1. The cartesian equation of the circle C is

$$x^2 + y^2 - 8x - 6y + 16 = 0.$$

(a) Find the coordinates of the centre of C and the radius of C.

(4)

(b) Sketch C.

(2)

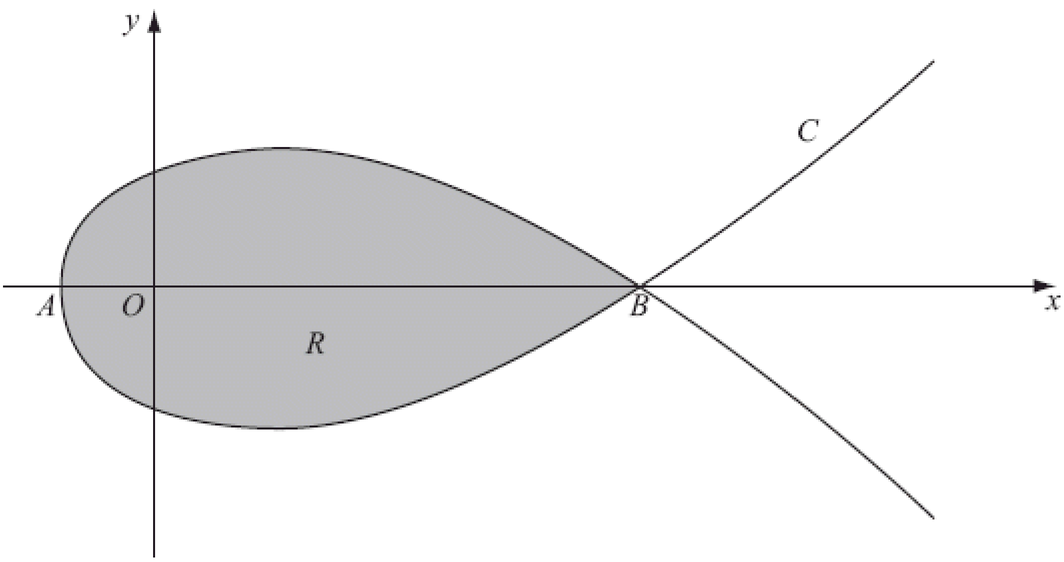
(c) Find parametric equations for C.

(3)

(d) Find, in cartesian form, an equation for each tangent to C which passes through the origin O.

(5)
(Total 14 marks)

2.



The diagram above shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

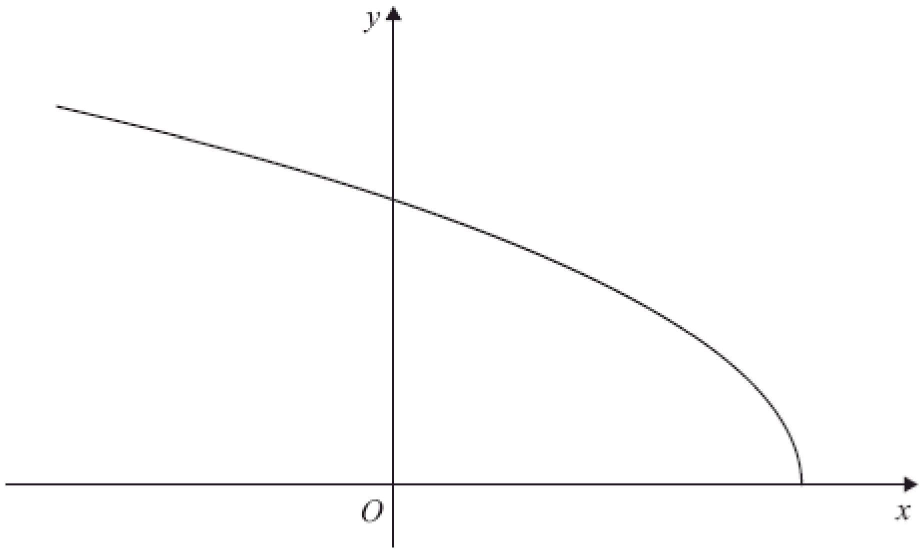
(3)

The region R, as shown shaded in the diagram above, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)
(Total 9 marks)

3.



The diagram above shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \qquad y = 6\sin t, \qquad 0 \leq t \leq \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

(b) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), -k \leq x \leq k,$$

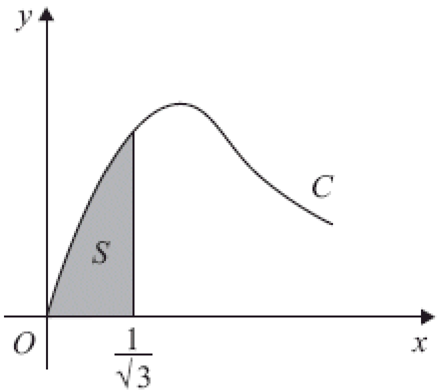
stating the value of the constant k.

(4)

(c) Write down the range of f (x).

(2)
(Total 10 marks)

4. (a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \, d\theta$.



(2)

The diagram above shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = 2\sin 2\theta, \qquad 0 \le \theta < \frac{\pi}{2}$$

The finite shaded region S shown in the diagram is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta$$

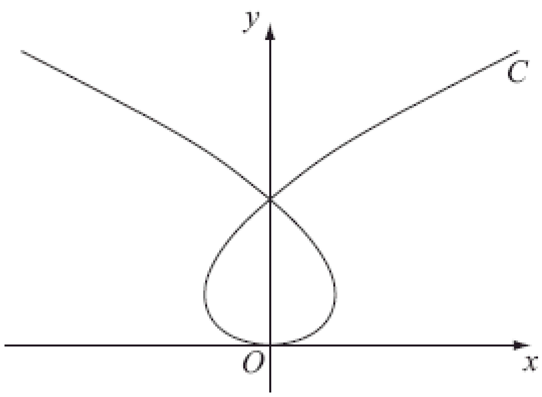
where k is a constant.

(5)

(c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)
(Total 10 marks)

5.



The curve C shown above has parametric equations

$$x = t^3 - 8t,\ y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

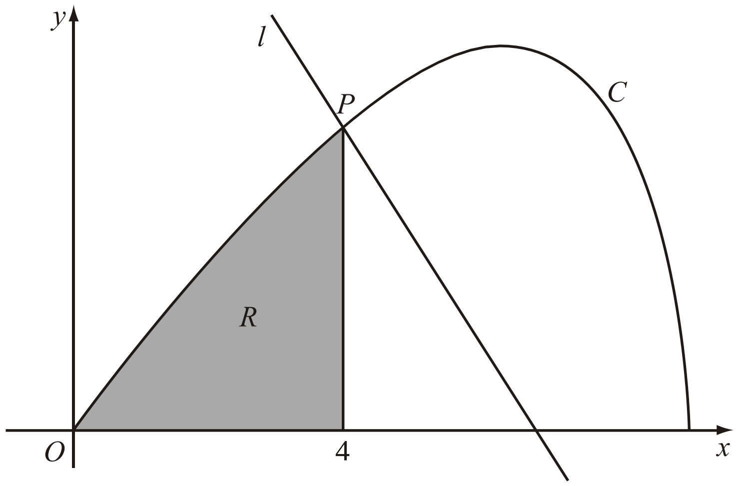
(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)
(Total 12 marks)

6.



The diagram above shows the curve C with parametric equations

$$x = 8\cos t, \quad y = 4\sin 2t, \quad 0 \le t \le \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in the diagram above.

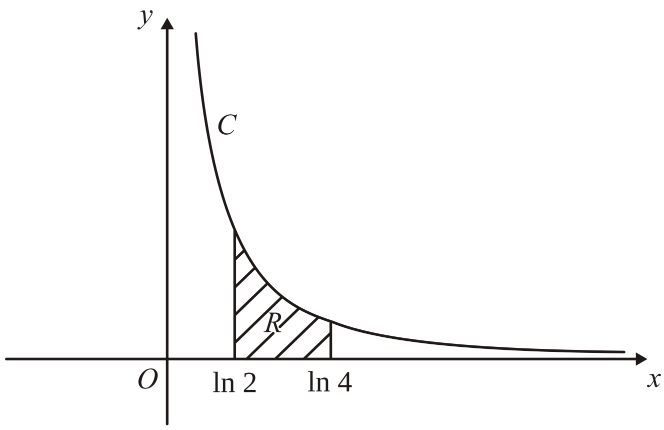
(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t\,\mathrm{d}t$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)
(Total 16 marks)

7.



The curve C has parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{(t+1)}, \quad t > -1$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = 1\text{n } 2$ and $x = 1\text{n } 4$, is shown shaded in the diagram above.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)} \, \mathrm{d}t \, .$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)
(Total 15 marks)

8. A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.

(3)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined.

