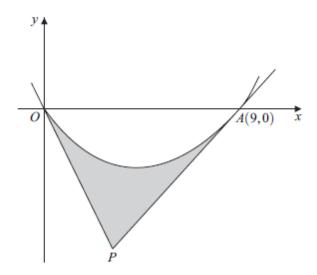
## **Differentiation Questions**

1 Given that  $y = 16x + x^{-1}$ , find the two values of x for which  $\frac{dy}{dx} = 0$ . (5 marks)

8 A curve, drawn from the origin O, crosses the x-axis at the point A(9,0). Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve, defined for  $x \ge 0$ , has equation

$$y = x^{\frac{3}{2}} - 3x$$

(a) Find  $\frac{dy}{dx}$ . (2 marks)

(b) (i) Find the value of  $\frac{dy}{dx}$  at the point O and hence write down an equation of the tangent at O. (2 marks)

(ii) Show that the equation of the tangent at A(9, 0) is 2y = 3x - 27. (3 marks)

(iii) Hence find the coordinates of the point P where the two tangents meet. (3 marks)

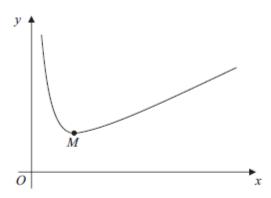
(c) Find  $\int \left(x^{\frac{3}{2}} - 3x\right) dx$ . (3 marks)

(d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP. (5 marks) 7 At the point (x, y), where x > 0, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

- (a) (i) Verify that  $\frac{dy}{dx} = 0$  when x = 4. (1 mark)
  - (ii) Write  $\frac{16}{x^2}$  in the form  $16x^k$ , where k is an integer. (1 mark)
  - (iii) Find  $\frac{d^2y}{dx^2}$ . (3 marks)
  - (iv) Hence determine whether the point where x = 4 is a maximum or a minimum, giving a reason for your answer. (2 marks)
- (b) The point P(1, 8) lies on the curve.
  - (i) Show that the gradient of the curve at the point P is 12. (1 mark)
  - (ii) Find an equation of the normal to the curve at P. (3 marks)
- (c) (i) Find  $\int (3x^{\frac{1}{2}} + \frac{16}{x^2} 7) dx$ . (3 marks)
  - (ii) Hence find the equation of the curve which passes through the point P(1, 8).(3 marks)

6 A curve C is defined for x > 0 by the equation  $y = x + 1 + \frac{4}{x^2}$  and is sketched below.



(a) (i) Given that 
$$y = x + 1 + \frac{4}{x^2}$$
, find  $\frac{dy}{dx}$ . (3 marks)

- (ii) The curve C has a minimum point M. Find the coordinates of M. (4 marks)
- (iii) Find an equation of the normal to C at the point (1, 6). (4 marks)

(b) (i) Find 
$$\int \left(x+1+\frac{4}{x^2}\right) dx$$
. (3 marks)

- (ii) Hence find the area of the region bounded by the curve C, the lines x = 1 and x = 4 and the x-axis. (2 marks)
- 5 A curve is defined for x > 0 by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point P lies on the curve where x = 2.

- (a) Find the y-coordinate of P. (1 mark)
- (b) Expand  $\left(1+\frac{2}{x}\right)^2$ . (2 marks)

(c) Find 
$$\frac{dy}{dx}$$
. (3 marks)

- (d) Hence show that the gradient of the curve at P is −2. (2 marks)
- (e) Find the equation of the normal to the curve at P, giving your answer in the form x + by + c = 0, where b and c are integers. (4 marks)

## **Differentiation Answers**

	Solution	Marks	Total	Comments
1	$y'(x) = 16 - x^{-2}$	M1		One term correct
		A1		Both correct
	$y'(x) = 16 - \frac{1}{x^2}$	B1		$x^{-2} = \frac{1}{x^2} \text{ OE PI}$
	$y'(x) = 0 \Rightarrow 16x^2 = 1;$ $\Rightarrow x = \pm \frac{1}{x}$	M1		c's $y'(x)=0$ and one relevant further step
	4	A1	5	Both answers required.
	Total		5	

