1. The circle C has centre $A(2,1)$ and passes through the point $B(10, 7)$.

(a) Find an equation for C.

(4)

The line l_1 is the tangent to C at the point B.

(b) Find an equation for l_1.

(4)

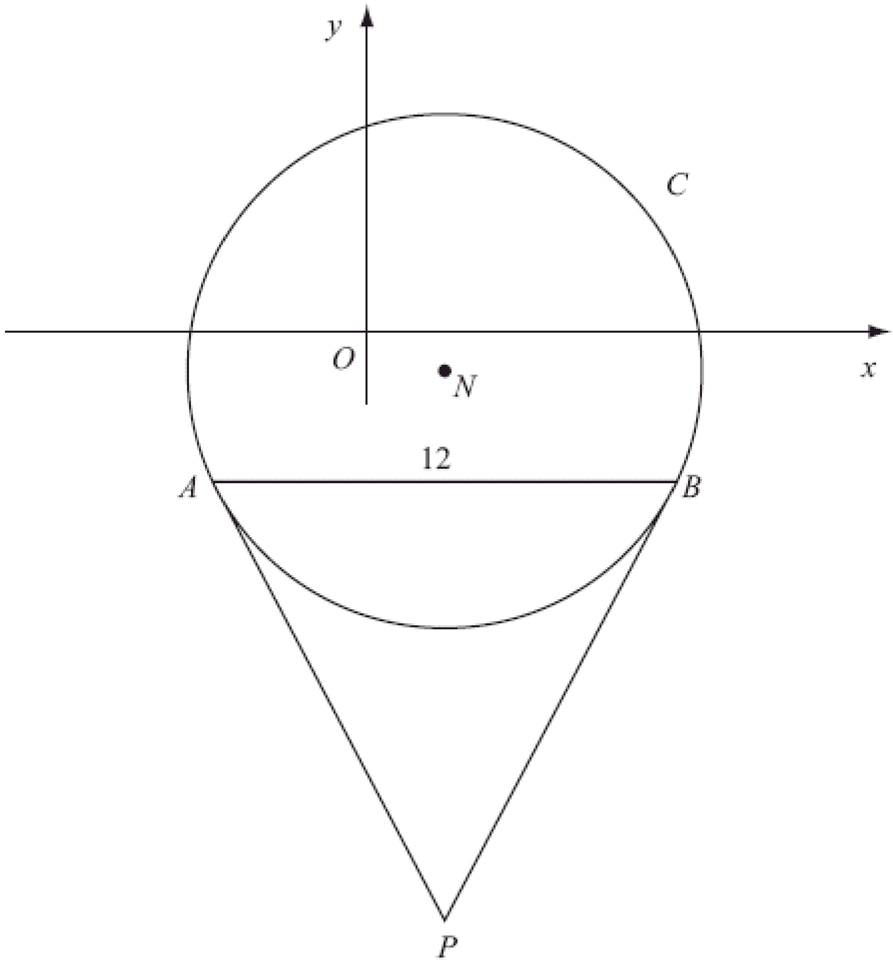
The line l_2 is parallel to l_1 and passes through the mid-point of AB.

Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form.

(3)
(Total 11 marks)

2.



The diagram above shows a sketch of the circle *C* with centre *N* and equation

$$(x - 2)^2 + (y + 1)^2 = \frac{169}{4}$$

(a) Write down the coordinates of *N*.

(2)

(b) Find the radius of *C*.

(1)

The chord AB of C is parallel to the x-axis, lies below the x-axis and is of length 12 units as shown in the diagram above.

(c) Find the coordinates of A and the coordinates of B.

(5)

(d) Show that angle $ANB = 134.8°$, to the nearest 0.1 of a degree.

(2)

The tangents to C at the points A and B meet at the point P.

(e) Find the length AP, giving your answer to 3 significant figures.

(2)
(Total 12 marks)

3. The circle C has equation

$$x^2 + y^2 - 6x + 4y = 12$$

(a) Find the centre and the radius of C.

(5)

The point $P(-1, 1)$ and the point $Q(7, -5)$ both lie on C.

(b) Show that PQ is a diameter of C.

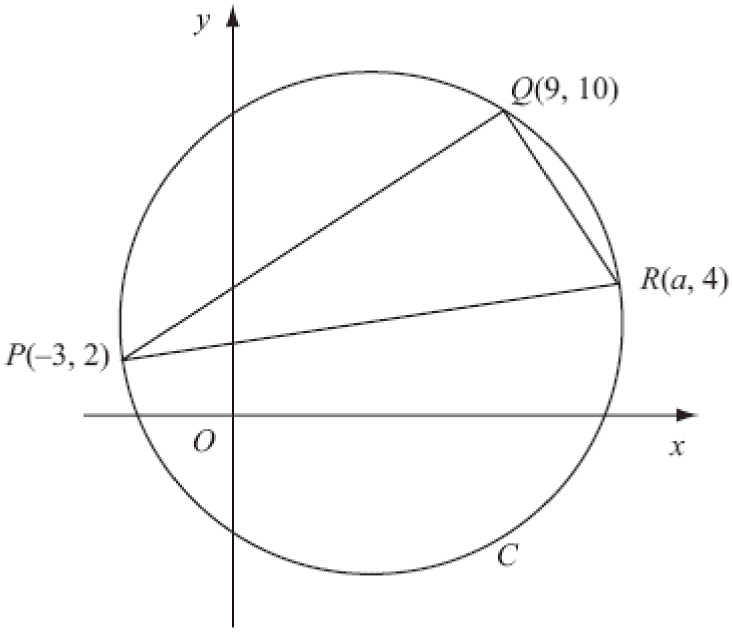
(2)

The point R lies on the positive y-axis and the angle $PRQ = 90°$.

(c) Find the coordinates of R.

(4)
(Total 11 marks)

4.



The points $P(-3, 2)$, $Q(9, 10)$ and $R(a, 4)$ lie on the circle C, as shown in the diagram above.

Given that PR is a diameter of C,

(a) show that $a = 13$,

(3)

(b) find an equation for C.

(5)
(Total 8 marks)

5. The circle C has centre $(3, 1)$ and passes through the point $P(8, 3)$.

(a) Find an equation for C.

(4)

(b) Find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

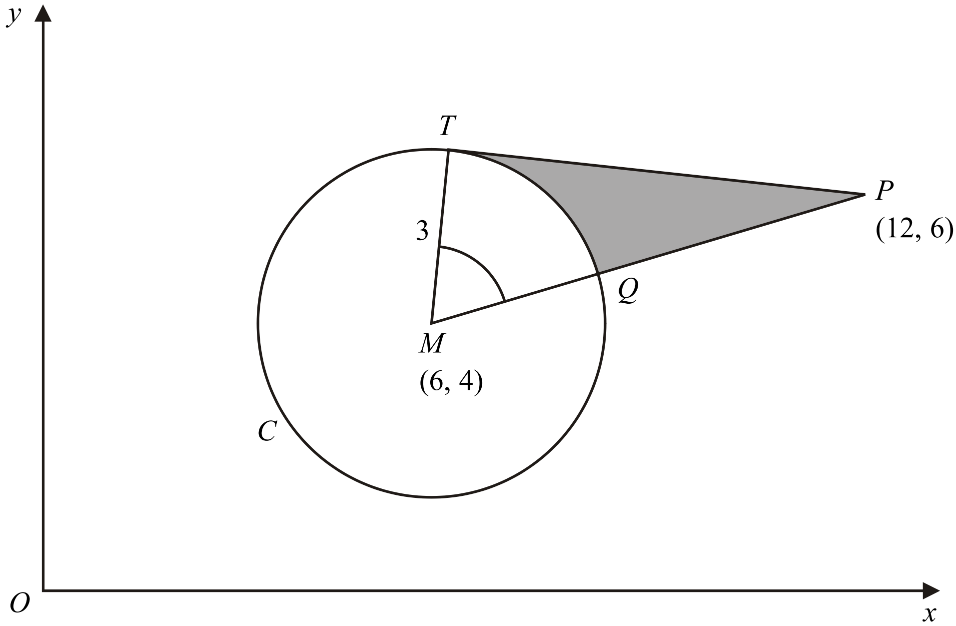
(5)
(Total 9 marks)

6. A circle C has centre M (6, 4) and radius 3.

(a) Write down the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = r^2.$$

(2)



The diagram above shows the circle C. The point T lies on the circle and the tangent at T passes through the point P (12, 6). The line MP cuts the circle at Q.