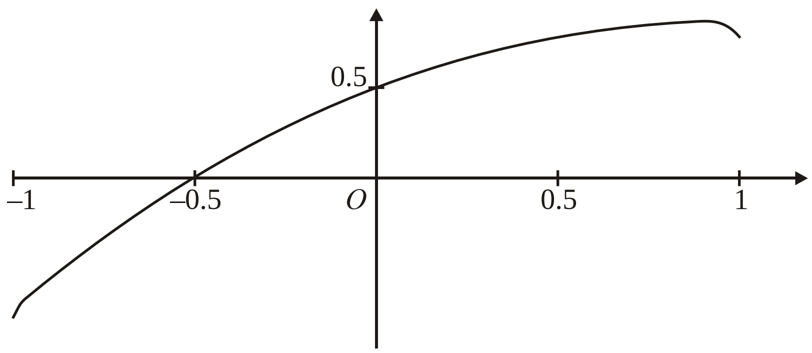
(5)

(c) Find a cartesian equation of the curve in the form $y^2 = f(x)$.

(4)

(Total 12 marks)

9.



The curve shown in the figure above has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \tfrac{\pi}{6}\right), \qquad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

(6)

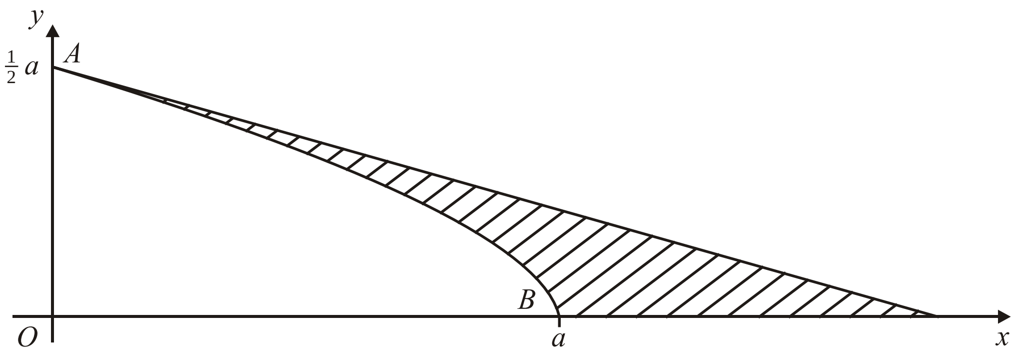
(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \quad -1 < x < 1$$

(3)

(Total 9 marks)

10.



The curve shown in the figure above has parametric equations

$$x = a\cos 3t,\ y = a\sin t,\quad 0 \le t \le \frac{\pi}{6}.$$

The curve meets the axes at points A and B as shown.

The straight line shown is part of the tangent to the curve at the point A.

Find, in terms of a,

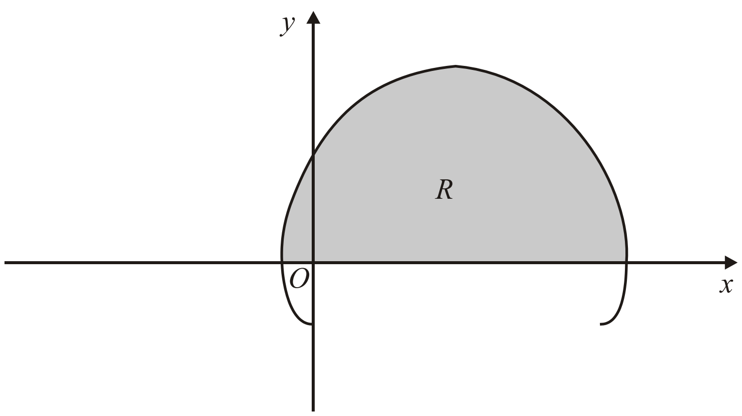
(a) an equation of the tangent at A,

(6)

(b) an exact value for the area of the finite region between the curve, the tangent at A and the x-axis, shown shaded in the figure above.

(9)
(Total 15 marks)

11.



The curve shown in the figure above has parametric equations

$$x = t - 2\sin t,\qquad y = 1 - 2\cos t,\qquad 0 \le t \le 2\pi$$

(a) Show that the curve crosses the x-axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x-axis, as shown shaded in the figure above.

(b) Show that the area of R is given by the integral

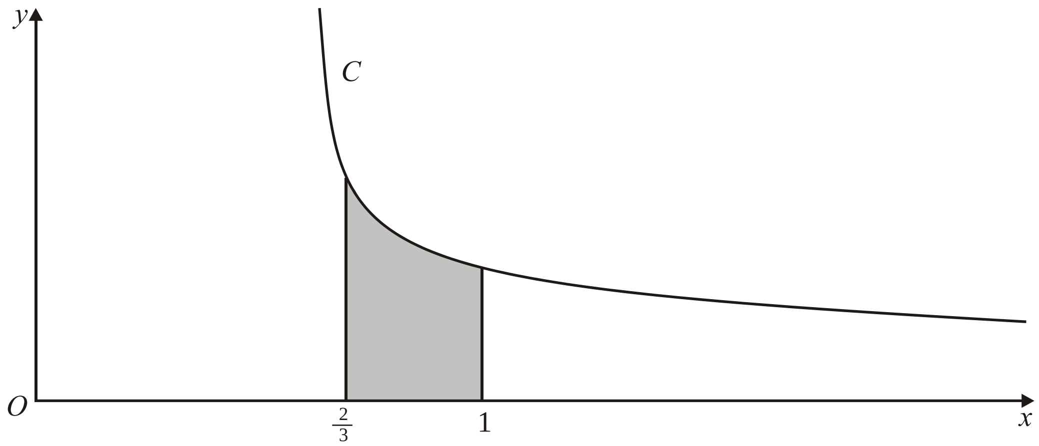
$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, dt.$$

(3)

(c) Use this integral to find the exact value of the shaded area.

(7)
(Total 12 marks)

12.



The curve C has parametric equations

$$x = \frac{1}{1+t}, \quad y = \frac{1}{1-t}, \quad |t| < 1.$$

(a) Find an equation for the tangent to C at the point where $t = \frac{1}{2}$.

(7)

(b) Show that C satisfies the cartesian equation $y = \dfrac{x}{2x-1}$.

(3)

The finite region between the curve C and the x-axis, bounded by the lines with equations $x = \frac{2}{3}$ and $x = 1$, is shown shaded in the figure above.

(c) Calculate the exact value of the area of this region, giving your answer in the form $a + b \ln c$, where a, b and c are constants.

(6)
(Total 16 marks)

13. A curve has parametric equations

$$x = 2 \cot t, \quad y = 2 \sin^2 t, \quad 0 < t \leq \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t.

(4)

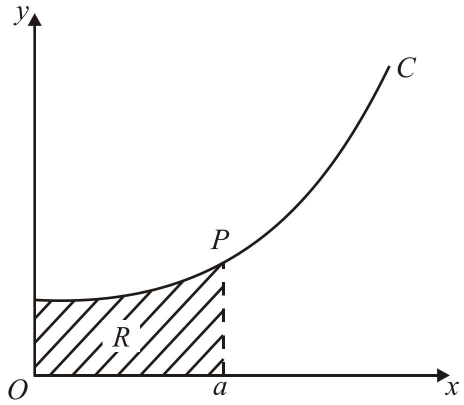
(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

(4)

(c) Find a cartesian equation of the curve in the form $y = f(x)$. State the domain on which the curve is defined.

(4)
(Total 12 marks)

14.



The diagram above shows a sketch of the curve C with parametric equations

$$x = 3t \sin t,\ y = 2 \sec t, \qquad 0 \leq t < \frac{\pi}{2}.$$

The point $P(a, 4)$ lies on C.

(a) Find the exact value of a.

(3)

The region R is enclosed by C, the axes and the line $x = a$ as shown in the diagram above.

(b) Show that the area of R is given by

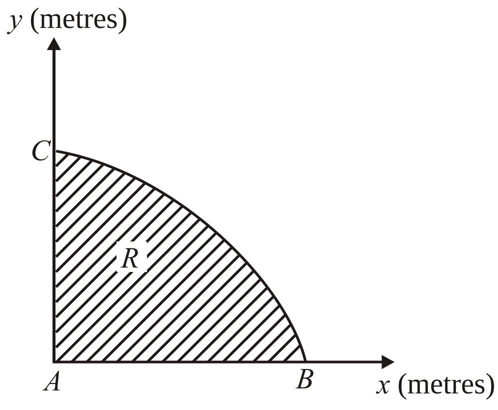
$$6\int_0^{\frac{\pi}{3}} (\tan t + t)\,dt.$$

(4)

(c) Find the exact value of the area of R.

(4)
(Total 11 marks)

15.



The diagram above shows a cross-section R of a dam. The line AC is the vertical face of the dam, AB is the horizontal base and the curve BC is the profile. Taking x and y to be the horizontal and vertical axes, then A, B and C have coordinates $(0, 0)$, $(3\pi^2, 0)$ and $(0, 30)$ respectively. The area of the cross-section is to be calculated.

Initially the profile BC is approximated by a straight line.

(a) Find an estimate for the area of the cross-section R using this approximation.

(1)

The profile BC is actually described by the parametric equations.

$$x = 16t^2 - \pi^2, \quad y = 30 \sin 2t, \quad \frac{\pi}{4} \le t \le \frac{\pi}{2}.$$

(b) Find the exact area of the cross-section R.