8(a) $\frac{3y}{dx} = \frac{2}{2}x^{2} - 3$ Al 2 Both correct  (b)(i) When $x = 0$ , $\frac{dy}{dx} = -3$ Eqn of tangent at $O$ is $y = -3x$ B1F $\checkmark$ Eqn tangent at $A$ is $y = 0 = y'(9)[x - 9]$ $\Rightarrow y = \frac{3}{2}(x - 9) \Rightarrow 2y = 3x - 27$ B1F $\checkmark$ Sign tangent at $A$ is $y = 0 = y'(9)[x - 9]$ $\Rightarrow y = \frac{3}{2}(x - 9) \Rightarrow 2y = 3x - 27$ Al 3 CSO. AG  (iii) Eliminating $y \Rightarrow -6x = 3x - 27$ $9x = 27 \Rightarrow x = 3$ When $x = 3$ , $y = -9$ . { $F(3, -9)$ }  (c) $\int (x^{\frac{3}{2}} - 3x) dx = \frac{2}{5}x^{\frac{3}{2}} - \frac{3x^{2}}{2}$ (+c)  M1 A2,1,0  A1 B1F $\checkmark$ Cone power correct Condone absence of "+c" and unsimplified forms  (d) $\int_{0}^{y} (x^{\frac{3}{2}} - 3x) dx = \frac{1}{2}x + 9x = 1$ A1 B1 Correct use of limits following integration  Sh. Area of triangle $OPA = \frac{1}{2}x + 9x = 1$ A1 B1 OE  A2 B1F $\checkmark$ Correct use of limits following integration  OE  OE  The provided answer < 0.  OE Ft on $y'(0)$ Attempt to find $y'(9)$ OE  CSO. AG  OE method to one variable (eg $2y = -y - 27$ )  [A1F for each coordinate; only ft on $y = kx$ tangent in (b)(i) for $k < 0$ ]  The provided answer < 0.  OE  The provided answer < 0.  The		4. 2.1	M1		One term correct
(b)(i) When $x = 0$ , $\frac{dy}{dx} = -3$ Eqn of tangent at $O$ is $y = -3x$ B1F $\checkmark$ 2 OE Ft on $y'(0)$ (ii) At $(9,0)$ $\frac{dy}{dx} = \frac{3}{2}(9)^{\frac{1}{2}} - 3$ M1 Attempt to find $y'(9)$ OE $\Rightarrow y = \frac{3}{2}(x - 9) \Rightarrow 2y = 3x - 27$ A1 3 CSO. AG  (iii) Eliminating $y \Rightarrow -6x = 3x - 27$ M1 OE method to one variable $(eg \ 2y = -y - 27)$ [A1F for each coordinate; only ft on $y = x$ 0 tangent in (b)(i) for $k < 0$ ]  (c) $\int \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{2}{5}x^{\frac{3}{2}} - \frac{3x^2}{2}$ (+c) M1 A2,1,0 3 One power correct Condone absence of "+c" and unsimplified forms  (d) $\int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{2}{5}x^{\frac{3}{2}} - \frac{3}{2}x^{9} - 0$ M1 Correct use of limits following integration $\frac{3}{2}$ Area of triangle $OPA = \frac{1}{2}x > 9 \times  y_p $ M1 Sh. Area $= \frac{1}{2}x > 9 \times  y_p  -  \int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx $ M1 OE $= 40.5 - 24.3 = 16.2$	8(a)	$\frac{dy}{dy} = \frac{3}{5}x^{\frac{1}{2}} - 3$		2	
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(ii) At $(9,0) \frac{dy}{dx} = \frac{3}{2}(9)^{\frac{1}{2}} - 3$ M1 Eqn tangent at $A$ is $y - 0 = y'(9)[x - 9]$ m1 OE $y = \frac{3}{2}(x - 9) \Rightarrow 2y = 3x - 27$ M1 OE method to one variable $(eg \ 2y = -y - 27)$ [AIF for each coordinate; only ft on $y = kx$ tangent in (b)(i) for $k < 0$ ]  (c) $\int \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{2}{5}x^{\frac{3}{2}} - \frac{3x^2}{2}$ (+c) M1 A2,1,0 3 One power correct Condone absence of "+c" and unsimplified forms  (d) $\int_{0}^{y} \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{1}{2}x + y = 0$ M1 Correct use of limits following integration $x = 2x + 3x = 0$ M1 Correct use of limits following integration $x = 2x + 3x = 0$ M1 OE		u.			
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(iii) Eliminating $y\Rightarrow -6x=3x-27$ M1 OE method to one variable (eg $2y=-y-27$ ) [A1F for each coordinate; only ft on $y=kx$ tangent in (b)(i) for $k<0$ ]  (c) $\int \left(x^{\frac{3}{2}}-3x\right) dx = \frac{2}{5}x^{\frac{3}{2}} - \frac{3x^2}{2}$ (+c) M1 A2,1,0 3 One power correct Condone absence of "+c" and unsimplified forms  (d) $\int_{0}^{x} \left(x^{\frac{3}{2}}-3x\right) dx = \frac{2}{5}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{3}{2}} = 0$ M1 Correct use of limits following integration $x=2$ and $x=2$ and $x=3$ $x=2$ $x=3$	( <b>n</b> )	$\frac{dx}{dx} = \frac{2}{2}$	MI		Attempt to find y (9)
(iii) Eliminating $y \Rightarrow -6x = 3x - 27$ M1 $A1F$ $9x = 27 \Rightarrow x = 3$ A1F $A1F$ When $x = 3$ , $y = -9$ . $\{P(3, -9)\}$ A1F $A1F$ $A1$		Eqn tangent at A is $y-0=y'(9)[x-9]$	ml		OE
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$9x = 27 \Rightarrow x = 3$ $\text{When } x = 3, \ y = -9. \ \{P(3, -9)\}$ $(c) \int \left(\frac{x^{\frac{3}{2}} - 3x}{2}\right) dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{3x^2}{2} \ (+c)$ $(d) \int \left(\frac{x^{\frac{3}{2}} - 3x}{2}\right) dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{3x^2}{2} \ (+c)$ $= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$ $Area of triangle OPA = \frac{1}{2} \times 9 \times  y_p  = 40.5 - 24.3 = 16.2 A1F A1F$	(iii)	Eliminating $y \Rightarrow -6x = 3x - 27$	M1		1
When $x = 3$ , $y = -9$ . $\{P(3, -9)\}$ A1F  3 $y = kx \text{ tangent in (b)(i) for } k < 0$ ]  (c) $\int \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{3x^2}{2}$ (+c)  M1 A2,1,0  3  One power correct Condone absence of "+c" and unsimplified forms  B1  PI $= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ M1 Sh.Area $= \frac{1}{2} \times 9 \times  y_p  -  \int_0^{\pi} \left(x^{\frac{3}{2}} - 3x\right) dx \right $ M1 Sh.Area $= \frac{1}{2} \times 9 \times  y_p  -  \int_0^{\pi} \left(x^{\frac{3}{2}} - 3x\right) dx \right $ M1 OE					
When $x = 3$ , $y = -9$ . $\{P(3, -9)\}$ A1F  3  (c) $\int \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{3x^2}{2}$ (+c)  M1 A2,1,0  3  One power correct Condone absence of "+c" and unsimplified forms  B1  PI $= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ M1  Sh.Area $= \frac{1}{2} \times 9 \times  y_p  -  \int_0^{9} \left(x^{\frac{3}{2}} - 3x\right) dx $ M1  One power correct Condone absence of "+c" and unsimplified forms  PI  Correct use of limits following integration  OE		$9x = 27 \implies x = 3$	A1F		