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

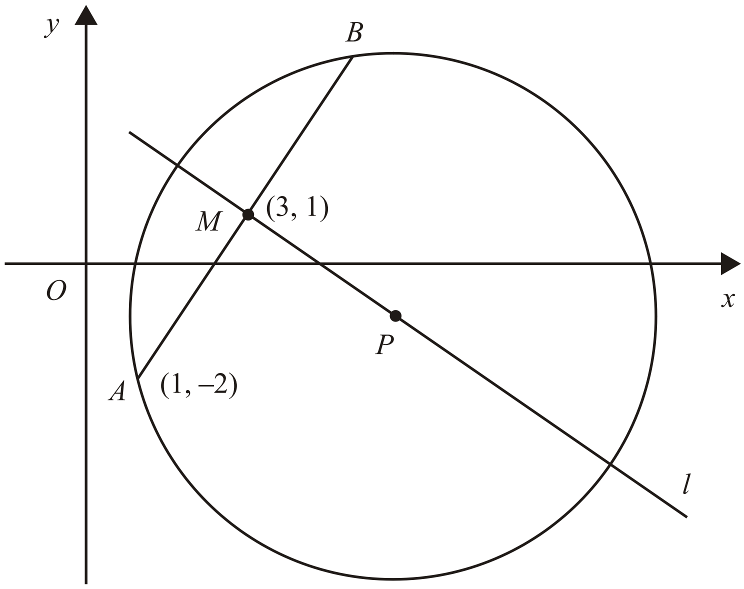
(4)

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in the diagram above.

(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

(5)
(Total 11 marks)

7.



The points A and B lie on a circle with centre P, as shown in the diagram above.
The point A has coordinates (1, –2) and the mid-point M of AB has coordinates (3, 1).
The line l passes through the points M and P.

(a) Find an equation for l.

(4)

Given that the x-coordinate of P is 6,

(b) use your answer to part (a) to show that the y-coordinate of P is –1,

(1)

(c) find an equation for the circle.

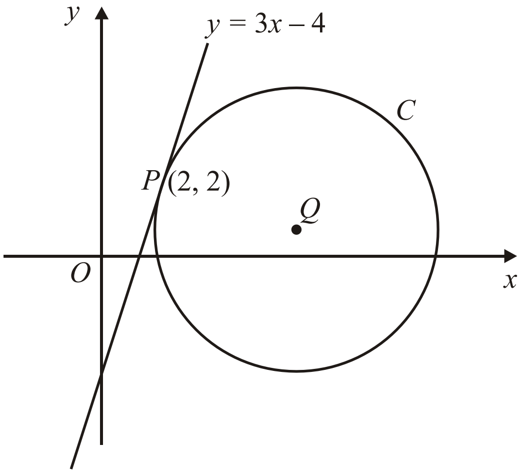
(4)
(Total 9 marks)

8. The line joining the points (–1, 4) and (3, 6) is a diameter of the circle C.

Find an equation for C.

(Total 6 marks)

9.



The line $y = 3x - 4$ is a tangent to the circle C, touching C at the point $P(2, 2)$, as shown in the figure above.

The point Q is the centre of C.

(a) Find an equation of the straight line through P and Q.

(3)

Given that Q lies on the line $y = 1$,

(b) show that the x-coordinate of Q is 5,

(1)

(c) find an equation for C.

(4)
(Total 8 marks)

10. A circle C has radius $\sqrt{5}$ and has its centre at the point with coordinates (4, 3).

(a) Prove that an equation of the circle C is $x^2 + y^2 - 8x - 6y + 20 = 0$.

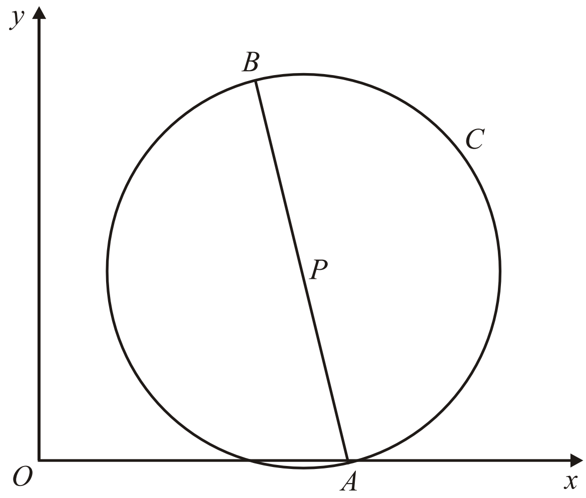
(3)

The line l, with equation $y = 2x$, is a tangent to the circle C.

(b) Find the coordinates of the point where the line l touches C.

(4)
(Total 7 marks)

11.



In the figure above, $A(4, 0)$ and $B(3, 5)$ are the end points of a diameter of the circle C.

Find

(a) the exact length of AB,

(2)

(b) the coordinates of the midpoint P of AB,

(2)

(c) an equation for the circle C.

(3)
(Total 7 marks)

12. Two circles C_1 and have equations

$$(x - 2)^2 + y^2 = 9 \text{ and } (x - 5)^2 + y^2 = 9$$

respectively.

(a) For each of these circles state the radius and the coordinates of the centre.

(3)

(b) Sketch the circles C_1 and on the same diagram.

(3)

(c) Find the exact distance between the points of intersection of C_1 and

(3)
(Total 9 marks)

13. The circle C, with centre at the point A, has equation $x^2 + y^2 - 10x + 9 = 0$.

Find

(a) the coordinates of A,

(2)

(b) the radius of C,

(2)

(c) the coordinates of the points at which C crosses the x-axis.

(2)

Given that the line l with gradient $\frac{7}{2}$ is a tangent to C, and that l touches C at the point T,

(d) find an equation of the line which passes through A and T.

(3)
(Total 9 marks)

14. A circle C_1 has equation

$$x^2 + y^2 - 12x + 4y + 20 = 0.$$

(a) Find the coordinates of the centre of C_1.

(2)

(b) Find the radius of C_1.

(2)

The circle C_1 cuts the x-axis at the points A and B.

(c) Find an equation of the circle with diameter AB.

(6)
(Total 10 marks)

15. The points A and B have coordinates $(5, –1)$ and $(13, 11)$ respectively.

(a) Find the coordinates of the mid-point of AB.

(2)

Given that AB is a diameter of the circle C,

(b) find an equation for C.

(4)
(Total 6 marks)

16. The circle C has centre $(5, 13)$ and touches the x-axis.

(a) Find an equation of C in terms of x and y.

(2)

(b) Find an equation of the tangent to C at the point $(10, 1)$, giving your answer in the form $ay + bx + c = 0$, where a, b and c are integers.

(5)
(Total 7 marks)

17. The point A has coordinates $(2, 5)$ and the point B has coordinates $(–2, 8)$.

Find, in cartesian form, an equation of the circle with diameter AB.

(Total 4 marks)

18. A circle C has equation

$$x^2 + y^2 - 6x + 8y - 75 = 0.$$

(a) Write down the coordinates of the centre of C, and calculate the radius of C.

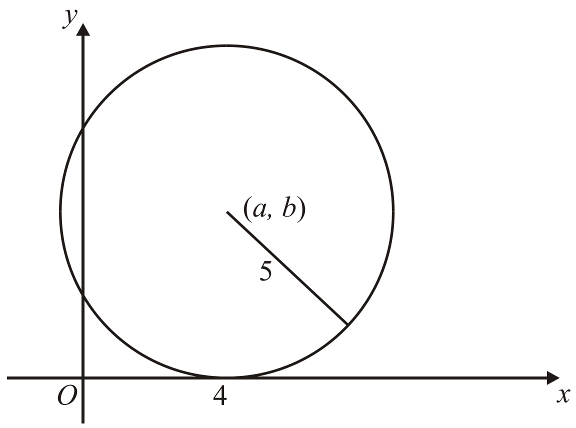
(3)

A second circle has centre at the point (15, 12) and radius 10.

