1. A circle with centre C has equation $x^2 + y^2 - 2x + 10y - 19 = 0$.

i. Find the coordinates of C and the radius of the circle.

[3]

ii. Verify that the point (7, –2) lies on the circumference of the circle.

[1]

iii. Find the equation of the tangent to the circle at the point (7, –2), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[5]

2. A circle C has equation $x^2 + y^2 + 8y - 24 = 0$.

i. Find the centre and radius of the circle.

ii. The point A (2, 2) lies on the circumference of C. Given that AB is a diameter of the circle, find the coordinates of B.

3. A circle with centre C has equation $(x - 2)^2 + (y + 5)^2 = 25$.

(i) Show that no part of the circle lies above the x-axis.

[3]

(ii) The point P has coordinates (6, k) and lies inside the circle. Find the set of possible values of k.

[5]

(iii) Prove that the line $2y = x$ does not meet the circle.

[4]

4. A circle with centre C has equation $x^2 + y^2 - 10x + 4y + 4 = 0$.

i. Find the coordinates of C and the radius of the circle.

[3]

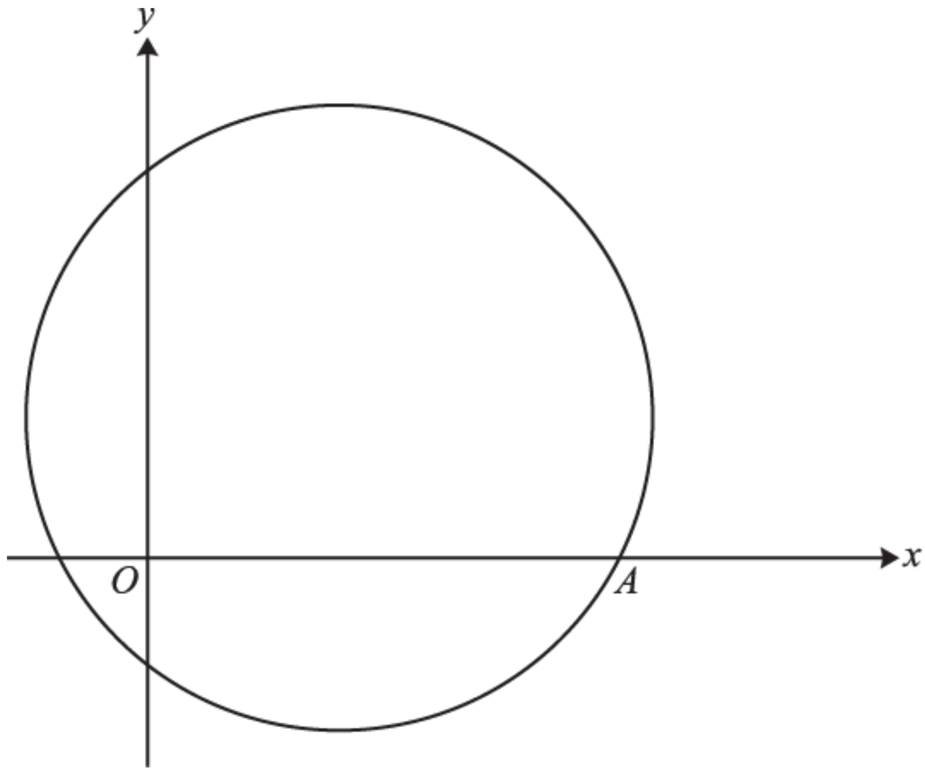
ii. Show that the tangent to the circle at the point P (8, 2) has equation $3x + 4y = 32$.

[5]

iii. The circle meets the y-axis at Q and the tangent meets the y-axis at R. Find the area of triangle PQR.

[4]

5.



The diagram shows the circle with equation $x^2 + y^2 - 8x - 6y - 20 = 0$.

i. Find the centre and radius of the circle.

[3]

The circle crosses the positive x-axis at the point A.

ii. Find the equation of the tangent to the circle at A.

[6]

iii. A second tangent to the circle is parallel to the tangent at A. Find the equation of this second tangent.

[3]

iv. Another circle has centre at the origin O and radius r. This circle lies wholly inside the first circle. Find the set of possible values of r.

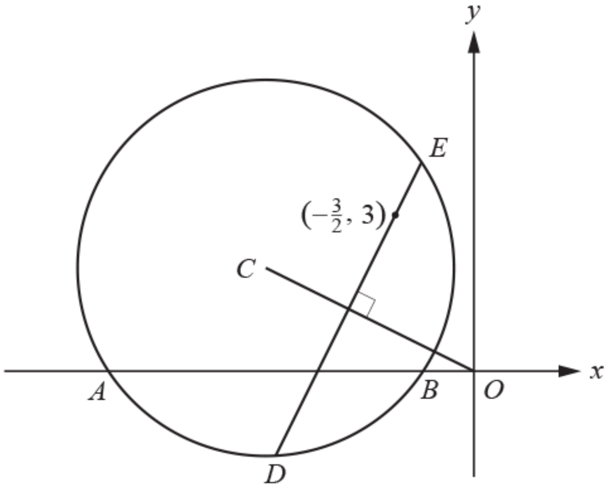
[2]

6. Points A and B have coordinates (3, 0) and (9, 8) respectively. The line AB is a diameter of a circle.

(a) Find the coordinates of the centre of the circle. [2]

(b) Find the equation of the tangent to the circle at the point B. [3]

7.



