (a) and (b) correct, they generally had (c) correct as well.

14. Part (a) was the worst answered part of the paper. Writing down a correct expression for the perimeter of the stage caused many problems: some had a 4*x* term whilst others thought the radius of the semicircle was 2*x*. Those that used correct expressions for both area and perimeter could often proceed to the printed result but some incurred sign errors on the way. The remainder of the question was handled quite well with most showing a clear understanding of the methods required. There was some poor algebraic manipulation in part (b) and this cost many candidates several accuracy marks, $x = \frac{80}{4-\pi}$ was a common incorrect answer. Most used the second derivative, with a comment, in part (c) as intended although some successfully examined the gradient either side of their stationary point. Whether through tiredness or genuine confusion, several candidates substituted an incorrect value in part (d), usually the value for their second derivative from part (c), and a number of candidates forgot to square their value in the second term of the expression for the area. Most of those who got to part (d) rounded to the nearest m^2 as requested.

15. Some of the solutions to this question were excellent, but others showed little understanding of the required mathematical techniques. While most candidates coped well with parts (a) and (b), some did not know the necessary formulae for the volume and surface area of a cylinder, so were unable to make progress.

Differentiation in part (c) was usually good, but then there were sometimes problems in coping with the algebra required to solve $1560r^{-2} + 4\pi r = 0$.

Part (d), in which candidates had to establish that *A* was a minimum, was often well done, although the significance of the second derivative was sometimes confused. To score full marks on this part of the question, it was necessary to produce a correct second derivative, then to proceed to a conclusion with a valid reason $\left(\frac{d^2A}{dr^2} > 0\right)$.

For the method of considering the sign of the gradient on either side of the turning point, examiners would expect to see evidence of calculation of the value of the gradient. Most candidates with a value for *r* (even when the value was negative) were able to attempt part (e), although numerical slips were not uncommon.

16. The given answer in part (a) enabled the vast majority of candidates to score the available marks without too much difficulty, and then most were able to differentiate successfully in part (b), where the most common mistake was to have zero as the derivative of *x*. Part (c) of this question proved the most demanding. Most candidates appeared to understand the meaning of "stationary point" and equated their derivative to zero, but rather than simply verifying that $\sqrt{3}$ and $-\sqrt{3}$ satisfied this equation, they tried to solve the resulting quartic equation. Some such attempts were successful but many others floundered. Sometimes the verification approach omitted $-\sqrt{3}$ and scored only one of the two available marks. Many recovered in part (d), finding the second derivative and correctly determining that $x = \sqrt{3}$ gave a minimum value. It should be noted here that examiners expect to see evidence of calculation of the value of the second derivative rather than the simple statement "positive, therefore minimum".

17. Although some weaker candidates made little progress with this question, most were able to pick up easy marks, particularly in parts (a) and (d). The differentiation in part (a) was completed correctly by the vast majority. In parts (b) and (c) however, many solutions showed evidence of confusion. Although a few candidates equated the *second* derivative to zero in part (b), methods were usually correct, and most candidates were able to solve the resulting quadratic equation to give $x = \frac{5}{3}$ or $x = 3$. Some then seemed to think that the point *P* had *x*-coordinate $\frac{5}{3}$ and the rest of their solution was similarly confused. Others began by finding the *y*-coordinate of *P* (12) and assuming that *PQ* was parallel to the *x*-axis, without addressing the fact that *Q* was a minimum. In part (c), having to show *PQ* to be parallel to the *x*-axis proved a little confusing for some, but most managed to explain about equal *y*-coordinates or equal gradients. Candidates should note that in a "show that…" question, a conclusion is expected. Most candidates demonstrated correct integration techniques in part (d), and most also subtracted the area of the appropriate rectangle. Apart from slips in calculation, there were many successful attempts to find the required area, scoring good marks.

18. Although there were many very good solutions to this question, a large number of candidates failed to cope with the parts requiring applications of differentiation.

Most scored the mark in part (a), either by solving an equation or by verification, for showing that $p = 6$, although arguments were occasionally incomplete.

Differentiation was required to answer parts (b) and (c) and most candidates scored the two marks for a correct derivative, seen in either of these parts. A significant number omitted part (b). Some found the gradient at *A* but did not proceed to find the equation of the tangent, some found the gradient of the normal, and some gave a non-linear tangent equation, failing to evaluate the derivative at $x = 6$. There was rather more success in part (c), although a common mistake here was to equate the second derivative to zero in the attempt to find the maximum turning point.

In part (d), most candidates appreciated the need for definite integration, and many completely correct solutions were seen. A few, however, used $x = 4$ (presumably taken from part (c)) instead of $x = 6$ as the upper integral limit.

19. Although some candidates produced excellent solutions to this question, it was clear that others were lacking in confidence or unfamiliar with this type of problem.

Formulae for the volume and/or surface area of a cylinder were not always known, so candidates sometimes struggled to make progress with part (a). Most knew that differentiation was required in part (b), however, the main problem here being the negative powers (either in differentiation or in solving the "derivative = 0" equation).

"Trial and improvement" was rare, and where it occurred candidates were given no credit in part (b), but were allowed to score marks for valid arguments and answers in (c) and (d). The second derivative approach in part (c) was successful for many candidates, and most then scored at least the method mark in part (d) for their attempt to calculate the minimum value of *A*.

20. No Report available for this question.

21. No Report available for this question.