(7)

(c) Calculate the percentage error in the estimate of the area of the cross-section R that you found in part (*a*).

(2)
(Total 10 marks)

16. The curve C is described by the parametric equations

$$x = 3 \cos t, \quad y = \cos 2t, \quad 0 \le t \le \pi.$$

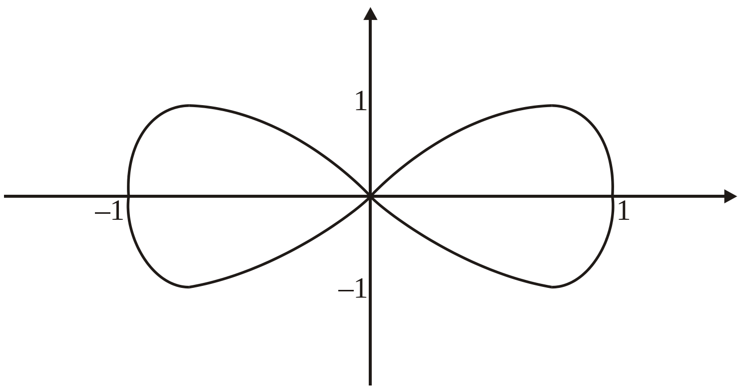
(a) Find a cartesian equation of the curve C.

(2)

(b) Draw a sketch of the curve C.

(2)
(Total 4 marks)

17.



The curve shown in the diagram above has parametric equations

$$x = \cos t,\ \ y = \sin 2t, \qquad 0 \leq t < 2\pi.$$

(a) Find an expression for $\dfrac{dy}{dx}$ in terms of the parameter t.

(3)

(b) Find the values of the parameter t at the points where $\dfrac{dy}{dx} = 0$.

(3)

(c) Hence give the exact values of the coordinates of the points on the curve where the tangents are parallel to the x-axis.

(2)

(d) Show that a cartesian equation for the part of the curve where $0 \leq t < \pi$ is

$$y = 2x\sqrt{(1 - x^2)}.$$

(3)

(e) Write down a cartesian equation for the part of the curve where $\pi \leq t < 2\pi$.

(1)

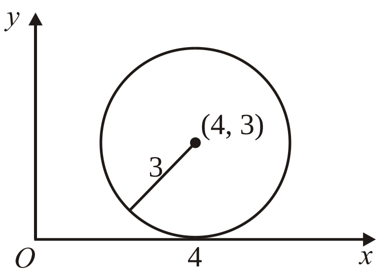
(Total 12 marks)

1. (a) $x^2 + y^2 - 8x - 6y + 16 = (x - 4)^2 - 16 + (y - 3)^2 - 9 + 16$

$(x - 4)^2 - (y - 3)^2 = 9$ A1

Centre (4, 3), radius 3 A1 A1 4

(b)

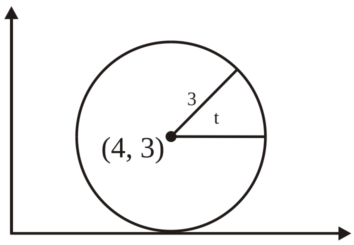


B1, B1 2

(c) $x = 4 + 3\cos t$

$y = 3 + 3\sin t$

$(0 \le t < 2\pi)$ A1 A1 3



(d) Line through origin $y = mx$

x-coordinate of points where this line cuts C satisfies

$(1 + m^2)x^2 - 8x - 6mx + 16 = 0$ A1

As line is tangent this equation has repeated roots

$(8 + 6m)^2 = 4(1 + m^2)16$

$16 + 9m^2 + 24m = 16 + 16m^2$

$24m = 7m^2$ A1

$m = 0, \; m = \dfrac{24}{7}$

Equations of tangents are $y = 0$, $y = \dfrac{24}{7}x$ A1 5

[14]

2.	(a)	$y = 0 \Rightarrow t(9 - t^2) = t(3 - t)(3 + t) = 0$			
		$t = 0, 3, –3$	Any one correct value	B1	
		At $t = 0$, $x = 5\,(0)^2 – 4 = –4$	Method for finding one value of x		
		At $t = 3$, $x = 5\,(3)^2 – 4 = 41$			
		$\left(\text{At } t = -3, x = 5(-3)^2 - 4 = 41\right)$			
		At A, $x = –4$; at B, $x = 41$	Both	A1	3
	(b)	$\dfrac{dx}{dt} = 10t$	Seen or implied	B1	
		$\int y\,dx = \int y\dfrac{dx}{dt}\,dt = \int t(9 - t^2)10t\,dt$		A1	
		$= \int (90t^2 - 10t^2)dt$			
		$\left[\dfrac{90t^3}{3} - \dfrac{10t^5}{5}\right]_0^3 = 30 \times 3^3 - 2 \times 3^5 (= 324)$			
		$A = 2\int y\,dx = 648$ (units^2)		A1	6

[9]

3.	(a)	$\dfrac{dx}{dt} = –4\sin 2t,\ \dfrac{dy}{dt}\ 6\cos t$		B1, B1	
		$\dfrac{dy}{dx} = -\dfrac{6\cos t}{4\sin 2t}\left(= -\dfrac{3}{4\sin t}\right)$			
		At t = $\dfrac{\pi}{3}$, $\quad m = -\dfrac{3}{4\times\frac{\sqrt{3}}{2}} = -\dfrac{\sqrt{3}}{2}$	accept equivalents, awrt –0.87	A1	4

Alternatives to (a) where the parameter is eliminated

1

$y = (18 - 9x)^{\frac{1}{2}}$		
$\dfrac{dy}{dx} = \dfrac{1}{2}(18 - 9x)^{-\frac{1}{2}} \times (–9)$	B1	
At $t = \dfrac{\pi}{3}, x = \cos\dfrac{2\pi}{3} = -1$	B1	
$\dfrac{dy}{dx} = \dfrac{1}{2} \times \dfrac{1}{\sqrt{(27)}} \times -9 = -\dfrac{\sqrt{3}}{2}$	A1	4

② $y^2 = 18 - 9x$

$2y\dfrac{dy}{dx} = -9$ B1

At $t = \dfrac{\pi}{3}$, $y = 6\sin\dfrac{\pi}{3} = 3\sqrt{3}$ B1

$\dfrac{dy}{dx} = -\dfrac{9}{2\times 3\sqrt{3}} = -\dfrac{\sqrt{3}}{2}$ A1 4

(b) Use of $\cos 2t = 1 - 2\sin^2 t$

$\cos 2t = \dfrac{x}{2}, \sin t = \dfrac{y}{6}$

$\dfrac{x}{2} = 1 - 2\left(\dfrac{y}{6}\right)^2$

Leading to $y = \sqrt{(18-9x)}\,(= 3\sqrt{(2-x)})$ cao A1

$-2 \le x \le 2$ $k = 2$ B1 4

(c) $0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$ B1

Fully correct. Accept $0 \le y \le 6$, [0, 6] B1 2

[10]

4. (a) $\int \sin^2\theta\, d\theta = \dfrac{1}{2}\int (1 - \cos 2\theta)\, d\theta = \dfrac{1}{2}\theta - \dfrac{1}{4}\sin 2\theta\,(+C)$ A1 2

(b) $x = \tan\theta \Rightarrow \dfrac{dx}{d\theta} = \sec^2\theta$

$\pi\int y^2\, dx = \pi\int y^2 \dfrac{dx}{d\theta}\, d\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta\, d\theta$ A1

$= \pi\int \dfrac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta}\, d\theta$

$16\pi\int \sin^2\theta\, d\theta$ $k = 16\pi$ A1

$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0$, $x = \dfrac{1}{\sqrt{3}} \Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \dfrac{\pi}{6}$ B1 5

$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta\, d\theta\right)$

	(c)	$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$			
		$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$	Use of correct limits	*	
		$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3}$	$p = \frac{4}{3}, q = -2$	A1	3
					[10]
5.	(a)	At A, $x = -1 + 8 = 7$ & $y = (-1)^2 = 1 \Rightarrow A(7,1)$	$A(7, 1)$	B1	1
	(b)	$x = t^3 - 8t$, $y = t^2$,			
		$\frac{dx}{dt} = 3t^2 - 8, \frac{dy}{dt} = 2t$			
		$\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$		
			Correct $\frac{dy}{dx}$	A1	
		At A, m(**T**) = $\underline{\frac{2(-1)}{3(-1)^2 - 8}} = \underline{\frac{-2}{3-8}} = \underline{\frac{-2}{-3}} = \underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:		
		T: y – (their 1) = m_r(x – (their 7)) or $1 = \frac{2}{5}(7) + c \Rightarrow c = 1 - \frac{14}{5} = -\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses y = (their gradient)x + "c" .	dM1	
		Hence **T**: $y = \frac{2}{5}x - \frac{9}{5}$			
		gives **T**: $\underline{2x - 5y - 9 = 0}$ **AG**		A1 cso	5
	(c)	$2(t^3 - 8t) - 5t^2 - 9 = 0$	Substitution of both $x = t^3 - 8t$ and $y = t^2$ into **T**		
		$2t^3 - 5t^2 - 16t - 9 = 0$			
		$(t + 1)\{(2t^2 - 7t - 9) = 0\}$			
		$(t + 1)\{(t + 1)(2t - 9) = 0\}$	A realisation that $(t + 1)$is a factor.	dM1	
		$\{t = -1(\text{at } A)\ t = \frac{9}{2} \text{ at } B\}$	$t = \frac{9}{2}$	A1	

