COORDINATE GEOMETRY — *Answers*

1 **a** $x^2 + y^2 = 25$ **b** $(x-1)^2 + (y-3)^2 = 4$ **c** $(x-4)^2 + (y+6)^2 = 1$

d $(x+1)^2 + (y+8)^2 = 9$ **e** $(x+\frac{1}{2})^2 + (y-\frac{1}{2})^2 = \frac{1}{4}$ **f** $(x+3)^2 + (y-9)^2 = 12$

2 **a** centre (0, 0) radius 4 **b** centre (6, 1) radius 9 **c** centre (–1, 4) radius 11

d centre (7, 0) radius 0.3 **e** centre (–2, –5) radius $4\sqrt{2}$ **f** centre (8, –9) radius $6\sqrt{3}$

3 **a** $x^2 + (y-2)^2 - 4 + 3 = 0$
$x^2 + (y-2)^2 = 1$
centre (0, 2) radius 1

b $(x-1)^2 - 1 + (y-5)^2 - 25 - 23 = 0$
$(x-1)^2 + (y-5)^2 = 49$
centre (1, 5) radius 7

c $(x+6)^2 - 36 + (y-4)^2 - 16 + 36 = 0$
$(x+6)^2 + (y-4)^2 = 16$
centre (–6, 4) radius 4

d $(x-1)^2 - 1 + (y+8)^2 - 64 = 35$
$(x-1)^2 + (y+8)^2 = 100$
centre (1, –8) radius 10

e $(x-4)^2 - 16 + (y+3)^2 - 9 = 0$
$(x-4)^2 + (y+3)^2 = 25$
centre (4, –3) radius 5

f $(x+5)^2 - 25 + (y-1)^2 - 1 - 19 = 0$
$(x+5)^2 + (y-1)^2 = 45$
centre (–5, 1) radius $3\sqrt{5}$

g $x^2 + y^2 - x - 6y + \frac{1}{4} = 0$
$(x-\frac{1}{2})^2 - \frac{1}{4} + (y-3)^2 - 9 + \frac{1}{4} = 0$
$(x-\frac{1}{2})^2 + (y-3)^2 = 9$
centre $(\frac{1}{2}, 3)$ radius 3

h $x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0$
$(x+\frac{1}{3})^2 - \frac{1}{9} + (y-\frac{4}{3})^2 - \frac{16}{9} + \frac{8}{9} = 0$
$(x+\frac{1}{3})^2 + (y-\frac{4}{3})^2 = 1$
centre $(-\frac{1}{3}, \frac{4}{3})$ radius 1

4 **a** radius = $\sqrt{9+16} = 5$ $\therefore$ $(x-1)^2 + (y+2)^2 = 25$

b radius = $\sqrt{25+4} = \sqrt{29}$ $\therefore$ $(x+5)^2 + (y-7)^2 = 29$

5 **a** centre $(\frac{1+3}{2}, -2) = (2, -2)$
radius = 1
$\therefore (x-2)^2 + (y+2)^2 = 1$

b centre $(\frac{-7+1}{2}, \frac{2+8}{2}) = (-3, 5)$
radius = $\sqrt{16+9} = 5$
$\therefore (x+3)^2 + (y-5)^2 = 25$

c centre $(\frac{1+4}{2}, \frac{1+0}{2}) = (\frac{5}{2}, \frac{1}{2})$
radius = $\sqrt{\frac{9}{4}+\frac{1}{4}} = \sqrt{\frac{5}{2}}$
$\therefore (x-\frac{5}{2})^2 + (y-\frac{1}{2})^2 = \frac{5}{2}$

6 **a** grad $PQ = \frac{10-1}{3-0} = 3$, grad $QR = \frac{9-10}{6-3} = -\frac{1}{3}$
grad $PQ \times$ grad $QR = 3 \times (-\frac{1}{3}) = -1$
$\therefore$ PQ and QR are perpendicular
$\therefore$ $\angle PQR$ is a right-angle

b $\angle PQR$ is a right-angle $\therefore$ PR is a diameter of C
$\therefore$ centre is $(\frac{0+6}{2}, \frac{1+9}{2}) = (3, 5)$
radius = 5
$\therefore (x-3)^2 + (y-5)^2 = 25$
$x^2 - 6x + 9 + y^2 - 10y + 25 - 25 = 0$
$x^2 + y^2 - 6x - 10y + 9 = 0$