A circle with centre C has equation $x^2 + y^2 + 8x - 4y + 7 = 0$, as shown in the diagram. The circle meets the
x-axis at A and B.

(a) Find

- the coordinates of C,
- the radius of the circle. [3]

(b) Find the coordinates of the points A and B. [2]

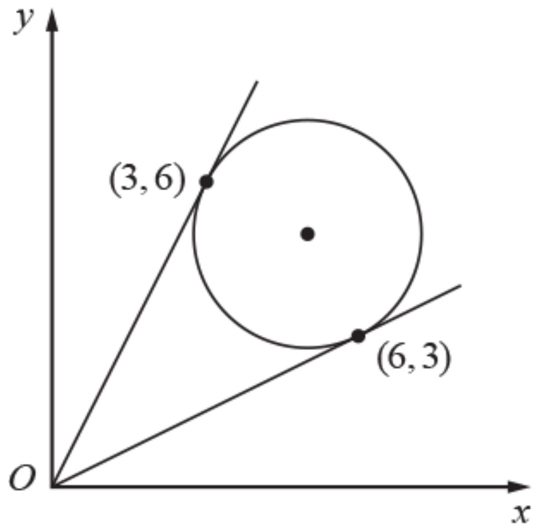
The chord DE passes through the point $\left(-\frac{3}{2}, 3\right)$ and is perpendicular to OC, where O is the origin.

(c) Find the coordinates of the points D and E. [7]

(d) Hence find the area of the quadrilateral $BEAD$. [2]

8. **In this question you must show detailed reasoning.**

A circle touches the lines $y = \frac{1}{2}x$ and $y = 2x$ at (6, 3) and (3, 6) respectively.



Find the equation of the circle. [7]

9. The circle $x^2 + y^2 - 8x + 2y = 0$ passes through the origin O. Line OA is a diameter to this circle.

(i) Find the equation of the line OA, giving your answer in the form $ax + by = 0$, where a and b are integers. [5]

(ii) The tangent to the circle at point A meets the x-axis at the point B. Find the area of triangle OAB. [6]

10. A circle with equation $x^2 + y^2 + 6x - 4y = k$ has a radius of 4.

(a) Find the coordinates of the centre of the circle. [2]

(b) Find the value of the constant k. [2]

11. The equation of a circle is $x^2 + y^2 + 6x - 2y - 10 = 0$.

(a) Find the centre and radius of the circle. [3]

(b) Find the coordinates of any points where the line $y = 2x - 3$ meets the circle $x^2 + y^2 + 6x - 2y - 10 = 0$. [4]

(c) State what can be deduced from the answer to part **(b)** about the line $y = 2x - 3$ and the circle
$x^2 + y^2 + 6x - 2y - 10 = 0$. [1]

12. A circle with centre C has equation $x^2 + y^2 + 8x - 2y - 7 = 0$.

Find

(a) the coordinates of C, [2]

(b) the radius of the circle. [1]

13. **In this question you must show detailed reasoning.**

The lines $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$ are tangents to a circle at (2, 1) and (–2, 1) respectively. Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [6]

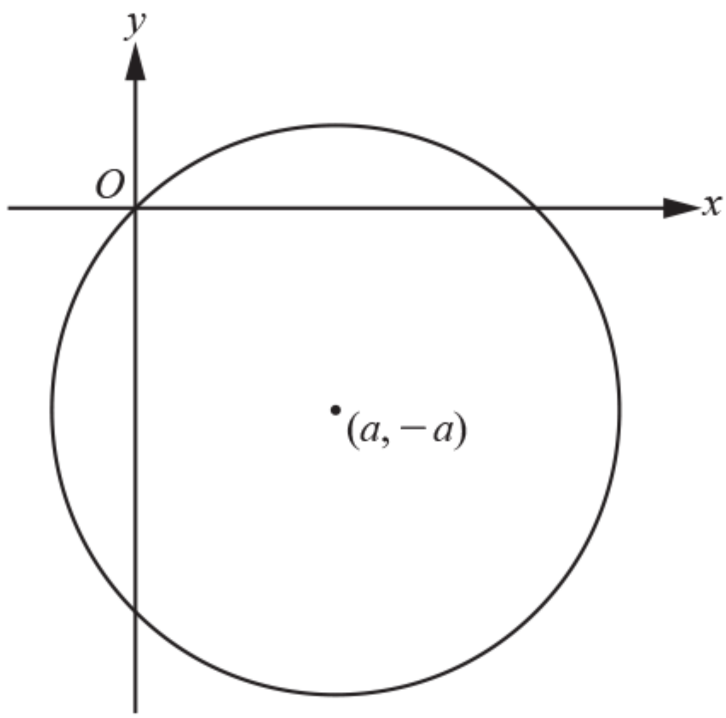
14. A line has equation $y = 2x$ and a circle has equation $x^2 + y^2 + 2x - 16y + 56 = 0$.