$x = \left(\frac{9}{2}\right)^2 - 8\left(\frac{9}{2}\right) = \frac{729}{8} - 36 = \frac{441}{8} = 55.125$ or awrt 55.1

Candidate uses their value of t to find either the x or y coordinate — ddM1

$y = \left(\frac{9}{2}\right)^2 = \frac{81}{4} = 20.25$ or awrt 20.3 — One of either x or y correct. — A1

Both x and y correct. — A1 — 6

Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$ — awrt

[12]

6. (a) At $P(4,\ 2\sqrt{3}\,)$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$

$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leq t \leq \frac{\pi}{2}$

$\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$

$\underline{t = \frac{\pi}{3}}$ or $\underline{\text{awrt } 1.05}$ (radians) only stated in the range $0 \leq t \leq \frac{\pi}{2}$ — A1 — 2

(b) $x = 8\cos t,\ y = 4\sin 2t$

$\frac{dx}{dt} = -8\sin t,\ \frac{dy}{dt} = 8\cos 2t$

At P, $\frac{dy}{dx} = \frac{8\cos\left(\frac{2\pi}{3}\right)}{\underline{-8\sin\left(\frac{\pi}{3}\right)}}$

$\left\{ = \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$

Hence m(**N**) $= -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$

N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$

N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ **AG**

or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$

so **N**: $\left\lfloor \underline{y = -\sqrt{3}x + 6\sqrt{3}} \right\rfloor$

	Attempt to differentiate both x and y wrt t to give $\pm p$ sin t and $\pm q$ cos $2t$ respectively		
	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1	
	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.		
	You may need to check candidate's substitutions for **Note the next two method marks are dependent on**		
	Uses m(**N**) = $-\frac{1}{\text{their m}(\mathbf{T})}$	dM1*	
	Uses $y - 2\sqrt{3}$ = (their m_N)($x - 4$) or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses y = (their m_N)x + "c".	dM1*	
	$\underline{y = -\sqrt{3}x + 6\sqrt{3}}$	A1 **cso AG**	6
(c)	$A = \int_0^4 y\,dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,dt$ $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,dt$		
	attempt at $A = \underline{\int y\frac{dx}{dt}}dt$		
	correct expression (ignore limits and dt)	A1	
	Seeing sin $2t$ = 2 sin t cos t anywhere in this part.		
	Correct proof. Appreciation of how the negative sign affects the limits. **Note that the answer is given in the question.**	A1 **AG**	4

(d) {Using substitution $u = \sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$}

{change limits:

when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$}

$$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \text{ or } A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$$

$$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$$

$$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$$

(Note that $a = \frac{64}{3}$, $b = -8$)

$k \sin^3 t$ or ku^3 with $u = \sin t$

Correct integration ignoring limits. A1

Substitutes limits of either ($t = \frac{\pi}{2}$ and $t = \frac{\pi}{3}$) or

($u = 1$ and $u = \frac{\sqrt{3}}{2}$) and subtracts the correct way round. dM1

$\underline{\frac{64}{3} - 8\sqrt{3}}$ A1 aef **isw** 4

Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that

cancels to $a = \frac{64}{3}$ and $b = -8$.

[16]

7 (a) $\left[x = \ln(t+2),\ y = \frac{1}{t+1}\right], \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{t+2}$

Area (R) = $\int_{\ln 2}^{\ln 4} \frac{1}{t+1}\mathrm{d}x; = \int_0^2 \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)\mathrm{d}t$

Changing limits, when:

$x = \ln 2 \Rightarrow \ln 2 = \ln(t + 2) \Rightarrow 2 = t + 2 \Rightarrow t = 0$

$x = \ln 4 \Rightarrow \ln 4 = \ln(t + 2) \Rightarrow 4 = t + 2 \Rightarrow t = 2$

Hence, Area (R) = $\int_0^2 \frac{1}{(t+1)(t+2)}\mathrm{d}t$

Must state $\dfrac{dx}{dt} = \dfrac{1}{t+2}$ B1

Area = $\displaystyle\int \frac{1}{t+1} dx$. Ignore limits.

$\displaystyle\int \left(\frac{1}{t+1}\right) \times \left(\frac{1}{t+2}\right) dt$. Ignore limits. A1 **AG**

changes limits $x \to t$ so that $\ln 2 \to 0$ and $\ln 4 \to 2$ B1 4

(b) $\left(\dfrac{1}{(t+1)(t+2)}\right) = \dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$

$1 = A(t+2) + B(t+1)$

Let $t = -1$, $1 = A(1) \Rightarrow \underline{A = 1}$

Let $t = -2$, $1 = B(-1) \Rightarrow \underline{B = -1}$

$\displaystyle\int_0^2 \frac{1}{(t+1)(t+2)} dt = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)} dt$

$= \left[\ln(t+1) - \ln(t+2)\right]_0^2$

$= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$

Takes out brackets: $= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\dfrac{3}{2}\right)$

$\dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$ with A and B found

Finds both A and B correctly.
Can be implied. A1

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} + \dfrac{1}{(t+2)}$ means first M1A0.

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}$ means first M1A1.

Either $\pm a \ln(t+1)$ or $\pm b \ln(t+2)$ dM1

Both ln terms correctly ft. A1ft

Substitutes ***both*** limits of 2 and 0 and subtracts the correct way round. ddM1

$\underline{\ln 3 - \ln 4 + \ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3 - \ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$

(must deal with ln 1) A1 aef isw 6

(c) $x = \ln(t + 2),\ y = \dfrac{1}{t+1}$

$e^x = t + 2 \Rightarrow t = e^x - 2$

$y = \dfrac{1}{e^x - 2 + 1} \Rightarrow y = \dfrac{1}{e^x - 1}$

Attempt to make $t = \ldots$ the subject

giving $t = e^x - 2$ A1

Eliminates t by substituting in y dM1

giving $y = \dfrac{1}{e^x - 1}$ A1 4

Aliter

Way 2

$t + 1 = \dfrac{1}{y} \Rightarrow t = \dfrac{1}{y} - 1 \text{ or } t = \dfrac{1-y}{y}$

$y(t + 1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \dfrac{1-y}{y}$

$x = \ln\left(\dfrac{1}{y} - 1 + 2\right) \text{ or } x = \ln\left(\dfrac{1-y}{y} + 2\right)$

$x = \ln\left(\dfrac{1}{y} + 1\right)$

$e^x = \dfrac{1}{y} + 1$

$e^x - 1 = \dfrac{1}{y}$

$y = \dfrac{1}{e^x - 1}$

Attempt to make $t = \ldots$ the subject

Giving either $t = \dfrac{1}{y} - 1$ or $t = \dfrac{1-y}{y}$ A1

Eliminates t by substituting in x dM1

giving $y = \dfrac{1}{e^x - 1}$ A1 4

Aliter
Way 3

$e^x = t + 2 \Rightarrow t + 1 = e^x - 1$

$y = \dfrac{1}{t+1} \Rightarrow y = \dfrac{1}{e^x - 1}$

Attempt to make $t + 1 = \ldots$ the subject

giving $t + 1 = e^x - 1$	A1	
Eliminates t by substituting in y	dM1	
giving $y = \dfrac{1}{e^x - 1}$	A1	4

Aliter
Way 4

$t + 1 = \dfrac{1}{y} \Rightarrow t + 2 = \dfrac{1}{y} + 1 \text{ or } t + 2 = \dfrac{1+y}{y}$

$x = \ln\left(\dfrac{1}{y} + 1\right) \text{ or } x = \ln\left(\dfrac{1+y}{y}\right)$

$x = \ln\left(\dfrac{1}{y} + 1\right)$

$e^x = \dfrac{1}{y} + 1 \Rightarrow e^x - 1 = \dfrac{1}{y}$

$y = \dfrac{1}{e^x - 1}$

Attempt to make $t + 2 = \ldots$ the subject

Either $t + 2 = \dfrac{1}{y} + 1 \text{ or } t + 2 = \dfrac{1+y}{y}$	A1	
Eliminates t by substituting in x	dM1	
giving $y = \dfrac{1}{e^x - 1}$	A1	4

(d) Domain: $\underline{x > 0}$

$\underline{x > 0}$ or just > 0	B1	1

[15]

8. (a) $x = \tan^2 t$, $y = \sin t$

$\frac{dx}{dt} = 2(\tan t)\sec^2 t, \frac{dy}{dt} = \cos t$ (*)

3

$\therefore \frac{dy}{dx} = \frac{\cos t}{2\tan t\sec^2 t}\left(= \frac{\cos^4 t}{2\sin t}\right)$

B1 Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$

$\frac{\pm\cos t}{\text{their } \frac{dx}{dt}}$

A1ft (*) $\frac{+\cos t}{\text{their } \frac{dx}{dt}}$

(b) When $t = \frac{\pi}{4}$, $x = 1$, $y = \frac{1}{\sqrt{2}}$ (need values)

When $t = \frac{\pi}{4}$, $\mathrm{m}(\mathbf{T}) = \frac{dy}{dx} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}$

$= \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{2}}\right)}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)(2)}} = \underline{\frac{1}{4\sqrt{2}}} = \underline{\frac{\sqrt{2}}{8}}$

B1, B1 The point $\underline{\left(1, \frac{1}{\sqrt{2}}\right)}$ or <u>(1, awrt 0.71)</u>

These coordinates can be implied.