(b) Sketch both circles on a single diagram and find the coordinates of the point where they touch.

(4)
(Total 7 marks)

19.



The circle C, with centre (a, b) and radius 5, touches the x-axis at (4, 0), as shown in the diagram above.

(a) Write down the value of a and the value of b.

(1)

(b) Find a cartesian equation of C.

(2)

A tangent to the circle, drawn from the point $P(8, 17)$, touches the circle at T.

(c) Find, to 3 significant figures, the length of PT.

(3)
(Total 6 marks)

20. The circle C, with centre A, has equation

$$x^2 + y^2 - 6x + 4y - 12 = 0.$$

(a) Find the coordinates of A.

(2)

(b) Show that the radius of C is 5.

(2)

The points P, Q and R lie on C. The length of PQ is 10 and the length of PR is 3.

(c) Find the length of QR, giving your answer to 1 decimal place.

(3)

(Total 7 marks)

1. (a) $(10-2)^2+(7-1)^2$ or $\sqrt{(10-2)^2+(7-1)^2}$ A1

$(x\pm 2)^2+(y\pm 1)^2=k$ (k a positive <u>value</u>)

$(x-2)^2+(y-1)^2=100$ (Accept 10^2 for 100) A1

(Answer only scores full marks) 4

(b) (Gradient of radius =) $\dfrac{7-1}{10-2}=\dfrac{6}{8}$ (or equiv.) Must be seen in part (b) B1

Gradient of tangent $=\dfrac{-4}{3}$ (Using perpendicular gradient method)

$y-7=m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞)

$y-7=\dfrac{-4}{3}(x-10)$ or equiv (ft gradient of <u>radius</u>, dep. on <u>both</u> M marks) A1ft

$\{3y=-4x+61\}$ (N.B. The A1 is only available as <u>ft</u> after B0)

The unsimplified version scores the A mark (isw if necessary...
subsequent mistakes in simplification are not penalised here.
The equation must at some stage be <u>exact</u>, not, e.g. $y=-1.3x+20.3$ 4

<u>**Note**</u>

2nd M: Using (10, 7) to find the equation, in any form, of a straight line through (10, 7), with any numerical gradient (except 0 or ∞).

<u>Alternative:</u> 2nd M: Using (10, 7) and an m value in $y = mx + c$ to find a value of c.

<u>Alternative</u> for first 2 marks (differentiation):

$2(x-2)+2(y-1)\dfrac{dy}{dx}=0$ or equiv. B1

Substitute $x = 10$ and $y = 7$ to find a value for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(This M mark can be awarded generously, even if the attempted ‘differentiation’ is not ‘implicit’).

(c) $\sqrt{r^2-\left(\dfrac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag.

$=\sqrt{10^2-5^2}$ or numerically exact equivalent A1

$PQ\left(=2\sqrt{75}\right)=10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark A1 3

<u>**Note**</u>

<u>Alternatives:</u>

To score must be a <u>fully</u> correct method to obtain $\frac{1}{2}PQ$ or PQ.

1st A1: For alternative methods that find PQ directly, this mark is for an <u>exact numerically correct version</u> of PQ.

[11]

2.	(a)	$N(2, -1)$	B1 B1	2
		<u>Note</u> B1 for 2 (α), B1 for –1		
	(b)	$r = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5$	B1	1
		<u>Note</u> B1 for 6.5 o.e.		
	(c)	Complete Method to find x coordinates, $x_2 - x_1 = 12$ and $\frac{x_1 + x_2}{2} = 2$ then solve		
		To obtain $x_1 = -4, x_2 = 8$	A1ft A1ft	
		Complete Method to find y coordinates, using equation of circle or Pythagoras i.e. let d be the distance below N of A then $d^2 = 6.5^2 - 6^2 \Rightarrow d = 2.5 \Rightarrow y = ..$		
		So $y_2 = y_1 = -3.5$	A1	5
		<u>Note</u> 1st for finding x coordinates – may be awarded if either x co-ord is correct A1ft,A1ft are for α –6 and α +6 if x coordinate of N is α 2nd for a method to find y coordinates – may be given if y co-ordinate is correct A marks is for –3.5 only.		
	(d)	Let $A\hat{N}B = 2\theta \Rightarrow \sin\theta = \frac{6}{"6.5"} \Rightarrow \theta = (67.38)......$		
		So angle ANB is 134.8 *	A1	2
		<u>Note</u> for a full method to find θ or angle ANB (eg sine rule or cosine rule directly or finding another angle and using angles of triangle.) **ft their 6.5 from radius or wrong *y*.** $\left(\cos ANB = \frac{"6.5"^2 + "6.5"^2 - 12^2}{2 \times "6.5" \times "6.5"} = -0.704\right)$		

A1 is a printed answer and must be 134.8 – do not accept 134.76.

(e) *AP* is perpendicular to *AN* so using triangle

ANP $\tan\theta = \dfrac{AP}{\text{"6.5"}}$

Therefore $AP = 15.6$ A1cao 2

Note

for a full method to find *AP*

Alternative Methods

N.B. May use triangle *AXP* where *X* is the mid point of *AB*. Or may use triangle ABP. From circle theorems may use angle $BAP = 67.38$ or some variation.

Eg $\dfrac{AP}{\sin 67.4} = \dfrac{12}{\sin 45.2}$, $AP = \dfrac{6}{\sin 22.6}$ or $AP = \dfrac{6}{\cos 67.4}$ are each worth

[12]

3. (a) $(x-3)^2 - 9 + (y+2)^2 - 4 = 12$ Centre is $(3, -2)$ A1, A1

$(x-3)^2 + (y+2)^2 = 12 + \text{"9"} + \text{"4"}$ $r = \sqrt{12 + \text{"9"} + \text{"4"}} = 5 (\text{or } \sqrt{25})$ A1 5

Note

1st for attempt to complete square. Allow $(x \pm 3)^2 \pm k$, or $(y \pm 2)^2 \pm k$, $k \neq 0$.

1st A1 *x*-coordinate 3, 2nd A1 *y*-coordinate –2

2nd for a full method leading to $r = ...$, with their 9 and their 4,

3rd A1 5 or $\sqrt{25}$

The 1st M can be <u>implied</u> by $(\pm 3, \pm 2)$ but a full method must be seen for the 2nd M.

Where the 'diameter' in part (b) has cle<u>arly</u> been used to answer part (a), no marks in (a), but in this case the (<u>not</u> the A1) for part (b) can be given for work seen in (a).

Alternative

1st for comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark.

2nd for using $r = \sqrt{g^2 + f^2 - c}$. Condone sign errors for this M mark.

(b) $PQ = \sqrt{(7 - -1)^2 + (-5 - 1)^2}$ or $\sqrt{8^2 + 6^2}$

= 10 = 2 × radius, $\therefore$ diam.(N.B. For A1, need a comment or conclusion) A1 2

[ALT: midpt. of PQ $\left(\frac{7+(-1)}{2}, \frac{1+(-5)}{2}\right)$: = (3, –2) = centre: A1]
[ALT: eqn. of PQ $3x + 4y - 1 = 0$: verify (3, –2) lies on this: A1]
[ALT: find two grads, e.g. PQ and P to centre: equal $\therefore$ diameter: A1]
[ALT: show that point S (–1, – 5) or (7, 1) lies on circle:
because $\angle PSQ = 90^\circ$, semicircle $\therefore$ diameter: A1]

(c) R must lie on the circle (angle in a semicircle theorem)… often implied by a diagram with R on the circle or by subsequent working) B1

$x = 0 \Rightarrow y^2 + 4y - 12 = 0$
$(y - 2)(y + 6) = 0$ y = ... (M is dependent on previous M) dM1

$y = -6$ or 2 (Ignore $y = -6$ if seen, and 'coordinates' are not required)) A1 4

Note

1st for setting $x = 0$ and getting a 3TQ in y by using eqn. of circle.
2nd (dep.) for attempt to solve a 3TQ leading to at least one solution for y.
Alternative 1: (Requires the B mark as in the main scheme)
1st M for using (3, 4, 5) triangle with vertices (3, – 2),(0, – 2),(0, y) to get a linear or quadratic equation in y (e.g. $3^2 + (y + 2)^2 = 25$).
2nd M (dep.) as in main scheme, but may be scored by simply solving a linear equation.
Alternative 2: (Not requiring realisation that R is on the circle)
B1 for attempt at $m_{PR} \times m_{QR} = -1$, (NOT m_{PQ}) or for attempt at Pythag. in triangle PQR.
1st for setting $x = 0$, i.e. (0, y), and proceeding to get a 3TQ in y. Then main scheme.
Alternative 2 by 'verification':
B1 for attempt at $m_{PR} \times m_{QR} = -1$, (NOT m_{PQ}) or for attempt at Pythag. in triangle PQR.
1st for trying (0, 2).
2nd (dep.) for performing all required calculations.
A1 for fully correct working and conclusion.