(a) Show that the line does not meet the circle. [3]

(b) (i) Find the equation of the line through the centre of the circle that is perpendicular to the line $y = 2x$. [4]

(ii) Hence find the shortest distance between the line $y = 2x$ and the circle, giving your answer in an exact form. [4]

15.



The diagram shows a circle with centre $(a, -a)$ that passes through the origin.

(a) Write down an equation for the circle in terms of a. [2]

(b) Given that the point $(1, -5)$ lies on the circle, find the exact area of the circle. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	Centre (1, –5)	B1	Correct centre	
		i	$(x-1)^2 + (y+5)^2 - 19 - 1 - 25 = 0$ $(x-1)^2 + (y+5)^2 = 45$	M1	Correct method to find r^2	$r^2 = (\pm5)^2 + (\pm1)^2 + 19$ for the M mark
		i	Radius $= \sqrt{45}$	A1	Correct radius. **Do not allow if wrong centre used in calculation of radius.** Examiner's Comments This standard piece of bookwork was generally done very well, with around three-quarters of candidates scoring all three marks. Only occasionally was the centre seen as (2, –10). The most common cause of errors was again dealing with negative numbers, particularly when squaring to find the radius, or not subtracting appropriately after completing the square.	**A0** if $\pm\sqrt{45}$
		ii	$7^2 + (-2)^2 - 14 - 20 - 19$ $= 0$	B1	Substitution of coordinates into equation of circle in any form or use of Pythagoras' theorem to calculate the distance of (7, –2) from *C* Examiner's Comments This was managed well by most candidates, with substitution of the point into the original equation generally a more successful approach than using Pythagoras' theorem.	**No follow through** for this part as **AG. Must be consistent –** do not allow finding the distance as $\sqrt{45}$ if no / wrong radius found in 9(i).
		iii	gradient of radius = $\dfrac{-5-(-2)}{1-7}$ or $\dfrac{-2-(-5)}{7-1}$	M1	uses $\dfrac{y_2-y_1}{x_2-x_1}$ with **their C** (3/4 correct)	**Follow through from 9(i) until final mark.**

		iii	$= \frac{1}{2}$	A1√	Follow through from their C allow unsimplified single fraction e.g. $\frac{-3}{-6}$	If (–1,5) is used for C, then expect
		iii	gradient of tangent = –2	B1√	Follow through from their gradient, even if M0 scored. Allow $\frac{-1}{\text{their fraction}}$ **B1**	Gradient of radius = $\frac{5-(-2)}{-1-7} = -\frac{7}{8}$
		iii	$y + 2 = -2\,(x - 7)$	M1	correct equation of straight line through (7, –2), any non-zero numerical gradient	Gradient of tangent = $\frac{8}{7}$
		iii	$2x + y - 12 = 0$	A1	oe 3 term equation in correct form i.e. $k(2x + y - 12) = 0$ where k is an integer **cao** Examiner's Comments A large number of candidates secured full marks on this question and almost all managed to secure partial credit. Some candidates simplified $\frac{-3}{-6}$ to $-\frac{1}{2}$. The incorrect simplification of $\frac{3}{6}$ to $\frac{1}{3}$ was also common. The majority remembered to find the negative reciprocal of their gradient and then substituted this correctly to find an equation of a straight line. Some candidates still miss the detail of the question and do not give the correct answer in the required form, needlessly losing the final mark. Another not uncommon error was to attempt to differentiate implicitly when candidates clearly had either not yet met this technique or did not understand the process; successful solutions using this approach were extremely rare.	Alternative markscheme for implicit differentiation: **M1** Attempt at implicit diff as evidenced by $2y\frac{dy}{dx}$ term A1 $2x + 2y\frac{dy}{dx} - 2 + 10\frac{dy}{dx} = 0$ **A1** Substitution of (7, –2) to obtain gradient of tangent = –2 Then **M1 A1** as main scheme
			Total	9		
2		i	Centre (0, –4)	B1		
		i	$x^2 + (y + 4)^2 - 16 - 24 = 0$	M1	$(y \pm 4)^2 - 4^2$ seen (or implied by correct answer)	Or attempt at $r^2 = f^2 + g^2 - c$
		i	Radius = $\sqrt{40}$	A1	Do not allow **A** mark from $(y - 4)^2$ Examiner's Comments	**A0** for $\pm\sqrt{40}$

					Over two-thirds of candidates secured all three marks interpreting the given equation of a circle correctly. Marks were lost mainly due to sign errors, both in attempting to find the centre and in attempts to complete the square.	
		ii ii	(−2, −10)	B1FT B1FT	FT through centre given in **(i)** FT through centre given in **(i)** **Examiner's Comments** Candidates who drew a diagram usually recognised that the coordinates of B could be found by simple addition or subtraction and were then usually successful in scoring both marks, especially as there was a follow-through from part (i). Those who tried to apply standard techniques involving Pythagoras' theorem and resulting quadratics were very rarely successful in finding either value.	i.e. (their $2x - 2$, their $2y - 2$) Apply same scheme if equation of diameter found and attempt to solve simultaneously; no marks until a correct value of x/y found.
		ii	2) If the candidate attempts to solve by using the formula a. If the formula is quoted incorrectly then **M0**. b. If the formula is quoted correctly then one sign slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**	c.		
			Total	**5**		
3		i i i	y coordinate of the centre is −5 Radius = 5 Centre is five units below x axis and radius is five, so just touches the x-axis	B1 B1 B1	Correct y value Correct radius Correct explanation based on the above — allow clear diagram **www** **Examiner's Comments** The simplest, and most common, approach to show that the circle did not go above the x axis was to identify the centre and radius from	Alt Shows only meets x axis at one point **B1** Correct y value for the centre **B1** Correct explanation **B1 www**