($y = \sin\left(\frac{\pi}{4}\right)$ is not sufficient for B1)

B1 aef any of the five underlined expressions or awrt 0.18

T: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x-1)$ (Note: The x and y coordinates must be the right way round.)

T: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$ (*)

or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \Rightarrow c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$

(*) A candidate who incorrectly differentiates $\tan^2 t$ to give $\frac{dx}{dt} = 2\sec^2 t$ or $\frac{dx}{dt} = \sec^4 t$ is then able to fluke the correct answer in part (b).
Such candidates can potentially get: (a) B0M1A1ft (b) B1B1B1M1A0 **cso**.
Note: cso means "correct solution only".
Note: part (a) not fully correct implies candidate can achieve a maximum of 4 out of 5 marks in part (b).

Hence **T**: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$ 5

M1ft aef Finding an equation of a tangent with *their point* and *their tangent gradient* or finds c using y = (their gradient)x + "c".

A1 aef cso Correct simplified EXACT equation of tangent

(c) **Way 1**

$x = \tan^2 t = \frac{\sin^2 t}{\cos^2 t}$ $y = \sin t$

$x = \frac{\sin^2 t}{1-\sin^2 t}$

$x = \frac{y^2}{1-y^2}$

$x(1 - y^2) = y^2 \Rightarrow x - xy^2 = y^2$
$x = y^2 + xy^2 \Rightarrow x = y^2(1 + x)$

$y^2 = \frac{x}{1+x}$ 4

Uses $\cos^2 t = 1 - \sin^2 t$

Eliminates 't' to write an equation involving x and y.

ddM1 Rearranging and factorising with an attempt to make y^2 the subject.

A1 $\frac{x}{1+x}$

Aliter
Way 2

$1 + \cot^2 t = \text{cosec}^2 t$

$= \dfrac{1}{\sin^2 t}$

Hence, $1 + \dfrac{1}{x} = \dfrac{1}{y^2}$

Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$

Uses $1 + \cot^2 t = \text{cosec}^2 t$

implied Uses $\text{cosec}^2 t = \dfrac{1}{\sin^2 t}$

ddM1 Eliminates 't' to write an equation involving x and y.

A1 $1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$

Aliter
Way 3

$x = \tan^2 t$ $\qquad$ $y = \sin t$

$1 + \tan^2 t = \sec^2 t$

$= \dfrac{1}{\cos^2 t}$

$= \dfrac{1}{1 - \sin^2 t}$

Hence, $1 + x = \dfrac{1}{1 - y^2}$

Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$

Uses $1 + \tan^2 t = \sec^2 t$

Uses $\sec^2 t = \dfrac{1}{\cos^2 t}$

ddM1 Eliminates 't' to write an equation involving x and y.

A1 $1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$

Aliter
Way 4

$y^2 = \sin^2 t = 1 - \cos^2 t$

$= 1 - \dfrac{1}{\sec^2 t}$

$= 1 - \dfrac{1}{(1 + \tan^2 t)}$

Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$

Uses $\sin^2 t = 1 - \cos^2 t$

Uses $\cos^2 t = \dfrac{1}{\sec^2 t}$

ddM1 then uses $\sec^2 t = 1 + \tan^2 t$

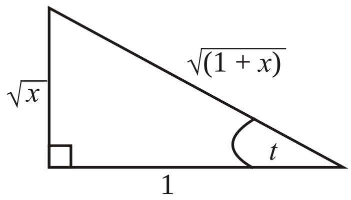
A1 $1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$

$\dfrac{1}{1+\dfrac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter
Way 5

$x = \tan^2 t \qquad\qquad y = \sin t$

$x = \tan^2 t \Rightarrow \tan t = \sqrt{x}$



Hence, $y = \sin t = \dfrac{\sqrt{x}}{\sqrt{1+x}}$

Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle

Uses Pythagoras to deduce the hypotenuse

ddM1 Eliminates 't' to write an equation involving x and y

A1 $\dfrac{x}{1+x}$

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c).
If a candidate produces a correct solution then please award all four marks. if they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.

[12]

9. (a) x = sin t $\qquad$ y = sin(t + $\frac{\pi}{6}$)

Attempt to differentiate both x and y wrt t to give two terms in cos

$\dfrac{dx}{dt} = \cos t, \ \dfrac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$ A1

Correct $\frac{dx}{dt}$ *and* $\frac{dy}{dt}$

When $t = \frac{\pi}{6}$,

$\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$ A1

Divides in correct way and substitutes for t to give any of the four underlined oe:
Ignore the double negative if candidate has differentiated $\sin \to -\cos$

when $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ B1

The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ *or* $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$

T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses
y = (their gradient) x + "c". dM1
Correct *__EXACT__* *equation of* *__tangent__*
oe. A1 oe

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or T: $\left\lfloor \underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}} \right\rfloor$ 6

(b) $y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6}$

Use of compound angle formula for sine.

Nb: $\sin^2 t + \cos^2 t = 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$

$\therefore$ $x = \sin t$ gives $\cos t = \sqrt{(1 - x^2)}$

Use of trig identity to find cos t in terms of x or $\cos^2 t$ *in terms of x.*

$\therefore\ y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$

gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$ AG A1 cso 3

Substitutes for sin *t,* cos $\frac{\pi}{6}$*, cost and sin* $\frac{\pi}{6}$ *to give y in terms of x.*

[9]

Aliter Way 2

(a) $x = \sin t$ $y = \sin\left(t + \frac{\pi}{6}\right) = \sin t + \cos\frac{\pi}{6} + \cos t\sin\frac{\pi}{6}$

(Do not give this for part (b))

Attempt to differentiate x and y wrt t to give $\frac{dx}{dy}$ *in terms of cos and* $\frac{dy}{dt}$ *in the form ± a cost ± b sin t*

$\frac{dx}{dt} = \cos t$; $\frac{dy}{dt} = \cos t\cos\frac{\pi}{6} - \sin t\sin\frac{\pi}{6}$ A1

Correct $\frac{dx}{dt}$ *and* $\frac{dy}{dt}$

When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\dfrac{\cos\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\sin\frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}}$ A1

Divides in correct way and substitutes for t to give any of the four underlined oe

When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ B1

The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ *or* $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$

T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses y = (their gradient)x + "c". dM1

Correct EXACT equation of *<u>tangent</u>* *oe.* A1 oe

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or **T**: $\left\lfloor \underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}} \right\rfloor$

Aliter Way 3

(a) $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$

$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$

Attempt to differentiate two terms using the chain rule for the second term.

Correct $\frac{dy}{dx}$ A1

$\underline{\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(1-(0.5)^2\right)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}}$ A1

Correct substitution of $x = \frac{1}{2}$ *into a correct* $\frac{dy}{dx}$

When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ B1

The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ *or* $\underline{(\frac{1}{2}, awrt\ 0.87)}$

T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x - \frac{1}{2})}$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses dM1
y = (their gradient) x + "c"

Correct <u>*EXACT*</u> *equation of* <u>*tangent*</u> *oe.* A1 oe

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or **T**: $\left\lfloor \underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}} \right\rfloor$ 6

Aliter Way 2

(b) $x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{(1-\sin^2 t)}$

Substitutes x = sin *t into the equation give in y.*

Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$

Cost = $\sqrt{(1-\sin^2 t)}$

Use of trig identity to deduce that cos t = $\sqrt{(1-\sin^2 t)}$

gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$

Hence $y = \sin t\cos\frac{\pi}{6} + \cos t\sin\frac{\pi}{6} = \sin(t+\frac{\pi}{6})$ A1 cso 3

Using the compound angle formula to prove $y = \sin(t+\frac{\pi}{6})$

10. (a) $\frac{dx}{dt} = -3a\sin 3t$, $\frac{dy}{dt} = a\cos t$ therefore $\frac{dy}{dx} = \frac{\cos t}{-3\sin 3t}$ A1

When $x = 0$, $t = \frac{\pi}{6}$ B1

Gradient is $-\frac{\sqrt{3}}{6}$

Line equation is $(y - \frac{1}{2}a) = -\frac{\sqrt{3}}{6}(x - 0)$ A1 6

(b) Area beneath curve is $\int a\sin t(-3a\sin 3t)dt$

$= -\frac{3a^2}{2}\int(\cos 2t - \cos 4t)dt$

$\frac{3a^2}{2}[\frac{1}{2}\sin 2t - \frac{1}{4}\sin 4t]$ A1

Uses limits 0 and $\frac{\pi}{6}$ to give $\frac{3\sqrt{3}a^2}{16}$ A1

Area of triangle beneath tangent is $\frac{1}{2}\times\frac{a}{2}\times\sqrt{3}a = \frac{\sqrt{3}a^2}{4}$ A1