(c) $\int \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{3x^2}{2} \text{ (+c)}$ $M1$ $A2,1,0$ $3$ One power correct Condone absence of "+c" and unsimplified forms  PI $= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ $Sh.Area = \frac{1}{2} \times 9 \times  y_p  -  \int_0^p \left(x^{\frac{3}{2}} - 3x\right) dx $ $= 40.5 - 24.3 = 16.2$ M1 One power correct Condone absence of "+c" and unsimplified forms  One power correct Condone absence of "+c" and unsimplified forms  One power correct Condone absence of "+c" and unsimplified forms  One power correct Condone absence of "+c" and unsimplified forms  One power correct Condone absence of "+c" and unsimplified forms  One power correct Condone absence of "+c" and unsimplified forms  One power correct Condone absence of "+c" and unsimplified forms  One power correct Condone absence of "+c" and unsimplified forms  One power correct Condone absence of "+c" and unsimplified forms		Water 2 0 (D/2 0))	AIT	2	y = kx tangent in (b)(1) for $k < 0$ ]
(d) $\int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx = B1$ $= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^{2} - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_{p} $ Sh.Area $= \frac{1}{2} \times 9 \times  y_{p}  -  \int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx $ M1 OE $= 40.5 - 24.3 = 16.2$ A2,1,0 $= B1$ M1 Correct use of limits following integration $= Correct use of limits following integration$ OE		when $x = 3$ , $y = -9$ . $\{P(3, -9)\}$	AIF	3	
(d) $\int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx = B1$ $= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^{2} - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_{p} $ Sh.Area $= \frac{1}{2} \times 9 \times  y_{p}  -  \int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx $ M1 OE $= 40.5 - 24.3 = 16.2$ A2,1,0 $= B1$ M1 Correct use of limits following integration $= Correct use of limits following integration$ OE		(3) 25 2-2			
(d) $\int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx = B1$ $= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^{2} - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_{p} $ Sh.Area $= \frac{1}{2} \times 9 \times  y_{p}  -  \int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx $ M1 OE $= 40.5 - 24.3 = 16.2$ A2,1,0 $= B1$ M1 Correct use of limits following integration $= Correct use of limits following integration$ OE	(c)	$\int  x^{2}-3x  dx = \frac{2}{5}x^{2} - \frac{3x}{2}$ (+c)	M1		One power correct
(d) $\int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx = B1$ $= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^{2} - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_{p} $ $Sh.Area = \frac{1}{2} \times 9 \times  y_{p}  -  \int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx $ $= 40.5 - 24.3 = 16.2$ B1 Correct use of limits following integration $M1$ OE		3 ( ) 3 2	A2,1,0	3	
$= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ $Sh. Area = \frac{1}{2} \times 9 \times  y_p  -  \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx $ $= 40.5 - 24.3 = 16.2$ M1 Correct use of limits following integration $M1$ OE					and unsimplified forms
$= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ $Sh.Area = \frac{1}{2} \times 9 \times  y_p  -  \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx $ $= 40.5 - 24.3 = 16.2$ M1 Correct use of limits following integration $M1$ OE					
$= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ $Sh.Area = \frac{1}{2} \times 9 \times  y_p  -  \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx $ $= 40.5 - 24.3 = 16.2$ M1 Correct use of limits following integration $M1$ OE		$\int_{1}^{9} \left( \frac{3}{\sqrt{2}} - 2 \right) dv =$			
$= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ $Sh.Area = \frac{1}{2} \times 9 \times  y_p  -  \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx $ $= 40.5 - 24.3 = 16.2$ M1 Correct use of limits following integration $M1$ OE	(d)	$\int_{0}^{\pi} \left( x^{2} - 5x \right) dx =$	Bl		PI
$= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ M1 $Sh.Area = \frac{1}{2} \times 9 \times  y_p  -  \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx  $ M1 $= 40.5 - 24.3 = 16.2$ A1 $5$					
$= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ M1 $Sh.Area = \frac{1}{2} \times 9 \times  y_p  -  \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx  $ M1 $= 40.5 - 24.3 = 16.2$ A1 $5$		$=\frac{2}{5} \times 9^{\frac{1}{2}} - \frac{3}{2} \times 9^2 - 0$	M1		Correct use of limits following integration
Area of triangle $OPA = \frac{1}{2} \times 9 \times  y_p $ M1  Sh. Area $= \frac{1}{2} \times 9 \times  y_p  -  \int_{0}^{9} \left(x^{\frac{3}{2}} - 3x\right) dx$ M1  OE  = 40.5 - 24.3 = 16.2 A1 5		J 2			
Sh.Area = $\frac{1}{2} \times 9 \times  y_p  -  \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx$   M1 OE = $40.5 - 24.3 = 16.2$ A1 5		24.3			
Sh.Area = $\frac{1}{2} \times 9 \times  y_p  -  \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx$   M1 OE = $40.5 - 24.3 = 16.2$ A1 5		Area of triangle $OPA = \frac{1}{2} \times 9 \times  v_{-} $	3.00		
= 40.5 - 24.3 = 16.2 A1 5		2 1 17 1	MI		
= 40.5 - 24.3 = 16.2 A1 5		Sh Arm $= \frac{1}{2} \times 0 \times 10^{-1} = \frac{9}{2} \left( \frac{3}{2^2} + 20 \right) = \frac{1}{2} \left( \frac{3}{2^2} + 20 \right$	3.01		05
AI 3		$\int_{0}^{3\pi} A_{1} dx = \frac{1}{2} A_{2} A_{1} A_{2} A_{1} A_{2} A_{1} A_{2} A_{2} A_{3} A_{4} A_{4} A_{4} A_{5} A_{5$	MI		OE
AI 3					
		= 40.5 - 24.3 = 16.2	A1	5	
10001		Total		18	