					the equation and state/show on a diagram that the circle just touched the axis at a single point. The majority of candidates showed clear solutions to this effect. Some, however, tested just a single point (usually $y = 1$) and showed this was not a point on the circle, which was of course insufficient.	
		ii	$CP^2 = (6 - 2)^2 + (k + 5)^2$ $CP^2 < 25 \Rightarrow 16 + k^2 + 10k + 25 < 25$	M1	Attempt to find CP or CP^2	Alternative Puts $x = 6$ to into equation of circle **M1**
		ii	$k^2 + 10k + 16 < 0$	A1	Correct three term quadratic expression*	Correct three term quadratic equation*, could be in terms of y **A1**
		ii	$(k + 2)(k + 8) < 0$	A1	$k = -2$ and $k = -8$ found	$k = -2$ and $k = -8$ found (allow y) **A1**
		ii	$-8 < k < -2$	M1	Chooses "inside region" for their roots of their quadratic	Then as main scheme
		ii		A1	Must be strict inequalities for the A mark * Or $(k + 5)^2 < 9$ **Examiner's Comments** Most candidates took the correct approach to this, substituting $x = 6$ and then solving the quadratic and finding the values of k that corresponded to the points on the circumference. A large number of candidates then stopped and failed to identify the correct range of values being any value between these. Those who carried on were usually correct, but it was fairly common to not give the answer as the strict inequalities required for the point to be inside the circle. There were some neat alternative solutions using Pythagoras' theorem to find the values of k from a good sketch.	* Or $(k + 5)^2 = 9$ **SC** Trial and improvement **B2** if final answer correct (**B1** if inequalities are not strict) Can only get 5/5 if fully explained
		iii	$(2y - 2)^2 + (y + 5)^2 = 25$ $5y^2 + 2y + 4 = 0$ $b^2 - 4ac = 4 - 4 \times 5 \times 4$	M1*	Attempts to eliminate x or y from equation of circle	If y eliminated: $5x^2 + 4x + 16 = 0$

			$= -76$ < 0, so line and circle do not meet			
		iii		A1	Correct three term quadratic obtained	$b^2 - 4ac = 16 - 4 \times 5 \times 16$ $= -304$
		iii		M1dep*	Correct method to establish quadratic has no roots e.g. considers value of $b^2 - 4ac$, tries to find roots from quadratic formula	**No marks for purely graphical attempts**
		iii		A1	Correct clear conclusion **www AG** **Examiner's Comments** There were a large number of fully correct solutions to the request to prove that the line and circle do not meet. Most performed the easier substitution for x, but $(2y)^2 = 2y^2$ was a fairly common error. Many were able to use the discriminant, or the quadratic formula, to explain their reasoning clearly. Only a few claimed that the line and circle did not meet because the quadratic could not be factorised. Some of the weaker candidates again resorted to testing a single point (sometimes the centre) or drawing a poor diagram.	
			Total	**12**		
4		i	$C = (5, -2)$	B1	Correct centre	
		i	$(x - 5)^2 + (y + 2)^2 - 25 = 0$	M1	$(x \pm 5)^2 - 5^2$ and $(y \pm 2)^2 - 2^2$ seen (or implied by correct answer)	Or attempt at $r^2 = f^2 + g^2 - c$
		i	Radius = 5	A1	Correct radius – do not allow **A** mark from $(x + 5)^2$ and / or $(y - 2)^2$ **Examiner's Comments** Apart from the usual sign error, most candidates were able to identify the centre and calculate the radius of the circle with little apparently difficulty.	± 5 or $\sqrt{25}$ A0.