Thus required area is $\frac{\sqrt{3}a^2}{4} - \frac{3\sqrt{3}a^2}{16} = \frac{\sqrt{3}a^2}{16}$ A1 9

N.B. The integration of the product of two sines is worth 3 marks
(lines 2 and 3 of to part (b))
If they use parts

$\int\sin t\sin 3t\,dt = -\cos t\sin 3t + \int 3\cos 3t\cos t\,dt$

$= -\cos t\sin 3t + 3\cos 3t\sin t + \int 9\sin 3t\sin t\,dt$

$8I = \cos t\sin 3t - 3\cos 3t\sin t$ A1

[15]

11. (a) Solves y = 0 $\Rightarrow$ $\cos t = \frac{1}{2}$ to obtain $t = \frac{\pi}{3}$ *or* $\frac{5\pi}{3}$

(need both for A1) A1 2

Or substitutes **both** values of t and shows that $y = 0$

(b) $\frac{dx}{dt} = 1 - 2\cos t$ A1

Area = $\int y\,dx = \int_{\pi/3}^{5\pi/3} (1 - 2\cos t)\ (1 - 2\cos t)dt$

$= \int_{\pi/3}^{5\pi/3} (1 - 2\cos t)^2\ dt$ AG B1 3

(c) Area $= \int 1 - 4\cos t + 4\cos^2 t\ dt$ 3 terms

$\int 1 - 4\cos t + 2(\cos 2t + 1)dt$ (use of correct double angle formula)

$= \int 3 - 4\cos t + 2\cos 2t dt$ A1

$= [3t - 4\sin t + \sin 2t]$ A1

Substitutes the two correct limits $t = \frac{5\pi}{3}$ and $\frac{\pi}{3}$ and subtracts.

$= 4\pi + 3\sqrt{3}$ A1A1 7

[12]

12. (a) $\frac{dx}{dt} = -\frac{1}{(1+t)^2}$ and $\frac{dy}{dt} = \frac{1}{(1-t)^2}$ B1, B1

$\therefore \frac{dy}{dx} = \frac{-(1+t)^2}{(1-t)^2}$ and at $t = ½$, gradient is -9 A1cao

requires their dy/dt / their dx/dt and substitution of t.

At the point of contact $x = \frac{2}{3}$ and $y = 2$ B1

Equation is $y - 2 = -9\left(x - \frac{2}{3}\right)$ A1 7

(b) **Either** obtain t in terms of x and y i,e, $t = \frac{1}{x} - 1$ or $t = 1 - \frac{1}{x}$ (or both)

Then substitute into other expression $y = f(x)$ or $x = g(y)$ and rearrange

(or put $\frac{1}{x} - 1 = 1 - \frac{1}{y}$ and rearrange)

To obtain $y = \frac{x}{2x-1}$ (*) a1 3

Or Substitute into $\frac{x}{2x-1} = \frac{\frac{1}{(1+t)}}{\frac{2}{1+t} - 1}$

$= \frac{1}{2-(1+t)} = \frac{1}{1-t}$ A1

$= y$ (*) 3

(c) Area $= \int_{\frac{2}{3}}^{1} \frac{x}{2x-1}\,dx$ B1

$= \int \frac{u+1}{2u}\frac{du}{2} = \frac{1}{4}\int 1 + \frac{1}{u}\,du$

putting into a form to integrate

$= \left[\frac{1}{4}u + \frac{1}{4}\ln u\right]_{\frac{1}{3}}^{1}$ A1

$= \frac{1}{4} - \left(\frac{1}{12} + \frac{1}{4}\ln\frac{1}{3}\right)$

$= \frac{1}{6} + \frac{1}{4}$ ln 3 or any correct equivalent. A1 6

[16]

Or Area $= \int_{\frac{2}{3}}^{1} \frac{x}{2x-1}\,dx$ B1

$= \int \frac{1}{2} + \frac{\frac{1}{2}}{2x-1}\,dx$

putting into a form to integrate

$= \left[\frac{1}{2}x + \frac{1}{4}\ln(2x-1)\right]_{\frac{2}{3}}^{1}$ A1

$= \frac{1}{2} - \frac{1}{3} - \frac{1}{4}\ln\frac{1}{3} = \frac{1}{6} - \frac{1}{4}\ln\frac{1}{3}$ dM1 A1 6

Or Area $= \int \frac{1}{1-t}\frac{-1}{(1+t)^2}\,dt$ B1

$= \int \frac{A}{(1-t)} + \frac{B}{(1+t)} + \frac{C}{(1+t)^2}\,dt$

putting into a form to integrate

$= \left[\frac{1}{4}\ln(1-t) - \frac{1}{4}\ln(1+t) + \frac{1}{2}(1+t)^{-1}\right]$ a1ft

= Using limits 0 and ½ and subtracting (either way round) dM1

$= \frac{1}{6} + \frac{1}{4}$ ln3 or any correct equivalent. A1 6

Or Area $= \int_{\frac{2}{3}}^{1} \frac{x}{2x-1}\,dx$ then use parts B1

$= \frac{1}{2}x\ln(2x-1) - \int_{\frac{2}{3}}^{1} \frac{1}{2}(2x-1)\,dx$

$= \frac{1}{2}x\ln(2x-1) - \left[\frac{1}{4}(2x-1)\ln(2x-1) - \frac{1}{2}x\right]$ M1A1

$= \frac{1}{2} - \left(\frac{1}{3}\ln\frac{1}{3} - \frac{1}{12}\ln\frac{1}{3} + \frac{1}{3}\right)$ DM1

$= \frac{1}{6} - \frac{1}{4}\ln\frac{1}{3}$ A1 6

13. (a) $\frac{dx}{dt} = -2\text{cosec}^2\, t,\ \frac{dy}{dt} = 4\sin t\cos t$ both A1

$\frac{dy}{dx} = \frac{-2\sin t\cos t}{\text{cosec}^2 t}$ ($= -2\sin^3 t\cos t$) A1 4

(b) At $t = \frac{\pi}{4}$, $x = 2$, $y = 1$ B1

both x and y

Substitutes $t = \frac{\pi}{4}$ into an attempt at $\frac{dy}{dx}$ to obtain gradient $\left(-\frac{1}{2}\right)$

Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$ A1 4

Accept $x + 2y = 4$ or any correct equivalent

(c) Uses $1 + \cot^2 t = \text{cosec}^2 t$, or equivalent, to eliminate t

$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ A1

correctly eliminates t

$y = \frac{8}{4 + x^2}$ cao A1

The domain ir $x \ldots 0$ B1 4

Alternative for (c):

$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}$; $\cos t = \frac{x}{2}\sin t = \frac{x}{2}\left(\frac{y}{2}\right)^{\frac{1}{2}}$

$\sin^2 t + \cos^2 t = 1 \Rightarrow \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$ A1

Leading to $y = \frac{8}{4 + x^2}$ A1

[12]

14. (a) $4 = 2\sec t \Rightarrow \cos t = \frac{1}{2}, \Rightarrow t = \frac{\pi}{3}$ A1

$\therefore\ a = 3 \times \frac{\pi}{3} \times \sin\frac{\pi}{3} = \underline{\frac{\pi\sqrt{3}}{2}}$ B1 3

(b) $A = \int_0^a y\,dx = \int y\frac{dx}{dt}\,dt$

Change of variable

$= \int 2\sec t \times [3\sin t + 3t\cos t]\,dt$

Attempt $\frac{dx}{dt}$

$= \int_0^{\frac{\pi}{3}} (6\tan t, + 6t)\,dt$ (*) A1, A1cso 4

Final A1 requires limit <u>*stated*</u>

(c) $A = [6\ln\sec t + 3t^2]_0^{\frac{\pi}{3}}$ A1

Some integration (both correct (A1) ignore lim.