[11]

4. (a) PQ: $m_1 = \frac{10-2}{9-(-3)} \left(= \frac{2}{3}\right)$ **and** QR: $m_2 = \frac{10-4}{9-a}$

Alt

(a) (Alternative method (Pythagoras) Finds **all three** of the following

$(9 - (-3))^2 + (10 - 2)^2$, (*i.e.*208), $(9 - a)^2 + (10 - 4)^2$,
$(a - (-3))^2 + (4 - 2)^2$

Using Pythagoras (correct way around) e.g. $a2 + 6a + 9 = 240 + a2 - 18a + 81$
to form equation

Solve (or verify) for *a*, $a = 13$ (*) A1 3

(b) Centre is at (5, 3) B1

$(r^2 =)\ (10 - 3)^2 + (9 - 5)^2$ or equiv., or $(d^2 =)\ (13 - (-3))^2 + (4 - 2)^2$ A1

$(x - 5)^2 + (y - 3)^2 = 65$ **or** $x^2 + y^2 - 10x - 6y - 31 = 0$ A1 5

Notes

considers gradients of *PQ* and *QR* -must be y difference / *x* difference (or considers three lengths as in alternative method)

Substitutes gradients into product = –1 (or lengths into Pythagoras' Theorem the correct way round)

A1 Obtains $a = 13$ with no errors by solution or verification. Verification can score 3/3.

(b) $m_1 m_2 = -1$: $\frac{8}{12} \times \frac{6}{9-a} = -1$ $a = 13$ (*) A1 3

Alt

Uses $(x - a)^2 + (y - b)^2 = r^2$ **or** $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes
(–3, 2), (9, 10) and (13, 4) then eliminates one unknown
Eliminates second unknown

Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3, r^2 = 65$ A1, A1, B1cao 5

Notes

Geometrical method: **B1** for coordinates of centre – can be implied by use in part (b)

for attempt to find r^2, d^2, r or d(allow one slip in a bracket).

A1 cao. These two marks may be gained implicitly from circle equation

for $(x \pm 5)^2 + (y \pm 3)^2 = k^2$ or $(x \pm 3)^2 + (y \pm 5)^2 = k^2$ ft their (5,3)
Allow k^2 non numerical.

A1 cao for whole equation and rhs must be 65 or $\left(\sqrt{65}\right)^2$, (similarly B1 must be 65 or $\left(\sqrt{65}\right)^2$, in alternative method for (b))

Further alternatives

(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is –3/2 This is
They then proceed using equations of lines through point Q or by using

gradient QR to obtain equation such as $\frac{4-10}{a-9} = -\frac{3}{2}$ (may still have x in this equation rather than a and there may be a small slip)

They then complete to give (a) = 13 **A1** A1

(ii) A long involved method has been seen finding the coordinates of the centre of the circle first.
This can be done by a variety of methods
Giving centre as (c, 3) and using an equation such as
$(c-9)^2 + 7^2 = (c+3)^2 + 1^2$ (equal radii)

or $\frac{3-6}{c-3} = -\frac{3}{2}$ (perpendicular from centre to chord bisects chord)

Then using c (= 5) to find a is

Finally a = 13 **A1** A1

(iii) Vector Method:
States **PQ. QR** = 0, with vectors stated 12i +8j and
$(9 - a)$**i** + **6**j is
Evaluates scalar product so $108 - 12a + 48 = 0$ **(**
solves to give a = 13 (**A1**) A1

[8]

5. (a) $(8-3)^2 + (3-1)^2$ or $\sqrt{(8-3)^2 + (3-1)^2}$ M1A1

$(x \pm 3)^2 + (y \pm 1)^2 = k$ or $(x \pm 1)^2 + (y \pm 3)^2 = k$ (k a positive <u>value</u>)

$(x-3)^2 + (y-1)^2 = 29$ (<u>Not</u> $(\sqrt{29})^2$ or 5.39^2) A1 4

For the M mark, condone <u>one</u> slip <u>inside</u> a bracket,
e.g. $(8-3)^2 + (3+1)^2$, $(8-1)^2 + (1-3)^2$
The first two marks may be gained implicitly from the circle equation.

(b) Gradient of radius = $\frac{2}{5}$ (or exact equiv.) Must be seen or used in (b) B1

Gradient of tangent = $\frac{-5}{2}$ (Using perpendicular gradient method)

$y - 3 = \frac{-5}{2}(x-8)$ (ft gradient of radius, dependent upon <u>both</u> M marks) M1A1ft

$5x + 2y - 46 = 0$ (Or equiv., equated to zero, e.g. $92 - 4y - 10x = 0$) A1 5
(Must have <u>integer</u> coefficients)

2nd M: Eqn. of line through (8, 3), in any form, with any grad. (except 0 or ∞).
If the 8 and 3 are the 'wrong way round', this M mark is only given if a correct general formula, e.g. $y - y_1 = m(x - x_1)$, is quoted.

Alternative:
2nd M: Using (8, 3) and an m value in $y = mx + c$ to find a value of c.
A1ft: as in main scheme.
(Correct substitution of 8 and 3, then a wrong c value will still score the A1ft)

Alternatives for the first 2 marks: (but in these 2 cases the 1st A mark is not ft)
(i) Finding gradient of tangent by implicit differentiation

$2(x - 3) + 2(y - 1)\frac{dy}{dx} = 0$ (or equivalent) B1

Subs. $x = 8$ and $y = 3$ into a 'derived' expression to find a value for dy / dx
(ii) Finding gradient of tangent by differentiation of
$y = 1 + \sqrt{20 + 6x - x^2}$

$\frac{dy}{dx} = \frac{1}{2}(20 + 6x - x^2)^{-\frac{1}{2}}(6 - 2x)$ (or equivalent) B1

Subs. $x = 8$ into a 'derived' expression to find a value for dy / dx

Another alternative:
Using $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
$x^2 + y^2 - 6x - 2y - 19 = 0$ B1
$8x + 3.y, -3(x + 8) - (y + 3) - 19 = 0$ A1ft (ft from circle eqn.)
$5x + 2y - 46 = 0$ A1

[9]

6. (a) $(x - 6)^2 + (y - 4)^2 =; 3^2$ B1; B1 2

Allow 9 for 3^2.

(b) Complete method for MP: $= \sqrt{(12 - 6)^2 + (6 - 4)^2}$

$= \sqrt{40}$ or awrt 6.325 A1

[These first two marks can be scored if seen as part of solution for (c)]

Complete method for $\cos\theta$, $\sin\theta$ or $\tan\theta$

e.g. $\cos\theta = \frac{MT}{MP} = \frac{3}{\text{candidate's }\sqrt{40}}$ (= 0.4743) (θ = 61.6835°)

[If TP = 6 is used, then M0]
θ = 1.0766 rad **AG** A1 4

First can be implied by √40 or √31
For second

May find TP = $\sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either

$\sin\theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}}$ (= 0.8803...) or $\tan\theta = \frac{\sqrt{31}}{3}$ (1.8859..) or cos rule

NB. Answer is given, but allow final A1 if all previous work is correct.