		ii	Gradient $PC = \frac{2--2}{8-5} = \frac{4}{3}$	M1	Attempt to find gradient of radius (3/4 correct)	See also alternative methods on next page
		ii		A1		
		ii	Gradient of tangent $= -\frac{3}{4}$	B1ft	$\frac{-1}{\text{their gradient}}$ processed	
		ii	$y-2=-\frac{3}{4}(x-8)$	M1	Equation of straight line through P, using their perpendicular gradient (not from rearrangement)	Do not allow use of gradient of radius instead of tangent
		ii	$4y+3x=32$ Gradient of radius $= \frac{2--2}{8-5} = \frac{4}{3}$ **M1A1** Attempts to rearrange equation of line to find gradient of line $= -\frac{3}{4}$ and compares with gradient of radius **M1** Multiply gradients to get –1 **B1** Check (8, 2) lies on line **B1**	A1	Rearrange to required form **www AG** Substitute for *x*/*y* or attempt to get an equation in 1 variable only **M1** $k(x^2-16x+64)=0$ or $k(y^2-4y+4)=0$ **A1** Correct method to solve quadratic — **see appendix 1 M1** $x=8$, $y=2$ found **A1** States one root implies tangent **B1** **Examiner's Comments** Unsuccessful attempts at implicit differentiation notwithstanding, most candidates were able to present a clear accurate solution to this part of the question. The expected approach of finding the gradient of the radius, its negative reciprocal and then the equation of the line through (8, 2) was performed very well. Some candidates merely rearranged the given equation to find its gradient and re-substituted; this gained no credit.	Ignore order of terms **M*1** Attempt at implicit differentiation as evidenced by $2y\frac{dy}{dx}$ term **A1** $2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$ **A1** Substitution of (8, 2) to obtain $-\frac{3}{4}$ Then as main scheme **OR** Attempts to rearrange equation of line to find gradient of line $= -\frac{3}{4}$ **M1dep** Check (8, 2) lies on line **B1**
		iii	$Q=(0,-2)$	B1	*Q* found correctly	For the M mark, allow splitting into two triangles $\frac{1}{2}\times6\times8+\frac{1}{2}\times4\times8$
		iii	$R=(0,8)$	B1	*R* found correctly	
		iii	Area $= \frac{1}{2}\times(8--2)\times8$	M1	Attempt to find area of triangle with their *Q*, *R* and height 8 i.e. $\frac{1}{2}\times(y_R-y_Q)\times8$	If using PQ as base then expect to see $\frac{1}{2}\times\sqrt{80}\times\sqrt{80}$ www

		iii	40	A1	**Examiner's Comments** Most candidates were able to find both points on the y-axis and the best solutions to this included a sketch diagram to aid candidates on their way. Some chose to find the lengths of all the sides of the triangle and multiply together sides that were not perpendicular before halving. Although full marks to this part were comparatively rare, it was noticeable that some lower attaining candidates who did use a good sketch were able to outscore many of the higher attaining students on this particular part.	
			Total	12		
5		i	Centre of circle (4, 3)	B1	Correct centre	
		i	$(x-4)^2 - 16 + (y-3)^2 - 9 - 20 = 0$	M1	$(x \pm 4)^2 - 4^2$ and $(y \pm 3)^2 - 3^2$ seen (or implied by correct answer)	Or $r^2 = 4^2 + 3^2 + 20$ **soi**
		i	$r^2 = 45$		$\sqrt{45}$ or better **www**	**ISW** after $\sqrt{45}$
		i	$r = \sqrt{45}$	A1		**Examiner's Comments** This proved to be a very successfully answered question, with around nine in ten candidates securing all three marks.
		ii	At A, $y = 0$ so $x^2 - 8x - 20 = 0$ $(x-10)(x+2) = 0$	M1	Valid method to find A e.g. put $y = 0$ and attempt to solve quadratic (allow slips) or Pythagoras' theorem	Alterative for finding gradient: **M1** Attempt at implicit differentiation as evidenced by $2y\frac{dy}{dx}$ term
		ii	A = (10, 0)	A1	Correct answer found	
		ii	Gradient of radius = $\frac{3-0}{4-10} = -\frac{1}{2}$	M1	Attempts to find gradient of radius (3 out of 4 terms correct for their centre, their A)	

		ii	Gradient of tangent = 2	B1		A1 $2x+2y\frac{dy}{dx}-8-6\frac{dy}{dx}=0$ **and** substitution of (10, 0) to obtain 2.
		ii	$y - 0 = 2(x - 10)$	M1	Equation of line **through their A**, any non-zero gradient	
		ii	$y = 2x - 20$	A1	Correct answer in any three-term form	**Examiner's Comments** Just over half of candidates obtained full marks in this part, with errors appearing at all stages. Some put x rather than y equal to 0 when trying to find A and the alternative method of using Pythagoras' theorem often led to slips. There were a significant number of problems finding the gradient and errors such as $-\frac{3}{6}=-\frac{1}{3}$ were commonly seen.
		iii	A' = (−2, 6)	B1	Finds the opposite end of the diameter	
		iii	$y - 6 = 2(x + 2)$	M1	Line through their A' parallel to their line in (ii)	Not through centre of circle
		iii	$y = 2x + 10$	A1	Correct answer in any three-term form	**Examiner's Comments** Many candidates did not realise that the point required for the parallel line was the opposite end of the diameter. Most did use the same gradient as in (ii), but some used the negative reciprocal. An interesting method sometimes seen was consideration of translation of the original line.
		iv	OC = $\sqrt{3^2+4^2}$ = 5	M1	Attempts to find the distance from O to their centre **and subtract from their radius**	**ISW** incorrect simplification
		iv	(0 <) $r < \sqrt{45} - 5$	A1	Correct inequality, condone ≤	**Examiner's Comments**