$= (6\ln 2 + 3 \times \frac{\pi^2}{9}) - (0)$ Use of $\frac{\pi}{3}$

$= \underline{6\ln 2 + \frac{\pi^2}{3}}$ A1 4

[11]

15. (a) Area of triangle = $\frac{1}{2} \times 30 \times 3\pi^2$ (= 444.132) B1 1

Accept 440 or 450

(b) **Either** Area shaded = $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 30\sin 2t.32t\mathrm{d}t$ A1

$= [-480t\cos 2t + \int 480\cos 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ A1

$= [-480t\cos 2t + 240\sin 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ A1 ft

$= 240(\pi - 1)$ A1 7

or $\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 60\cos 2t.(16t^2 - \pi^2)\mathrm{d}t$ A1

$= [(30\sin 2t\,(\pi^2 - 16t^2) - 480t\cos 2t + \int 480\cos 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ A1

$= [-480t\cos 2t + 240\sin 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ A1 ft

$= 240(\pi - 1)$ A1 7

(c) Percentage error = $\frac{240(\pi - 1) - \text{estimate}}{240(\pi - 1)} \times 100 = 13.6\%$ A1 2

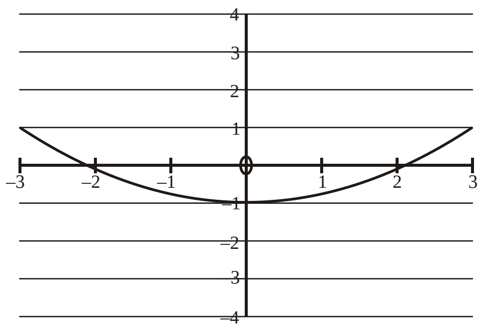
(Accept answers in the range 12.4% to 14.4%)

[10]

16. (a) Attempt to use correctly stated double angle formula $\cos 2t = 2\cos^2 t - 1$, or complete method using other double angle formula for $\cos 2t$ with $\cos^2 t + \sin^2 t = 1$ to eliminate t and obtain $y =$

$y = 2\left(\frac{x}{3}\right)^2 - 1$ or any correct equivalent.(even $y = \cos 2(\cos^1 \left(\frac{x}{3}\right))$ A1 2

(b)



shape	B1	
position including restricted domain $-3 < x < 3$	B1	2

[4]

17. (a) $\dfrac{dx}{dt} = -\sin t,\quad \dfrac{dy}{dt} = 2\cos 2t \quad \therefore \dfrac{dy}{dx} = \dfrac{2\cos 2t}{-\sin t}$ A1 A1 3

(b) $2\cos 2t = 0 \quad \therefore 2t = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \dfrac{7\pi}{2}$

So $t = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$ A1 A1 3

(c) $\left(\dfrac{1}{\sqrt{2}}, 1)\right) \left(\dfrac{1}{\sqrt{2}}, -1)\right) \left(-\dfrac{1}{\sqrt{2}}, 1)\right) \left(-\dfrac{1}{\sqrt{2}}, -1)\right)$ A1 2

(d) $y = 2\sin t\cos t$

$= 2\sqrt{1-\cos^2 t}\cos t = 2x\sqrt{1-x^2}$ A1 3

(e) $y = -2x\sqrt{1-x^2}$ B1 1

[12]

1. No Report available for this question.

2. Part (a) was well done. The majority of candidates correctly found the x-coordinates of A and B, by putting $y = 0$, solving for t and then substituting in $x = 5t^2 – 4$. Full marks were common. Part (b) proved difficult. A substantial minority of candidates failed to substitute for the dx when substituting into $\int y\,\mathrm{d}x$ or used $\frac{\mathrm{d}t}{\mathrm{d}x}$ rather than $\frac{\mathrm{d}y}{\mathrm{d}x}$. A surprising feature of the solutions seen was the number of candidates who, having obtained the correct $\int t\left(9-t^2\right)10t\,\mathrm{d}t$, were unable to remove the brackets correctly to obtain $\int\left(90t^2-10t^4\right)\mathrm{d}t$. Weaknesses in elementary algebra flawed many otherwise correct solutions. Another source of error was using the x-coordinates for the limits when the variable in the integral was t. At the end of the question, many failed to realise that $\int_0^3\left(90t^2-10t^4\right)\mathrm{d}t$ gives only half of the required area.

Some candidates made either the whole of the question, or just part (b), more difficult by eliminating parameters and using the cartesian equation. This is a possible method but the indices involved are very complicated and there were very few successful solutions using this method.

3. Nearly all candidates knew the method for solving part (a), although there were many errors in differentiating trig functions. In particular $\frac{d}{dt}$ (2cos2t) was often incorrect. It was clear from both this question and question 2 that, for many, the calculus of trig functions was an area of weakness. Nearly all candidates were able to obtain an exact answer in surd form. In part (b), the majority of candidates were able to eliminate t but, in manipulating trigonometric identities, many errors, particularly with signs, were seen. The answer was given in a variety of forms and all exact equivalent answers to that printed in the mark scheme were accepted. The value of k was often omitted and it is possible that some simply overlooked this. Domain and range remains an unpopular topic and many did not attempt part (c). In this case, inspection of the printed figure gives the lower limit and was intended to give candidates a lead to identifying the upper limit.

4. The responses to this question were very variable and many lost marks through errors in manipulation or notation, possibly through mental tiredness. For examples, many made errors in manipulation and could not proceed correctly from the printed $\cos 2\theta = 1 – 2\sin^2\theta$ to $\sin^2 \theta = \frac{1}{2}-\frac{1}{2}\cos 2\theta$ and the answer $\frac{x}{2}-\frac{1}{4}\sin 2\theta$ was often seen, instead of $\frac{\theta}{2}-\frac{1}{4}\sin 2\theta$. In part (b), many never found $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ or realised that the appropriate form for the volume was

$$\pi\int y^2\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta.$$

However the majority did find a correct integral in terms of θ although some were unable to use the identity $\sin 2\theta = 2\sin\theta\cos\theta$ to simplify their integral. The incorrect value $k = 8\pi$ was very common, resulting from a failure to square the factor 2 in $\sin 2\theta = 2\sin\theta\cos\theta$. Candidates were expected to demonstrate the correct change of limits. Minimally a reference to the result tan

$\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, or an equivalent, was required. Those who had complete solutions usually gained the two method marks in part (c) but earlier errors often led to incorrect answers.

5. Part (a) was answered correctly by almost all candidates. In part (b), many candidates correctly applied the method of finding a tangent by using parametric differentiation to give the answer in the correct form. Few candidates tried to eliminate t to find a Cartesian equation for C, but these candidates were usually not able to find the correct gradient at A.

In part (c), fully correct solutions were much less frequently seen. A significant number of candidates were able to obtain an equation in one variable to score the first method mark, but were then unsure about how to proceed. Successful candidates mostly formed an equation in t, used the fact that $t + 1$ was a factor and applied the factor theorem in order for them to find t at the point B. They then substituted this t into the parametric equations to find the coordinates of B. Those candidates who initially formed an equation in y only went no further. A common misconception in part (c), was for candidates to believe that the gradient at the point B would be the same as the gradient at the point A and a significant minority of candidates attempted to solve $\frac{2t}{3t^2 - 8} = \frac{2}{5}$ to find t at the point B.

6. In part (a), many candidates were able to give $t = \frac{\pi}{3}$. Some candidates gave their answer only in degrees instead of radians. Other candidates substituted the y-value of P into $y = 4 \sin 2t$ and found two values for t, namely $t = \frac{\pi}{6}, \frac{\pi}{3}$. The majority of these candidates did not go on to reject $t = \frac{\pi}{6}$. In part (b), many candidates were able to apply the correct formula for finding $\frac{dy}{dx}$ in terms of t, although some candidates erroneously believed that differentiation of $4 \sin 2t$ gave either $-8 \cos 2t$, $8\cos t$ or $2 \cos 2t$. Some candidates who had differentiated incorrectly, substituted their value of t into $\frac{dy}{dx}$ and tried to "fudge" their answer for $\frac{dy}{dx}$ as $\frac{1}{\sqrt{3}}$, after realising from the given answer that the gradient of the normal was $-\sqrt{3}$. The majority of candidates understood the relationship between the gradient of the tangent and its normal and many were able to produce a fully correct solution to this part.

A few candidates, however, did not realise that parametric differentiation was required in part (b) and some of these candidates tried to convert the parametric equations into a Cartesian equation. Although some candidates then went on to attempt to differentiate their Cartesian equation, this method was rarely executed correctly.

Few convincing proofs were seen in part (c) with a significant number of candidates who were not aware with the procedure of reversing the limits and the sign of the integral. Therefore, some candidates conveniently differentiated $8 \cos t$ to give "positive" $8\sin t$, even though they had previously differentiated $8 \cos t$ correctly in part (a). After completing this part, some candidates had a 'crisis of confidence' with their differentiation rules and then went on to amend their correct solution to part (a) to produce an incorrect solution. Other candidates differentiated $8 \cos t$ correctly but wrote their limits the wrong way round giving

$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 4\sin 2t\,.(-8\sin t)\,dt$ and after stating $\sin 2t = 2\cos t \sin t$ (as many candidates were able to do in this part) wrote $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} -64\sin^2 t\cos t\,dt$. These candidates then wrote down the given answer by arguing that all areas should be positive.

Part (d) was unstructured in the sense that the question did not tell the candidates how to integrate the given expression. Only a minority of candidates spotted that a substitution was required, although some candidates were able to integrate $64\sin^2 t\cos t$ by inspection. Many candidates replaced $\sin^2 t$ with $\frac{1}{2}(1 - \cos 2t)$ but then multiplied this out with $\cos t$ to give $\frac{1}{2}\cos t - \frac{1}{2}\cos t\cos 2t$. Very few candidates correctly applied the sum-product formula on this expression, but most candidates usually gave up at this point or went on to produce some incorrect integration. Other candidates replaced $\sin^2 t$ with $1 - \cos^2 t$, but did not make much progress with this. A significant number of candidates used integration by parts with a surprising number of them persevering with this technique more than once before deciding they could make no progress. It is possible, however, to use a 'loop' method but this was very rarely seen. It was clear to examiners that a significant number of stronger candidates spent much time trying to unsuccessfully answer this part with a few candidates producing at least two pages of working.