(c) Complete method for area *TMP*; e.g. = $\frac{1}{2} \times 3 \times \sqrt{40}\sin\theta$

= $\frac{3}{2}\sqrt{31}$ (= 8.3516..) allow awrt 8.35 — A1

Area (sector)*MTQ* = 0.5 × 3^2 × 1.0766 (= 4.8446…)
Area *TPQ* = candidate's (8.3516.. – 4.8446..)
= 3.507 awrt — A1 — 5
[Note: 3.51 is A0]

First (alternative) $\frac{1}{2} \times 3 \times \sqrt{40-9}$

Second allow even if candidate's value of θ used.
(Despite being given!)

[11]

7. (a) Gradient of *AM*: $\frac{1-(-2)}{3-1} = \frac{3}{2}$ or $\frac{-3}{-2}$ — B1

Gradient of *l*: $= -\frac{2}{3}$ M: use of $m_1m_2 = -1$, or equiv.

$y - 1 = -\frac{2}{3}(x-3)$ or $\frac{y-1}{x-3} = -\frac{2}{3}$ [$3y = -2x + 9$] (Any equiv. form) — M1A1 — 4

2nd eqn. of a straight line through (3, 1) with any gradient except 0 or ∞.

Alternative: Using (3, 1) in $y = mx + c$ to find a value of c
scores but an equation (general or specific) must be seen.

Having cords the wrong way round, e.g. $y - 3 = -\frac{2}{3}(x-1)$,

loses the 2nd M mark unless a correct general formula is seen,
e.g. $y - y_1 = m(x - x_1)$.

If the point *P*(6, –1) is used to find the gradient of *MP*, maximum marks are (a) B0 M0 A1 (b) B0.

(b) $x = 6$: $3y = -12 + 9 = -3$ $y = -1$ (or show that for $y = -1$, $x = 6$)(*) — B1 — 1
(A conclusion is not required)

(c) (r^2 =) $(6-1)^2 + (-1-(-2))^2$ M: Attempt r^2 or r — M1A1
N.B. Simplification is not required to score M1A1
$(x \pm 6)^2 + (y \pm 1)^2 = k$, $k \neq 0$ (Value for k not needed, could be r^2 or r)
$(x-6)^2 + (y+1)^2 = 26$ (or equiv.) — A1 — 4

Allow $(\sqrt{26})^2$ or other exact equivalents for 26.
(But... $(x-6)^2 + (y--1)^2 = 26$ scores M1A0)

(Correct answer with no working scores full marks)

1st Condone <u>one</u> slip, numerical or sign, <u>inside</u> a bracket.

Must be attempting to use points $P(6, -1)$ and $A(1, -2)$, or perhaps P and B.
(Correct coordinates for B are (5, 4)).

1st M: alternative is to use a complete Pythag. method on triangle MAP, n.b. $MP = MA = \sqrt{13}$.

<u>Special case:</u>
If candidate persists in using <u>their</u> value for the y-coordinate of P instead of the given –1, allow the M marks in part (c) if earned.

[9]

8. Centre $\left(\frac{-1+3}{2}, \frac{6+4}{2}\right)$, i e (1, 5) A1

$r = \frac{\sqrt{(3-(-1))^2 + (6-4)^2}}{2}$

or $r^2 = (1-(-1))^2 + (5-4)^2$ or $r^2 = (3-1)^2 + (6-5)^2$ o.e.

$(x-1)^2 + (y-5)^2 = 5$ A1, A1 6

Some use of correct formula in x *or* y coordinate. Can be implied.

Use of $\left(\frac{1}{2}(x_A - x_B), \frac{1}{2}(y_A - y_B)\right) \to (-2, -1)$ or (2, 1) is M0 A0

but watch out for use of $x_A + \frac{1}{2}(x_A - x_B)$ etc which is okay.

(1, 5)
(5, 1) gains A0. A1

Correct method to find r or r^2 using given points or f.t. from their centre.
Does not need to be simplified.

Attempting radius = $\sqrt{\frac{(\text{diameter})^2}{2}}$ is an incorrect method, so M0.

N.B. Be careful of labelling: candidates may not use d for diameter and r for radius.
Labelling should be ignored.
Simplification may be incorrect – mark awarded for correct method.
Use of $\sqrt{(x_1 - x_2)^2 - (y_1 - y_2)^2}$ is M0.

Write down $(x \pm a)^2 + (y \pm b)^2$ = any constant (a letter or a number).
Numbers do not have to be substituted for a, b and if they are they can be wrong.

LHS is $(x-1)^2 + (y-5)^2$. Ignore RHS. A1

RHS is 5.
Ignore subsequent working. Condone use of decimals that leads to exact 5. A1

Or correct equivalents, e.g. $x^2 + y^2 - 2x - 10y + 21 = 0$.

Alternative – note the order of the marks needed for ePEN.

As above.

As above. A1

$x^2 + y^2$ + (constant)x + (constant)y + constant = 0. Numbers do not have to be substituted for the constants and if they are they can be wrong. 3rd

Attempt an appropriate substitution of the coordinates of their centre (i.e. working with coefficient of x and coefficient of y in equation of circle) and substitute (–1, 4) or (3, 6) into equation of circle. 2nd

$-2x - 10y$ part of the equation $x^2 + y^2 - 2x - 10y + 21 = 0$. A1

$+21 = 0$ part of the equation $x^2 + y^2 - 2x - 10y + 21 = 0$. A1

Or correct equivalents, e.g. $(x - 1)^2 + (y - 5)^2 = 5$.

[6]

9. (a) Gradient of PQ is $-\frac{1}{3}$ B1

$y - 2 = -\frac{1}{3}(x - 2)$ $(3y + x = 8)$ A1 3

eqn. of a straight line through (2, 2) with any gradient except 3, 0 or ∞.

<u>Alternative</u>: Using (2, 2) in y = mx + c to find a value of c scores but an equation (general or specific) must be seen.

If the given value x = 5 is used to find the gradient of PQ, maximum marks are (a) B0 A1 (b) B0.

(b) $y = 1$: $3 + x = 8$ $x = 5$ B1 1

(c) $(\text{"5"} - 2)^2 + (1 - 2)^2$ M: Attempt PQ^2 or PQ A1

$(x - 5)^2 + (y - 1)^2 = 10$ M: $(x \pm a)^2 + (y \pm b)^2 = k$ A1 4

For the first condone <u>one</u> slip, numerical or sign, <u>inside</u> a bracket.

The first can be scored if <u>their</u> x-coord. is used instead of 5.

For the second allow any equation in this form, with non-zero a, b and k.

[8]

10.	(a)	Uses circle equation		
		$(x-4)^2 + (y-3)^2 = (\sqrt{5})^2$	A1	
		Multiplies out to give $x^2 - 8x + 16 + y^2 - 6y + 9 = 5$ and thus		
		$x^2 + y^2 - 8x - 6y + 20 = 0$	A1	3
		Alternative		
		Or states equation of circle is $x^2 + y^2 + 2gx + fy + c = 0$		
		has centre (–g, –f) and so g = – 4 and f = – 3		
		Uses $g^2 + f^2 - c = r^2$ to give $c = 3^2 + 4^2 - \sqrt{5}$, i.e. $c = 20$	A1	
		$x^2 + y^2 - 8x - 6y + 20 = 0$	A1	3
	(b)	$y = 2x$ meets the circle when $x^2 + (2x)^2 - 8x - 6(2x) + 20 = 0$		
		$5x^2 - 20x + 20 = 0$	A1	
		Solves and substitutes to obtain $x = 2$ and $y = 4$. Coordinates are (2, 4)	A1	4
		Or Implicit differentiation attempt , $2x + 2y\dfrac{dy}{dx} - 8 - 6\dfrac{dy}{dx} = 0$	A1	
		Uses $y = 2x$ and $\dfrac{dy}{dx} = 2$ to give $10x - 20 = 0$.		
		Thus $x = 2$ and $y = 4$	A1	4

[7]

11.	(a)	$(AB)^2 = (4-3)^2 + (5)^2$ [= 26]		
		$AB = \underline{\sqrt{26}}$	A1	2
	(b)	$p = \left(\dfrac{4+3}{2}, \dfrac{5}{2}\right)$		
		$= \underline{\left(\dfrac{7}{2}, \dfrac{5}{2}\right)}$	A1	2
	(c)	$(x - x_p) + (y - y_p) = \left(\dfrac{AB}{2}\right)^2$	LHS RHS	
		$(x - 3.5)^2 + (y - 2.5)^2 = 6.5$	oe A1 c.a.o	3