						This proved very demanding, with many candidates unable to start; those who drew a diagram were generally more successful but less than a quarter of candidates secured both marks. Even amongst those who found the maximum length of the radius to be $\sqrt{45} - 5$, it was quite rare to see the correct inequality.
			Total	14		
6		a	$\left(\frac{3+9}{2}, \frac{0+8}{2}\right)$ (6,4)	M1 (AO1.1a) A1 (AO1.1) [2]	Correct working for either coordinate May be implied by $x = 6$ or $y = 4$	
		b	Gradient of radius through B is $\frac{8-4}{9-6} = \frac{4}{3}$ Gradient of tangent is $-\frac{3}{4}$ So equation of tangent is $y = -\frac{3}{4}x + \frac{59}{4}$ oe	M1 (AO1.1) M1 (AO1.1) A1 (AO2.2a) [3]	FT their gradient	
			Total	5		
7		a	Centre of circle is (–4, 2) $(x+4)^2 - 16 + (y-2)^2 - 4 + 7 = 0$ $r^2 = 13 \Rightarrow r = \sqrt{13}$	B1 (AO1.1) M1 (AO1.1) A1 (AO1.1) [3]	Correct centre $(x \pm 4)^2 - 16 + (y \pm 2)^2 - 4$ seen OR $r^2 = 4^2 + 2^2 - 7$	

					$r = 3.61$ or better www	
		b	$y = 0 \Rightarrow x^2 + 8x + 7 = 0$ $A(-7, 0)$ and $B(-1, 0)$	M1 (AO1.1a) A1 (AO1.1) [2]	Substitute $y=0$ and attempt to solve BC	
		c	$m_{OC} = -\frac{1}{2}$ Hence $m_{DE} = 2$ $y - 3 = 2\left(x + \frac{3}{2}\right) \Rightarrow y = 2x + 6$ $(x + 4)^2 + (2x + 4)^2 = 13$ $5x^2 + 24x + 19 = 0 \Rightarrow x = \ldots$ $x = -\frac{19}{5},\ -1$	M1 (AO3.1a) A1FT (AO1.2) M1 (AO1.1) M1 (AO3.1a) M1 (AO1.1) A1 (AO1.1) A1 (AO3.2a)	Identify gradient of line *OC* Use of $m_1 m_2 = -1$ with their m_{OC} Form equation of line *DE* Substitute to get quadratic in one variable Expand and attempt to solve their 3-term quadratic	

			D is $\left(-\frac{19}{5},\ -\frac{8}{5}\right)$ and E is $(-1, 4)$	[7]	BC	
		d	$\text{Area} = \frac{1}{2}(6)(4) + \frac{1}{2}(6)\left(\frac{8}{5}\right)$ $= \frac{84}{5}$	M1 (AO1.1a) A1 (AO1.1) [2]	$\text{Area} = \frac{1}{2}(\text{their } (7-1))\left(\text{their } \left(4+\frac{8}{5}\right)\right)$	
			Total	14		
8			DR Grad of rad = −2 or $-\frac{1}{2}$ $y - 3 = -2(x - 6)$ or $y - 6 = -\frac{1}{2}(x - 3)$ $y = -2x + 15$ or $y = -\frac{1}{2}x + 7\frac{1}{2}$ Equation of line from O to centre is $y = x$	B1(AO3.1a) M1(AO1.1a) M1(AO1.2) M1(AO2.1) A1(AO1.1) M1(AO1.1)	Attempt equation of either radius or attempt equation of other radius	

			$x = -2x + 15$ or $x = -\frac{1}{2}x + 7\frac{1}{2}$ C is (5, 5) $r^2 = (5-3)^2 + (5-6)^2$ (= 5) $(x-5)^2 + (y-5)^2 = 5$	A1(AO1.1) [7]	Solve their equation of radius with $y = x$ ISW	or equns of both radii
			Total	7		
9		i	$(x-4)^2 - 16 + (y+1)^2 - 1 = 0$ $(x-4)^2 + (y+1)^2 = 17$	M1	Correct method to find centre of circle	e.g. $(x \pm 4)^2$ and $(y \pm 1)^2$ seen (or implied by correct answer) **M** can be implied by correct centre.
			Centre = (4, −1)	A1	Correct centre soi.	Note: Centre (– 4, 1) leads to "correct" answer. **M1A0B0M1A0** Max 2/5
			$m = -\frac{1}{4}$	B1	Gradient of OA correct (could use OC or CA) [A = (8, −2) is not required for this part,	
			$y = -\frac{1}{4}x$	M1		

			$x + 4y = 0$	A1 [5]	but may be used] Attempts equation of straight line through O or A or centre of the circle with their calculated gradient. **www** Correct equation in required form i.e. $k(x + 4y) = 0$ for integer k, allow $0 = 4y + x$ etc. <u>Alternative for first three marks:</u> **M1** Attempt at implicit differentiation as evidenced by $2y\frac{dy}{dx}$ term **A1** $2x + 2y\frac{dy}{dx} - 8 + 2\frac{dy}{dx} = 0$ and substitutes O to obtain $\frac{dy}{dx} = 4$ **B1** Find correct negative reciprocal **Examiner's Comments** There were many full accurate solutions to this question, although the testing of the identification of the centre of a circle but without that specific request proved taxing to some. There were both sign errors and division errors, with (4, −2) being frequently seen. Slips also followed in finding the gradient of the line. Many candidates chose the origin as the point to use to find the equation, although the use of the centre of the circle or even A itself were not uncommon. Some candidates failed to give the final answer in the required form.	