7. The majority of candidates were able to show the integral required in part (a). Some candidates, however, did not show evidence of converting the given limits, whilst for other candidates this was the only thing they were able to do.

In part (b), it was disappointing to see that some strong candidates were unable to gain any marks by failing to recognise the need to use partial fractions. Those candidates who split the integral up as partial fractions usually gained all six marks, while those candidates who failed to use partial fractions usually gave answers such as $\ln(t^2 + 3t + 2)$ or $\ln(t + 1) \times \ln(t + 2)$ after integration. A few candidates only substituted the limit of 2, assuming that the result of substituting a limit of 0 would be zero Few candidates gave a decimal answer instead of the exact value required by the question.

Part (c) was well answered by candidates of all abilities with candidates using a variety of methods as identified in the mark scheme. Occasionally some candidates were able to eliminate t but then failed to make y the subject.

The domain was not so well understood in part (d), with a significant number of candidates failing to correctly identify it.

8. In part (a), a significant number of candidates struggled with applying the chain rule in order to differentiate $\tan^2 t$ with respect to t. Some candidates replaced $\tan^2 t$ with $\frac{\sin^2 t}{\cos^2 t}$ and proceeded to differentiate this expression using both the chain rule and the quotient rule. Very few candidates incorrectly differentiated sin t to give – cos t. A majority of candidates were then able to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ by dividing their $\frac{\mathrm{d}y}{\mathrm{d}t}$ by their $\frac{\mathrm{d}x}{\mathrm{d}t}$.

In part (b), the majority of candidates were able to write down the point $\left(1, \frac{1}{\sqrt{2}}\right)$, and find the equation of the tangent using this point and their tangent gradient. Some candidates found the incorrect value of the tangent gradient using $t = \frac{\pi}{4}$ even though they had correctly found $\frac{\mathrm{d}y}{\mathrm{d}x}$ in part (a). There were a significant number of candidates, who having correctly written down the equation of the tangent as $y - \frac{1}{\sqrt{2}} = \frac{3\sqrt{2}}{8}(x - 1)$ were unable to correctly rearrange this equation into the form. $y = ax + b$.

In part (c), there were many ways that a candidate could tackle this question and there were many good solutions seen. Errors usually arose when candidates wrote down and used incorrect trigonometric identities. It was disappointing to see a number of candidates who used trigonometric identities correctly and reached $y^2 = x(1 - y^2)$ but were then unable to rearrange this or, worse still, thought that this was the answer to the question.

9. Part (a) was surprisingly well done by candidates with part (b) providing more of a challenge even for some candidates who had produced a perfect solution in part (a).

In part (a), many candidates were able to apply the correct formula for finding $\frac{dy}{dx}$ in terms of t, although some candidates erroneously believed that differentiation of a sine function produced a negative cosine function. Other mistakes included a few candidates who either cancelled out "cos" in their gradient expression to give $\frac{t + \frac{\pi}{6}}{t}$ or substituted t = $\frac{\pi}{6}$ into their x and y expressions before proceeding to differentiate each with respect to t. Other candidates made life more difficult for themselves by expanding the y expression using the compound angle formula, giving them more work, but for the same credit. Many candidates were able to substitute t=$\frac{\pi}{6}$ into their gradient expression to give $\frac{1}{\sqrt{3}}$, but it was not uncommon to see some candidates who simplified $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ incorrectly to give $\sqrt{3}$ The majority of candidates wrote down the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and understood how to find the equation of the tangent using this point and their tangent gradient.

Whilst some candidates omitted part (b) altogether, most realised they needed to use the compound angle formula, though it was common to see that some candidates who believed that sin $(t + \frac{\pi}{6})$could be rewritten as ' sin$t$ + sin $\frac{\pi}{6}$ '. Many candidates do not appreciate that a proof requires evidence, as was required in establishing that cos $t = \sqrt{1 - x^2}$, and so lost the final two marks. There were, however, a significant number of candidates who successfully obtained the required Cartesian equation.

10. This question proved a significant test for many candidates with fully correct solutions being rare. Many candidates were able to find $\frac{dx}{dt}$ and $\frac{dy}{dt}$, although confusing differentiation with integration often led to inaccuracies. Some candidates attempted to find the equation of the tangent but many were unsuccessful because they failed to use $t = \frac{\pi}{6}$ in order to find the gradient as $-\frac{\sqrt{3}}{6}$.

Those candidates who attempted part (b) rarely progressed beyond stating an expression for the area under the curve. Some attempts were made at integration by parts, although very few candidates went further than the first line. It was obvious that most candidates were not familiar with integrating expressions of the kind $\int \sin at \sin bt \, dt$. Even those who were often spent time deriving results rather than using the relevant formula in the formulae book.

Those candidates who were successful in part (a) frequently went on to find the area of a triangle and so were able to gain at least two marks in part (b).

11. The majority of candidates gained the marks in part (a) and a good proportion managed to produce the given result in part (b). Some candidates suggested that the area of R was $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} y^2 \, dt$, which made the question rather trivial; although that happened to be true here as $y = \frac{dx}{dt}$, working was needed to produce that statement.

The integration in part (c), although well done by good candidates, proved a challenge for many; weaker candidates integrating $(1\text{-}2\cos t)^2$ as $\frac{(1-2\cos t)^3}{3}$ or something similar. It may have been that some candidates were pressed for time at this point but even those who knew that a cosine double-angle formula was needed often made a sign error, forgot to multiply their expression for $\cos^2 t$ by 4, or even forgot to integrate that expression. It has to be mentioned again that the limits were sometimes used as though $\pi = 180$,

so that $[3t - 4\sin t + \sin 2t]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$ became [900 – …….] – [180 – ….].

12. (a) A number of candidates lost marks in this question. Some confused differentiation with integration and obtained a logarithm, others made sign slips differentiating y, and a number who obtained the correct gradient failed to continue to find the equation of the tangent using equations of a straight line.

(b) There was a lack of understanding of *proof* with a number of candidates merely substituting in values. Better candidates were able to begin correctly but some did not realise that if the answer is given it is necessary to show more working.

(c) Very few got this correct. There was a tendency to use parts and to be unable to deal with the integral of ln(2x–1). The most successful methods involved dividing out, or substituting for (2x-1). Those who tried a parametric approach rarely recognised the need for partial fractions.

13. This proved a testing question and few could find both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correctly. A common error was to integrate x, giving $\frac{\mathrm{d}x}{\mathrm{d}t} = 2\ln(\sin t)$. Most knew, however, how to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ from $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ and were able to pick up marks here and in part (b). In part (b), the method for finding the equation of the tangent was well understood. Part (c) proved very demanding and only a minority of candidates were able to use one of the trigonometric forms of Pythagoras to eliminate t and manipulate the resulting equation to obtain an answer in the required form. Few even attempted the domain and the fully correct answer, x□ 0, was very rarely seen

14. Part (a) was often answered well but some candidates who worked in degrees gave the final answer as $90\sqrt{3}$. Part (b) proved more challenging for many; some did not know how to change the variable and others failed to realize that $\frac{\mathrm{d}x}{\mathrm{d}t}$ required the chain rule. Most candidates made some progress in part (c) although a surprising number thought that $\int \tan t \,\mathrm{d}t$ was $\sec^2 t$. The examiners were encouraged to see most candidates trying to give an exact answer (as required) rather than reaching for their calculators.

15. This was found to be the most difficult question on the paper. Some excellent candidates did not appear to have learned how to find the area using parametric coordinates and could not even write down the first integral. A few candidates used the formula $\frac{1}{2}\int x\frac{dy}{dt} - \frac{1}{2}\int y\frac{dx}{dt}$ from the formula book. The integration by parts was tackled successfully, by those who got to that stage and there were few errors seen. The percentage at the end of the question was usually answered well by the few who completed the question.

16. Part (a) was generally answered quite well and most of the candidates used the correct double angle formula, as on the mark scheme.

Quite a few used $\sin^2 x + \cos^2 x = 1$ to obtain $x^2/9 + (1 - y)/2 = 1$. A sizeable minority also eliminated t to obtain $y=\cos2(\cos^{-1}(x/3))$. Some of these candidates arrived at a straight line equation, e.g. $3y = 2x$. If the double angle formula was mis-quoted it tended to be by stating $\cos 2x = \cos^2 x - 1$ or $\cos 2x = 1 - \sin^2 x$.

Quite a few candidates, however found $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ and then stopped. Presumably they differentiated before they read the question! These candidates seemed to have no concept of trying to eliminate *t.*

In Part (b) very few correct graphs were seen, and a large number of answers were on graph paper. Many candidates achieved a B1 for the shape, but very few realised the importance of the restricted domain.. Some candidates sketched their derived Cartesian equation, and others worked successfully from the original parametric equations.

17. No Report available for this question.