[7]

(a) for an expression for AB or AB^2 N.B. $(x_1 + x_2)^2 + ...$ is M0

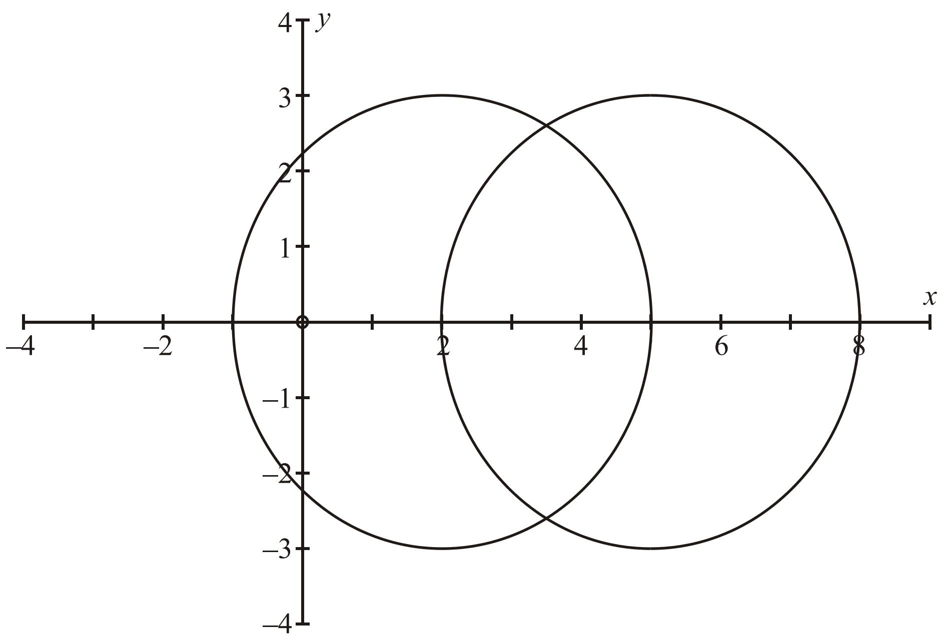
(b) for a full method for x_p

(c) 1st for using their x_p and y_p in LHS
2nd for using their AB in RHS

N.B. $x^2 + y^2 - 7x - 5y + 12 = 0$ scores, of course, 3/3 for part (c).
Condone use of calculator approximations that lead to correct answer given.

12. (a) $r = 3$ (both circles) B1
Centres are at (2, 0) and (5, 0) B1, B1 3

(b)



1st circle correct quadrants centre on x axis B1
2nd circle correct quadrants centre on x axis B1
circles same size and passing through centres of other circle B1 3

	(c)	Finds circles meet at $x = 3.5$, by mid point of centres or by solving algebraically		
		Establishes $y = \pm\frac{3\sqrt{3}}{2}$, and thus distance is $3\sqrt{3}$.	A1	3
		Or uses trig or Pythagoras with lengths 3, angles 60 degrees, or 120 degrees. Complete and accurate method to find required distance		
		Establishes distance is $3\sqrt{3}$.	A1	3

[12]

13.	(a)	Centre (5, 0) (or $x = 5, y = 0$)	B1 B1	2
		(0, 5) scores B1 B0		
	(b)	$(x \pm a)^2 \pm b \pm 9 + (y \pm c)^2 = 0 \Rightarrow r^2 = \ldots$ or $r = \ldots$, Radius = 4	A1	2
	(c)	(1, 0), (9, 0)	B1ft, B1ft	2
		allow just $x = 1$, $x = 9$		
	(d)	Gradient of $AT = -\frac{2}{7}$	B1	
		$y = -\frac{2}{7}(x - 5)$	m1 A1ft	3
		Equation of straight line through centre, <u>any</u> gradient (except 0 or ∞) *(The equation can be in any form).*		
		A1ft: Follow through centre, but gradient must be $-\frac{2}{7}$		

[9]

14.	(a)	$x^2 + y^2 - 12x + 4y + 20 = 0$ $(x - 6)^2 + (y + 2)^2 + k = 0$	A1	2
	(b)	$(x - 6)^2 + (y + 2)^2 = 20$ Radius = <u>$\sqrt{20}$</u>	A1	2

Use of formulae

(a) 2g = –12, 23f = 4, c = 20

Centre (–g, –f)

Centre <u>(6, –2)</u> A1 2

(b) Radius = $\sqrt{36 + 4 - 20}$

Radius <u>$\sqrt{20}$</u> A1 2

(c) $x^2 + - 12x + 20 = 0$

$\Rightarrow x = 2;\ 10$ A1

Centre of is <u>(6,0)</u> B1ft

Radius of is 6 – 2 or 10 – 6 = <u>4</u> B1ft

Equation of is $(x - 6)^2 + y^2 = 4^2$ M1ft A1 6

centered radius

o.e.

[14]

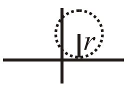
15. (a) $\left(\dfrac{5+13}{2}, \dfrac{-1+11}{2}\right), = \underline{\underline{(9,5)}}$ A1 2

(b) $r^2 = (9 - 5)^2 + (5 - - 1^2)$ (=52) or $r^2 = (13 - 9)^2 + (11 - 5)^2$ (=52) (or equiv.)

Equation of circle: $(x - 9)^2 + (y - 5)^2 = 52$ (or equiv.) A1ft A1 4

[6]

16. (a)



Eqn: $(x - 5)^2 + (y - 13)^2 = r^2$

$r = 13$ $\underline{(x - 5)^2 + (y - 13)^2 = 13^2}$ A1 2

(b) Differentiate: $2(x - 5) + 2(y - 13)\dfrac{dy}{dx} = 0$ Attempt to diff.

At (10, 1) $(2 \times 5) + 2 \times (-12)\dfrac{dy}{dx} = 0$ Use of (10, 1)

$\dfrac{dy}{dx} = \dfrac{10}{24}$ or $\dfrac{5}{12}$ A1

Eqn. of tangent: $y - 1 = \dfrac{5}{12}(x - 10)$ f.t. on their m

<u>$5x - 12y - 38 = 0$</u> A1 5

[7]

17. **Either**

Obtains centre (0, 6.5) B1

Finds radius or diameter by Pythagoras Theorem, to obtain

$r = 2.5$ or $r^2 = 6.25$ A1

$x^2 + (y - 6.5)^2 = 2.5^2$ or $x^2 + y^2 - 13y + 36 = 0$ B1 4

Or

$\frac{y-8}{x+2} \times \frac{y-5}{x-2} = -1$

For either gradient B1
Gradients multiplied and put = to –1 M1A1

$X^2 + y^2 - 13y + 36 = 0$ B1 4

Or

Obtains centre (0, 6.5) B1

$x^2 + (y - 6.5)^2 = r^2$ or $x^2 + y^2 - 13y + c = 0$ B1

substitutes either (2, 5) or (–2, 8)

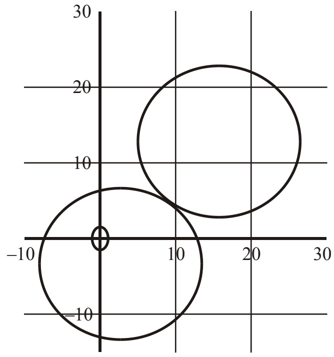
$x^2 + (y - 6.5)^2 = 2.5^2$ or $x^2 + y^2 - 13y + 36 = 0$ A1 4

[4]

18. (a) Centre is at (3,–4) B1

radius = $\sqrt{(3^2 + (-4)^2 - -75)} = 10$ A1 3

(b)



1st circle
2st circle
Circles touching
at (9, 4)

1st circle B1
2nd circle B1
Circles touching B1
At (9 , 4) B1 4

[7]

19. (a) $a = 4$, $b = 5$ (both are required) B1 1

(b) $(x - 4)^2 + (y - 5)^2 = 25$ A1 ft 2

(c) Finding the distance between centre and (8, 17), $\sqrt{[(8-a)^2 + (17-b)]^2}$

Complete method to find PT, i.e. use Pythagoras theorem and subtraction.