				B1ft B1ft	Must be seen / used in (ii); ft their centre ft their gradient in (i)	
	ii		A = (8, − 2) $m' = 4$ $y + 2 = 4(x - 8)$ When $y = 0, x = \frac{17}{2}$ Area $= \frac{1}{2} \times \frac{17}{2} \times 2 = \frac{17}{2}$	M1 M1 M1 A1 [6]	Attempts equation of perpendicular line through their A. (**Not** (4, −1).) Attempt to find *x* value of point B from their equation of perpendicular line Attempt to find area of OAB e.g. $\frac{1}{2} \times$ their OB × their 2, or $\frac{1}{2} \times$ their OA ×	If centre used here, max **B1B1**, 2/6. Equation of line/B may not be seen explicitly. Must have used a valid method to find B. $\text{OA} = \sqrt{68}, \text{AB} = \sqrt{\frac{17}{4}}$ Look out for "correct" answer from wrong coordinates – **A0**.

					their AB, or split into two triangles Accept 8.5 or equivalent fractions but not unsimplified surds. **www** Examiner's Comments Candidates who made errors in the first part of the question were still able to score 4 out of 5 in this part as follow through and method marks were allowed from wrong centres. Just over a quarter of candidates were able to secure all 5 marks, but a significant minority were unable to access this part, either through not realising A was the opposite end of the diameter to O or not realising that the perpendicular gradient was needed to find the coordinates of B. Several found B through the use of a sketch rather than finding the equation of the perpendicular line. In general candidates who used a sketch were more likely to access, and succeed in, this question than those who did not.	
			Total	11		
10		a	$(x \pm 3)^2 + (y \pm 2)^2 \ldots$ $(-3,2)$	M1(AO1.1a) A1(AO1.1) [2]	Attempt to complete the square State correct centre www Ignore constant term(s)	

		b	$13 + k = 16$ $k = 3$	M1(AO1.1a) A1(AO1.1) [2]	Attempt to link 9, 4, 16 and k Obtain $k = 3$	
			Total	4		
11		a	centre is (–3, 1) $(x + 3)^2 - 9 + (y - 1)^2 - 1 - 10 = 0$ $(x + 3)^2 + (y - 1)^2 = 20$ radius = $2\sqrt{5}$ or $\sqrt{20}$	B1 (AO 1.1) M1 (AO 1.1a) A1 (AO 1.1) [3]	Correct centre of circle: Allow $x = -3$, $y = 1$ Attempt to complete the square twice: Allow for $(x \pm 3)^2 \pm 9 + (y \pm 1)^2 \pm 1$ seen $(x \pm 3)^2 + (y \pm 1)^2 - 10 = 0$ is M0 as no evidence of subtracting the constant terms to complete the squares. Or attempt to use $r^2 = g^2 + f^2 - c$ Correct radius: From correct working only, including correct factorisation. Allow $r = 4.47$, or better <u>Examiner's Comments</u> Solutions to this question were nearly always correct, with most	

					candidates choosing to write the equation in factorised form. There were a few sign errors when stating the centre of the circle, and also a few errors when subtracting the constant term when completing the square each time.	
		b	$x^2 + (2x - 3)^2 + 6x - 2(2x - 3) - 10 = 0$ OR $(x + 3)^2 + (2x - 4)^2 = 20$ $x^2 - 2x + 1 = 0$ $x = 1$ $(1, -1)$	M1 (AO 3.1a) A1 (AO 1.1) A1 (AO 1.1) A1 (AO 2.1) [4]	Substitute the linear equation into the quadratic equation — Either substitute for y, or an attempt at x. Either use the given expanded equation or their attempt at a factorised equation Correct three term quadratic — Must be three terms, but not necessarily on same side of equation BC, or from any valid method — A0 if additional incorrect x value A0 if additional points also given — Allow $x = 1$, $y = -1$ **Examiner's Comments** All candidates attempted to solve the equations simultaneously, either using the expanded equation of the circle or the factorised equation. As this question did not specify 'detailed reasoning', it was expected that candidates would solve the ensuing quadratic on their calculator but instead most still showed the factorisation.	

		c	The line is a tangent to the circle at (1, −1)	B1ft (AO 2.2a) [1]	<table><tr><td>Correct deduction Strict follow-through on their number of roots from **(b)**</td><td>Allow just mention of 'tangent' Allow other correct statements such as the line and the circle only touch once</td></tr></table> **Examiner's Comments** Part (b) shows one point of intersection so it was expected that candidates would put this information into context and conclude that the line was a tangent to the circle. If an error had happened in part (b) resulting in other than one point of intersection then candidates could still get this mark for a correct deduction from their answer.	
			Total	8		
12		a	$(x + 4)^2 - 16 + (y - 1)^2 - 1 - 7 = 0$ $(x + 4)^2 + (y - 1)^2 = 24$ $C(-4,1)$	M1(AO 1.1)E A1(AO 1.1)E [2]	<table><tr><td>Correct method to find centre of circle</td><td>e.g. $(x \pm 4)^2$ and $(y \pm 1)^2$ seen (or implied)</td></tr></table>	