7 **a** centre (0, 0) radius 8
dist. pt to centre = 9
$\therefore$ outside circle

b $(x-1)^2 - 1 + (y-3)^2 - 9 - 26 = 0$
$(x-1)^2 + (y-3)^2 = 36$
centre (1, 3) radius 6
dist. pt to centre = $\sqrt{9+16} = 5$
$\therefore$ inside circle

c $(x+5)^2 - 25 + (y-2)^2 - 4 = 140$
$(x+5)^2 + (y-2)^2 = 169$
centre (–5, 2) radius 13
dist. pt to centre = $\sqrt{144+25} = 13$
$\therefore$ on circle

d $(x+1)^2 - 1 + (y+4)^2 - 16 - 13 = 0$
$(x+1)^2 + (y+4)^2 = 30$
centre (–1, –4) radius $\sqrt{30}$
dist. pt to centre = $\sqrt{9+25} = \sqrt{34}$
$\therefore$ outside circle

8 $(x+6)^2 - 36 + (y-3)^2 - 9 + 27 = 0$
$(x+6)^2 + (y-3)^2 = 18$
centre (–6, 3) radius $3\sqrt{2}$
dist. Q to centre = $\sqrt{196+4} = 10\sqrt{2}$
min. $PQ = 10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$

9 x-coord of centre = $\frac{2+8}{2} = 5$
y-coord of centre = 4 $\therefore$ centre (5, 4)
radius = dist. (0, 4) to (5, 4) = 5
$\therefore$ $(x-5)^2 + (y-4)^2 = 25$

10 $(x+4)^2 - 16 + (y-6)^2 - 36 + k = 0$
$(x+4)^2 + (y-6)^2 = 52 - k$
centre (–4, 6) $r^2 = 52 - k$
$r > 0$ $\therefore$ $k < 52$
also require $r < 4$
$\therefore$ $52 - k < 16$
$k > 36$
$\therefore$ $36 < k < 52$

11 **a** mid-point $PQ = (\frac{-2+2}{2}, \frac{-2+(-4)}{2}) = (0, -3)$
grad $PQ = \frac{-4+2}{2+2} = -\frac{1}{2}$
perp. grad = 2
$\therefore$ $y = 2x - 3$

b mid-point $PR = (\frac{-2+7}{2}, \frac{-2+1}{2}) = (\frac{5}{2}, -\frac{1}{2})$
grad $PR = \frac{1+2}{7+2} = \frac{1}{3}$
perp. grad = –3
perp. bisector $y + \frac{1}{2} = -3(x - \frac{5}{2})$
$y = 7 - 3x$
centre where intersect $2x - 3 = 7 - 3x$
$x = 2$ $\therefore$ (2, 1)

c radius = dist. (2, 1) to (7, 1) = 5
$\therefore$ $(x-2)^2 + (y-1)^2 = 25$

12 **a** $(x-2)^2 - 4 + (y-2)^2 - 4 - 28 = 0$
$(x-2)^2 + (y-2)^2 = 36$
centre (2, 2) radius 6
dist. = $\sqrt{64+36} = 10$

b tangent perp to radius
$\therefore$ $AB^2 = 10^2 - 6^2 = 64$
$AB = 8$

13 $(x+3)^2 - 9 + (y-1)^2 - 1 = 0$
$(x+3)^2 + (y-1)^2 = 10$
centre (–3, 1) radius $\sqrt{10}$
dist. centre to (2, 6) = $\sqrt{25+25} = \sqrt{50}$
$PQ^2 = (\sqrt{50})^2 - (\sqrt{10})^2 = 40$
$PQ = \sqrt{40} = 2\sqrt{10}$

14 **a** $(x-3)^2 - 9 + (y-5)^2 - 25 + 16 = 0$

$\therefore$ centre (3, 5)

b grad = $\frac{5-2}{3-6} = -1$

c $y - 2 = -(x - 6)$ [$y = 8 - x$]

15 **a** $(x+2)^2 - 4 + y^2 = 13$

$\therefore$ centre (−2, 0)

grad = $\frac{0-4}{-2+1} = 4$

$\therefore$ $y - 4 = 4(x + 1)$ [$y = 4x + 8$]

b $(x+1)^2 - 1 + (y+2)^2 - 4 - 40 = 0$

$\therefore$ centre (−1, −2)

grad normal = $\frac{-2-1}{-1-5} = \frac{1}{2}$

$\therefore$ grad tangent = −2

$\therefore$ $y - 1 = -2(x - 5)$ [$y = 11 - 2x$]

c $(x-5)^2 - 25 + (y+2)^2 - 4 + 4 = 0$

$\therefore$ centre (5, −2)

grad normal = $\frac{-2-2}{5-2} = -\frac{4}{3}$

$\therefore$ grad tangent = $\frac{3}{4}$

$\therefore$ $y - 2 = \frac{3}{4}(x - 2)$ [$3x - 4y + 2 = 0$]