$PT = 11.6$ A1 3

[6]

20.	(a)	$(x - 3)^2 + (y + 2)^2$ (= 9 + 4 + 12) *Attempt to complete the square*		
		$\therefore$ Centre is at (3, – 2)	A1	2
	(b)	$(\ldots)^2 + (\ldots)^2 = 12 + 4 + 9 = 25 = 5^2$ *ft their centre*		
		$\therefore$ Radius = 5	A1	2
	(c)	PQ = 10 means PQ is a diameter and so angle PRQ is 90°		
		Pythagoras' Theorem gives $QR^2 = 10^2 - 3^2 = 91$		
		So QR = 9.54…= 9.5 (1 d.p.)	A1	3

[7]

1. In part (a), most candidates were able to write down an expression for the radius of the circle (or the square of the radius). Most were also familiar with the form of the equation of a circle, although some weaker candidates gave equations of straight lines. Sometimes radius was confused with diameter, sometimes (10, 7) was used as the centre instead of (2, 1) and sometimes the equation of the circle was given as $(x-2)^2+(y-1)^2=10$ instead of $(x-2)^2+(y-1)^2=10^2$.

Many candidates knew the method for finding the equation of the tangent at (10, 7). Typical mistakes here were to invert the gradient of the radius *AB*, to find a line parallel to the radius and to find a line through the centre of the circle.

Part (c), for which there was a very concise method, proved difficult even for able candidates. Those who drew a simple sketch were sometimes able to see that the length of the chord could easily be found by using Pythagoras' Theorem, but a more popular (and very lengthy) approach was to find the equation of the chord, to find the points of intersection between the chord and the circle, and then to find the distance between these points of intersection. This produced a great deal of complicated algebra and wasted much time. Mistakes usually limited candidates to at most 1 mark out of 3, but a few produced impressively accurate algebra. Weaker candidates often made little progress with this question.

2. (a) and (b) Most candidates obtained the first three marks for giving the centre and the radius of the circle, but some gave the centre as (–2, 1) and a few failed to find the square root of 169/4 and gave 42.25 as the radius.

(c) Diagrams and use of geometry helped some candidates to find the coordinates of *A* and *B* quickly and easily. Others used algebraic methods and frequently made sign errors. A common mistake was to put $y = 0$ in the equation of the circle. This was not relevant to this question.

(d) Use of the cosine rule on triangle *ANB* was a neat method to show this result. Others divided triangle *ANB* into two right angled triangles and obtained an angle from which *ANB* could be calculated.

(e) This part was frequently omitted and there were some long methods of solution produced by candidates. It was quite common to see candidates obtain equations of lines, coordinates of *P* and use coordinate geometry to solve this part even though there were only two marks available for this. Simple trigonometry was quicker and less likely to lead to error. $6.5 \times \tan ANP$ gave the answer directly.

3. Many candidates had difficulty with this question, with part (c) being particularly badly answered.

In part (a) the method of completing the square was the most popular approach, but poor algebra was often seen, leading to many incorrect answers. Although the correct centre coordinates (3, – 2) were often achieved (not always very convincingly), the radius caused rather more problems and answers such as $\sqrt{12}$ appeared frequently. Some candidates inappropriately used the information about the diameter in part (b) to find their answers for part (a), scoring no marks.

There were various possible methods for part (b), the most popular of which were either to show that the mid-point of *PQ* was the centre of the circle or to show that the length of *PQ* was twice the radius of the circle. Provided that either the centre or the radius was correct in part (a), candidates therefore had at least two possible routes to success in part (b), and many scored both marks here. Some, however, thought that it was sufficient to show that both *P* and *Q* were on the circle.

Part (c) could have been done by using the fact that the point *R* was on the circle (angle in a semicircle result), or by consideration of gradients, or by use of Pythagoras' Theorem. A common mistake in the 'gradient' method was to consider the gradient of *PQ*, which was not directly relevant to the required solution.

Many candidates were unable to make any progress in part (c), perhaps omitting it completely, and time was often wasted in pursuing completely wrong methods such as finding an equation of a line perpendicular to *PQ* (presumably thinking of questions involving the tangent to a circle).

(e) This part was frequently omitted and there were some long methods of solution produced by candidates. It was quite common to see candidates obtain equations of lines, coordinates of *P* and use coordinate geometry to solve this part even though there were only two marks available for this. Simple trigonometry was quicker and less likely to lead to error. 6.5 × tan *ANP* gave the answer directly.

4. Part (a) caused much more of a problem than part (b). A large number of solutions did not really provide an adequate proof in the first part of this question. The original expected method, involving gradients, was the least frequently used of the three successful methods. Finding the three lengths and using Pythagoras was quite common although successful in a limited number of cases – there were many instances of equations being set up but abandoned when the expansion of brackets started to cause problems. Finding the gradient of QR as –3/2 and substituting to find the equation of the line for QR before using y = 4 to get a, was usually well done. Some used verification but in many cases this led to a circular argument.

In part (b) the centre was often calculated as (8, 3) or (8, 1) indicating errors with negative signs. There were several instances of (5, 4) arising from (4 + 2)/2 being thought to be 4 – maybe cancelling the 2's? The length of PQ was usually correct but frequently thought to be the radius rather than the diameter. The equation of a circle was well known but weaker candidates in some cases took points on the circumference as the centre of the circle in their equation, showing lack of understanding.

5. In part (a), most candidates were able to gain the first two marks for attempting to find the radius. The form of equation for a circle was generally well known, but occasionally radius and diameter were confused. A few candidates felt that a mid-point calculation was required at some stage of the working and a few offered $(x - 3)^2 + (x - 1) = 29$ as the circle equation.
Part (b) was less well done than part (a) but most candidates made some progress. There were a

few who could not accurately calculate the gradient of the radius, then others who did not seem to realise that the tangent was perpendicular to the radius. Candidates would have found a simple sketch beneficial here. Those who tried to find the gradient by differentiating the equation of the circle were almost always unsuccessful, since methods such as implicit differentiation were not known. The final mark was often lost through careless arithmetical errors or failure to understand the term integer. Good candidates, however, often produced concise, fully correct solutions to this question.

6. Part (a) provided 2 marks for the majority of candidates but it was surprising, as the form was given, to see such "slips" as $(x - 6) + (y - 4) = 9$ or $(x - 6)^2 - (y - 4)^2 = 9$. There were some good solutions to part (b) but this did prove to be quite discriminating: Many candidates did not really attempt it; some actually used the given answer to calculate TP or PM, and then used these results to show that angle TMQ = 1.0766; and a large number of candidates made the serious error of taking TP = 6. It was disappointingly to see even some of the successful candidates using the cosine rule in triangle TMP, having clearly recognised that it was right-angled.

Part (c) was answered much better, with most candidates having a correct strategy. However, there were some common errors: use of the wrong sides in ½absinC; careless use of Pythagoras to give TP = $7(\sqrt{40+9})$; mixing up the formulae for arc length and sector area; and through inaccuracy or premature approximation, giving answers like 3.51 or 3.505.

7. In general, this question was very well done with many candidates scoring full marks. Part (a) was usually correct, with most candidates realising that the required straight line l had to be perpendicular to the given chord. Some candidates unnecessarily found the coordinates of the point B, using a mid-point formula. Others, again unnecessarily, found the equation of the line AB. For most, part (b) provided useful verification of the accuracy of their equation of l, but a few persisted with a wrong y-coordinate for P despite $y = -1$ being given.
Those who failed in the first two parts of the question were still able to attempt the equation of the circle in part (c). This part was, however, where many lost marks. A common mistake was to calculate the length of PM and to use this as the radius of the circle, and even those who correctly identified PA as the radius sometimes made careless sign errors in their calculations. Some candidates knew the formula $(x - a)^2 + (y - b)^2 = r^2$ but seemed unsure of how to use it, while others gave a wrong formula such as $(x - a)^2 - (y - b)^2 = r^2$ or $(x - a)^2 + (x - b)^2 = r^2$ or $(x - a) + (y - b) = r^2$. The point (3, 1) was sometimes used as the 'centre'.