uestion	Solution	Marks	Total	Comments
7(a)(i)	When $x = 4$ , $\frac{dy}{dx} = 3(2) + \frac{16}{16} - 7 = 0$	B1	1	AG Be convinced
(ii)	$\frac{16}{x^2} = 16x^{-2}$	B1	1	Accept $k = -2$
(iii)	$\frac{d^2 y}{dx^2} = 3 \times \frac{1}{2} x^{-\frac{1}{2}} + 16 \times (-2) x^{-3} - 0$	M1		A power decreased by 1
	$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{1}{2}};  -32x^{-3}$	A1; A1√	3	candidate's negative integer k [-1 for >2 term(s)]
(iv)	When $x = 4$ , $\frac{d^2y}{dx^2} = \frac{3}{4} - \frac{32}{64} = \frac{1}{4}$	M1		Attempt to find $y''(4)$ reaching as far as two simplified terms
	Minimum since $y''(4) \ge 0$	E1√	2	candidate's sign of y"(4)
	[Alternative: Finds the sign of $y'(x)$ either statement: (M1) Correct ft conclusion with $y'(4)=0$ ]			
(b)(i)	At $P(1,8)$ , $\frac{dy}{dx} = 3(1)^{\frac{1}{2}} + \frac{16}{1^2} - 7 = 12$	В1	1	AG Be convinced
(ii)	Gradient of normal = $-\frac{1}{12}$	M1		Use of or stating $m \times m' = -1$
(ii)	Gradient of normal = $-\frac{1}{12}$	M1		Use of or stating $m \times m' = -1$
	Equation of normal is $y-8=m[x-1]$	M1		Can be awarded even if m=12
	$y-8 = -\frac{1}{12}(x-1) \Rightarrow 12y-96 = -x+1$ $\Rightarrow 12y+x=97$	A1	3	Any correct form of the equation
(c)(i)				
	$\dots = 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 16\frac{x^{-1}}{-1} - 7x + c$	M1 A2,1,0	3	One power correct.  A1 if 2 of 3 terms correct candidate's negative integer k  Condone absence of "+ c"
(ii)	$y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + c \qquad (*)$	В1√		y = candidate's answer to (c)(i) with tidied coefficients and with '+c'. (' $y$ =' PI by next line)
	When $x = 1$ , $y = 8 \implies 8 = 2-16-7+c$	M1		Substitute. (1,8) in attempt to find constant of integration
	$y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + 29$	A1	3	Accept $c = 29$ after (*), including $y =$ , stated
	Total		17	