16 $x = 0 \Rightarrow y^2 + 6y - 16 = 0$

$(y + 8)(y - 2) = 0$

$y = -8, 2$

$y = 0 \Rightarrow x^2 - 6x - 16 = 0$

$(x + 2)(x - 8) = 0$

$x = -2, 8$

$\therefore$ (0, −8), (0, 2), (−2, 0) and (8, 0)

17 **a** sub. $x^2 + (x-4)^2 = 10$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1, 3$

$\therefore$ (1, −3) and (3, −1)

b sub. $y = 17 - 3x$

$x^2 + (17 - 3x)^2 - 4x - 2(17 - 3x) - 15 = 0$

$x^2 - 10x + 24 = 0$

$(x - 4)(x - 6) = 0$

$x = 4, 6$

$\therefore$ (4, 5) and (6, −1)

c sub.

$4x^2 + 4(2x + 2)^2 + 4x - 8(2x + 2) - 15 = 0$

$4x^2 + 4x - 3 = 0$

$(2x + 3)(2x - 1) = 0$

$x = -\frac{3}{2}, \frac{1}{2}$

$\therefore$ $(-\frac{3}{2}, -1)$ and $(\frac{1}{2}, 3)$

18 sub.

$x^2 + (1 - x)^2 + 6x + 2(1 - x) = 27$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = -4, 3$

$\therefore$ (−4, 5) and (3, −2)

$AB = \sqrt{49 + 49} = 7\sqrt{2}$

19 sub.

$x^2 + (2x + 1)^2 - 8x - 8(2x + 1) + 27 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

repeated root $\therefore$ tangent

touch when $x = 2$ $\therefore$ at (2, 5)

20 sub.

$x^2 + (x + k)^2 + 6x - 8(x + k) + 17 = 0$

$2x^2 + (2k - 2)x + k^2 - 8k + 17 = 0$

tangent $\therefore$ repeated root $\therefore$ $b^2 - 4ac = 0$

$\Rightarrow$ $(2k - 2)^2 - 8(k^2 - 8k + 17) = 0$

$k^2 - 14k + 33 = 0$

$(k - 3)(k - 11) = 0$

$\therefore$ $k = 3$ or 11

21 sub.

$x^2 + m^2x^2 - 8x - 16mx + 72 = 0$

$(1 + m^2)x^2 - (8 + 16m)x + 72 = 0$

tangent $\therefore$ repeated root $\therefore$ $b^2 - 4ac = 0$

$\Rightarrow$ $(8 + 16m)^2 - 288(1 + m^2) = 0$

$m^2 - 8m + 7 = 0$

$(m - 1)(m - 7) = 0$

$\therefore$ $m = 1, 7$

22 sub. $x = \frac{k - 3y}{2}$

$(\frac{k - 3y}{2})^2 + y^2 + 6(\frac{k - 3y}{2}) + 4y = 0$

$(k - 3y)^2 + 4y^2 + 12(k - 3y) + 16y = 0$

$13y^2 - (6k + 20)y + k^2 + 12k = 0$

tangent $\therefore$ repeated root $\therefore$ $b^2 - 4ac = 0$

$\Rightarrow$ $(6k + 20)^2 - 52(k^2 + 12k) = 0$

$k^2 + 24k - 25 = 0$

$(k + 25)(k - 1) = 0$

$\therefore$ $k = -25, 1$

23 **a** $x = 0 \Rightarrow y^2 - 6y - 7 = 0$

$(y + 1)(y - 7) = 0$

$y = -1, 7$

$\therefore$ $(0, -1)$ and $(0, 7)$

b $(x - 2)^2 - 4 + (y - 3)^2 - 9 = 7$

$\therefore$ centre $(2, 3)$

grad normal at $(0, -1) = \frac{3+1}{2-0} = 2$

$\therefore$ grad tangent at $(0, -1) = -\frac{1}{2}$

$\therefore$ $y = -\frac{1}{2}x - 1$

grad normal at $(0, 7) = \frac{3-7}{2-0} = -2$

$\therefore$ grad tangent at $(0, 7) = \frac{1}{2}$

$\therefore$ $y = \frac{1}{2}x + 7$

intersect when $-\frac{1}{2}x - 1 = \frac{1}{2}x + 7$

$x = -8$

$\therefore$ $(-8, 3)$