					Examiner's Comments This proved to be a good start for nearly all candidates with the vast majority correctly completing the square (twice) to find the coordinates of the centre of the circle. When errors occurred these were nearly always down to sign errors inside the two brackets.	
	b		Radius = $\sqrt{24}$	B1(AO 1.1)E [1]	oe e.g. $2\sqrt{6}$ **Examiner's Comments** Nearly all candidates stated the radius of the circle correctly in either part (a) or part (b).	
			Total	3		
13			DR $y - 1 = -2(x - 2)$ — or $y = -2x + c$ & sub (2, 1) $y = -2x + 5$ — $c = 5$	M1 (AO3.1a) A1 (AO1.1) A1	**If no wking seen, no marks** or $y - 1 = 2(x-(-2))$ or solve $y = -2x + 5$ & $y = 2x+5$ $y = 2x + 5$ or $c = 5$ Alt method using proportion: Centre is on y-axis, not (0, 1) (may be implied) M1 $\frac{c-1}{2} = 2$ or $c = 1 + 2 \times 2$ $c = 5$ A1	

			Centre is (0, 5) $r = \sqrt{2^2 + 4^2}$ $= \sqrt{20}$ $x^2 + (y - 5)^2 = 20$ oe $x^2 + y^2 - 10y + 5 = 0$	(AO3.2a) M1 (AO1.1a) M1 (AO1.2) A1 (AO1.1) [6]	<table><tr><td>stated or implied or $r^2 = 2^2 + 4^2$ or ft their centre = 20 or $a = 0$, $b = -10$, $c = 5$ ft their centre and rad^2 ($\neq 0$), however found cao</td><td>Centre is (0, 5) A1</td></tr></table> <u>Examiner's Comments</u> A clear initial sketch proved to be extremely useful for the majority of candidates that successfully answered this question. Some failed to consider one or both normals and therefore were unable to make any significant progress. Some found the centre incorrectly, for example by finding the midpoint of the line joining (2, 1) and (–2, 1). Others treated (0, 0) as the centre of the circle. However, despite many false starts, many used their incorrect centre and radius in the equation of a circle, and gained at least one mark. However, some candidates found a centre (p, q) and radius r, and then wrote $x^2 + y^2 + px + qy + r$. A great many candidates started from the equation $x^2 + y^2 + ax + by + c = 0$, given in the question, and attempted to find a, b and c by	

					substituting coordinates and various other devices. Not surprisingly, these generally failed to gain any marks. (Although one candidate did actually succeed by this method, taking several pages to do so, and gained full credit.)	
			Total	6		
14		a	$x^2 + 4x^2 + 2x - 32x + 56 = 0$ $5x^2 - 30x + 56 = 0$ $30^2 - 4 \times 5 \times 56 = -220$ $b^2 - 4ac < 0$ hence no real roots so the circle and line do not intersect	M1 (AO 2.1) M1 (AO 2.4) A1 (AO 2.2a) [3]	Substitute $y = 2x$ into equation of circle and rearrange to three term quadratic Consider discriminant Conclude with no real roots	
		b	(i) Centre of circle is (–1, 8) Gradient of perpendicular is –0.5 $y - 8 = -0.5(x + 1)$ $x + 2y = 15$	B1 (AO 1.1) B1 (AO 2.2a) M1 (AO 1.1) A1 (AO 1.1) [4]	Seen or used For gradient of perpendicular Attempt equation of line through their circle centre with gradient of –0.5 Obtain correct equation Allow any 3 term	

(ii)	$5x = 15$ $x = 3,\ y = 6$ distance from centre to line is $\sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$ $(x + 1)^2 + (y - 8)^2 = 3^2$ hence shortest distance between line and circle is $2\sqrt{5} - 3$	M1 (AO 3.1b) M1 (AO 1.1) M1 (AO 1.1a) A1 (AO 3.2a) [4]	Attempt to solve simultaneously with $y = 2x$ Use Pythagoras to find distance between centre of circle and point of intersection Attempt to find radius of circle Obtain $2\sqrt{5} - 3$	equivalent Seen at any point in solution – allow back credit to part (a) if the radius is found at the same time as the centre of circle Allow any exact equiv	
	Total	11			

15		a	$(x - a)^2 + (y + a)^2 = K$ $K = 2a^2$	B1 (AO 1.1) B1 (AO 1.1) [2]	Correct LHS (accept if expanded: $x^2 + y^2 - 2ax + 2ay + 2a^2$) Correct RHS Allow full marks for any equivalent form, e.g. $x^2 + y^2 - 2ax + 2ay = 0$	
		b	$(1 - a)^2 + (-5 + a)^2 = 2a^2$ $a = \frac{13}{6} \Rightarrow \text{Area} = \pi \times 2\left(\frac{13}{6}\right)^2$ $= \frac{169}{18}\pi$	M1 (AO 1.1a) M1 (AO 1.1) A1 (AO 2.2a) [3]	Substitute (1, –5) into their circle equation Solve for a and substitute into πr^2 with their r^2	
			Total	5		