×	Solution	LVERT IND	10001	Commence
6(a)(i)	$y = x + 1 + 4x^{-2} \implies \frac{dy}{dx} = 1 - 8x^{-3}$	M1 A2,1,0	3	Power $p \rightarrow p-1$ (A1 if $1 + ax^n$ with $a = -8$ or $n = -3$ )
(ii)	$1 - 8x^{-3} = 0$	M1		Puts c's $\frac{dy}{dx} = 0$
	$x^3 = 8$	m1		Using $x^{-k} = \frac{1}{x^k}$ to reach $x^a = b$ , $a > 0$ or
	x=2	A1		correct use of logs.
	When $x = 2$ , $y = 4$	A1ft	4	
(iii)	At (1, 6), $\frac{dy}{dx} = 1 - 8 = -7$	M1		Attempt to find $y'(1)$
	Gradient of normal = $\frac{1}{7}$	M1		Use of or stating $m \times m' = -1$
	Equation of normal is $y-6=m[x-1]$	M 1		m numerical
	$y-6 = \frac{1}{7}(x-1)$ $\{\frac{y-6}{x-1} = \frac{1}{7}; \ 7y = x+41\}$	A1ft	4	OE ft on c's answer for (a)(i) provided at least A1 given in (a)(i) and previous 3M marks awarded
(b)(i)	$\int x \left( +1 + \frac{4}{x^2} \right) dx =$			
	$\dots = \frac{x^2}{2} + x - 4x^{-1} \ \{+c\}$	M1 A2,1,0	3	One of three terms correct.  For A2 need all <u>three</u> terms as printed or better  (A1 if 2 of 3 terms correct)
(ii)	{Area=} $\int_{1}^{4} x + 1 + \frac{4}{x^2} dx =$			
	$\left[\frac{x^2}{2} + x - \frac{4}{x}\right]_1^4 = (8 + 4 - 1) - \left(\frac{1}{2} + 1 - 4\right)$	M1		Dealing correctly with limits; F(4)-F(1) (must have integrated)
	= 13.5 Total	A1	2 16	
	Lotal	I	10	I