8. Most candidates were able to state the general equation of a circle in the form $(x - a)^2 + (y - b)= r^2$. However, it was more common to see the coordinates of a mid-point misquoted as $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$ and the formula for the distance between two points misquoted as $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$. Some candidates were not able to make any progress beyond stating the general equation of a circle and a few found equations of straight lines. However, most found the coordinates of the mid-point, attempted to find the radius and substituted their values into the equation of the circle. There was some confusion between the diameter and radius; a common error was to give r^2 as $\frac{d^2}{2}$. Some did not simplify their $(\sqrt{5})^2$ and others confused r and r^2. A few candidates successfully used $x^2 + y^2 + 2gx + 2fy + c = 0$ and occasionally a successful solution was obtained by using a general point $P(x, y)$ on the circle, the equations of two straight lines and the result that the angle subtended by a diameter is 90°.

9. There were many correct solutions to part (a), with most candidates realising that the required straight line *PQ* had to be perpendicular to the tangent. Inappropriately, a few looked ahead to the given information for part (b), immediately taking *Q* as (5, 1) and scoring no more than 2 marks out of 4 for parts (a) and (b) combined. For most, part (b) provided useful verification of the accuracy of their equation for *PQ* Those who failed in the first two parts of the question were still able to attempt the equation of the circle in part (c), but this part was not particularly well done, only about half the candidates being able to produce a completely correct equation. Some did not realise that they needed to calculate the radius of the circle, while others were unsure of the significance of a , b and r in $(a - x)^2 + (b - y)^2 = r^2$ Some used (2, 2) as the 'centre', some used (1, 5) instead of (5, 1) as the centre, and some confused radius and diameter.

10. In part (a) most candidates were familiar with the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ and showed sufficient steps in their working towards the given answer. Candidates using $x^2 + y^2 + 2gx + 2fy + c = 0$ were less successful, often giving muddled or incomplete solutions. Part (b) was also well done. Some candidates lost the final A mark when, having found $x = 2$, they substituted this into the equation of the circle (rather than into $y = 2x$). This gave them two values of y and they were required to pick out the correct value of y for the final mark. A few candidates used implicit differentiation to reach $\frac{dy}{dx} = \frac{8 - 2x}{2y - 6}$ but then did not proceed to substitute $\frac{dy}{dx} = 2$ and $y = 2x$ to find the required value of x.

11. Most candidates knew how to find the length of *AB* and they usually gave the exact surd form of the answer although a number went on to use a calculator approximation in part (c). Part (b) was very well done with over 90% of candidates giving the correct coordinates for *P*. The formula for the equation of the circle in part (c) though was not so well known. Most attempted to use $(x - x_p)2 + (y - y_p)^2 = r^2$ but some had $(x + x_p)^2$...or $(x - x_p)^2 - (y - y_p)^2$ and there were a number of candidates who did not appreciate that the right-hand side of the formula required r not *AB* or they forgot the square. Some realized that they required $\left(\frac{\sqrt{26}}{2}\right)^2$ but they were unable to evaluate this correctly with 13 being a common error. There were a number of fully correct solutions but this was one of the few places on the paper where the success rate fell below 50%.

12. Part (a) was found to be easy by most candidates though some did give themselves extra work when finding the centre and radius of the circles, especially if they used the f, g, c method rather than the $(x-a)^2 + (y-b)^2 = r^2$ form of the equation of a circle. The diagrams were well drawn in part (b), with circles clearly in the correct quadrants and in most cases appearing to be the same size and to pass through the centre of the other circle. In part (c) many did not give exact answers, instead resorting to the calculator. The usual method of solution was algebraic rather than geometric.

13. Although some candidates produced very good solutions to parts (a) and (b), others were clearly confused or produced poor algebra. Many were unable to perform the necessary manipulation to find the centre and radius of the circle, and seemed unwilling to accept that the centre could lie on the x-axis, sometimes interpreting the '9' in the equation as '$9y$'. The technique of 'completing the square' was often badly handled.

Fortunately, parts (c) and (d) were accessible despite failure in (a) and (b). The usual method in part (c) was to substitute $y = 0$ into the equation of the circle, then to solve the resulting quadratic equation. Many candidates completed this correctly, although $x = 0$ instead of $y = 0$ was sometimes seen. Success in part (d) was rather variable, but it was clear that a diagram helped to clarify the demand here. Those who realised that the required line was perpendicular to the tangent often proceeded to produce a correct equation, while others simply used the given gradient $\frac{7}{2}$ in their straight line equation. A few attempts involving differentiation were seen, but these were almost invariably unsuccessful.

14. This was a popular question. Candidates were fully prepared for this topic and few had difficulties at any stage. Even the weaker candidates were able to gain high marks, with clear precise methods, well presented. It was pleasing to see good diagrams clarifying solutions. A few candidates did have problems in part (b), confusing an initial good start of putting $y = 0$ into the original circle to solve for x, with finding new y values from the original circle after obtaining the x coordinates for the ends of the diameter.

15. Most candidates found the mid-point correctly in part (a). The commonest mistake was to use $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$ leading to (4, 6). There was some confusion in part (b) between the radius and the diameter and some candidates had difficulty in halving the diameter when in surd form. Most used $(x-9)^2 + (y-5)^2 = r^2$ to write down the equation of the circle as intended but some misquoted the formula with + signs, others used the diameter instead of the radius and some used A or B instead of the mid-point. A few candidates tried to use "f,g and c" formulae but usually without success. An interesting and successful alternative approach used the angle in a semicircle theorem. By defining a general point $P(x, y)$ on the circle and simply equating the product of the gradients AP and BP to -1, the equation of the circle can be found without using the mid-point or finding the radius.

16. In part (a) some confusion arose in determining the radius. A diagram would have helped many candidates see it was 13 and not 5. Candidates who used the formula $x^2 + y^2 + 2gx + 2fy + c = 0$ invariably ran into numerical difficulties when trying to establish the value of c. The majority of successful candidates simply wrote down $(x-5)^2 + (y-13)^2 = 13^2$ and moved on. There were two main approaches used in part (b). The most common involved implicit differentiation and was generally carried out confidently and accurately. As usual a few introduced an extra $\frac{dy}{dx} =$ on their first line but they ignored it in the subsequent working. The second popular approach to this part of the question involved finding the gradient of the radius and then using the perpendicular gradient rule to find the gradient of the tangent. A few candidates tried to quote formulae for the equation of a tangent to a circle but this approach is not recommended and was rarely successful.

17. This was a fairly accessible first question, but answers included a number of errors. The most common error was to use the radius as the distance between the given points (whereas this was the diameter). Sign errors were seen in mid-point and distance formulae and some candidates assumed that one of the given points was the centre of the circle.The $(x - a)^2 + (y - b)^2 = r^2$ form of equation of the circle resulted in more correct responses than starting with $x^2 + y^2 + 2gx + 2fy + c = 0$.

18. In part (a) candidates used completion of squares and the f, g, c method in roughly equal numbers, both methods being largely successful, with the occasional sign slip.

In part (b) the sketching was not good. Many drawings were far from circular, and candidates did not appreciate the hint that they had been given about the circles touching. Many marks were lost due to drawing incomplete circles and the two circles were often of very different sizes. Very few candidates noticed that all they needed to do to find the contact point was to find the midpoint of the line joining the centres. Far too many embarked on a page or more of work, setting up and failing to solve equations to find the point of intersection. Again a large number of the candidates used graph paper.

19. Parts (a) and (b) were usually answered well. Those who drew a diagram were at an advantage in part (c). They used a method which included use of Pythagoras' theorem to obtain their answer. It was disappointing that a number of these candidates misapplied Pythagoras, they squared and added instead of squaring and subtracting to find their final answer. Those who used the general equation of a tangent to a circle, or who began by differentiating the equation of the circle gave themselves a much more difficult task and usually made the error of finding the equation of a tangent at the point (8, 17), which was not a point on the circle and so did not yield a solution to the problem. A few candidates knew the formula for the distance from an external point.

20. No Report available for this question.