5(a)	$y_p = 4$	B1	1	
(b)	$y = 1 + \frac{2}{x} + \frac{2}{x} + \frac{4}{x^2}$ $y = 1 + 4x^{-1} + 4x^{-2}$ $\frac{dy}{dx} = -4x^{-2} - 8x^{-3}$	B2,1,0	2	(B1 if only one error in the expansion) For B2 the last line of the candidate's solution must be correct
(c)	$\frac{dy}{dx} = -4x^{-2} - 8x^{-3}$	M1 A1ft A1	3	Index reduced by 1 after differentiating x to a negative power  At least 1 term in x correct ft on expn  CSO Full correct solution. ACF
(d)	When $x = 2$ , $\frac{dy}{dx} = -4 \times 2^{-2} - 8 \times 2^{-3}$ Gradient = $-1 - 1 = -2$	M1 A1	2	Attempt to find y'(2).  AG (be convinced-no errors seen)
(e)	$-2 \times m' = -1$ $y - 4 = m(x - 2)$	M1 M1		$m_1 \times m_2 = -1$ OE stated or used. PI C's $y_P$ from part (a) if not recovered; m must be numerical.
	$y-4 = \frac{1}{2}(x-2)$ $x-2y+6=0$	A1ft A1	4	Ft on candidate's $y_P$ from part (a) if not recovered. CAO Must be this or $0 = x - 2y + 6$
	Total		12	