1. i. Points A and B have coordinates (– 2, 1) and (3, 4) respectively. Find the equation of the perpendicular bisector of AB and show that it may be written as $5x + 3y = 10$.

[6]

ii. Points C and D have coordinates (– 5, 4) and (3, 6) respectively. The line through C and D has equation $4y = x + 21$. The point E is the intersection of CD and the perpendicular bisector of AB. Find the coordinates of point E.

[3]

iii. Find the equation of the circle with centre E which passes through A and B. Show also that CD is a diameter of this circle.

[5]

2. The circle $(x - 3)^2 + (y - 2)^2 = 20$ has centre C.

i. Write down the radius of the circle and the coordinates of C.

[2]

ii. Find the coordinates of the intersections of the circle with the x- and y-axes.

[5]

iii. Show that the points A(1,6) and B(7,4) lie on the circle. Find the coordinates of the midpoint of AB. Find also the distance of the chord AB from the centre of the circle.

[5]

3. A circle has equation $(x - 2)^2 + (y + 3)^2 = 25$.

(a) Write down

- The radius of the circle.
- The coordinates of the centre of the circle. [2]

(b) Find, in exact form, the coordinates of the points of intersection of the circle with the y-axis. [3]

(c) Show that the point (1, 2) lies outside the circle. [2]

(d) The point P(–1, 1) lies on the circle. Find the equation of the tangent to the circle at P. [4]

4. **In this question you must show detailed reasoning.**
Determine for what values of k the graphs $y = 2x^2 - kx$ and $y = x^2 - k$ intersect. [6]

5. **In this question you must show detailed reasoning.**
Find the coordinates of the points of intersection of the curve $y = x^2 + x$ and the line $2x + y = 4$. [5]

6. A circle has centre C (–4, 3) and passes through the origin.
(a) Find the equation of the circle. [3]

(b) The circle crosses the positive y-axis at the point A. The tangent to the circle at A meets the x-axis at B. Find the x-coordinate of B. [5]

7. Point P lies on the positive x-axis and point Q lies on the positive y-axis. Triangle OPQ is isosceles and its area is 18 square units. Fig. 3 shows the circle which passes through points O, P and Q.

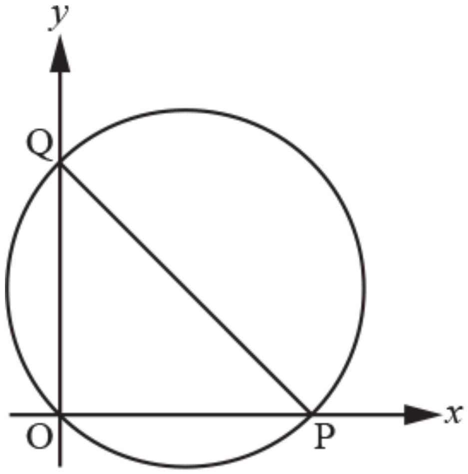


Fig. 3

Find the equation of the circle. [5]

8. (a) **In this question you must show detailed reasoning.**

Determine the exact values of k for which the curves $y = x^2 - kx$ and $y = 3(k + 1) + kx - x^2$ touch. [6]

(b) Determine whether or not there is a value of k for which the curves cross on the y-axis. [4]

9. The equation of a circle is $(x + 2)^2 + (y - 3)^2 = 5$.

(i) State the radius of this circle and the coordinates of its centre. [2]

(ii) Find the equation of the line through the centre of the circle which is parallel to the line $5x + y = 4$. [2]

10. The equation of a circle is

$$x^2 - 4x + y^2 + 6y - 12 = 0.$$

Find

(a) the coordinates of the centre of the circle, [2]

(b) the radius of the circle. [2]

11. In this question you must show detailed reasoning.

The centre of a circle C is at the point (–1, 3) and C passes through the point (1, –1). The straight line L passes through the points (1, 9) and (4, 3). Show that L is a tangent to C. [7]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	midpt of $AB = \left(\frac{1}{2}, \frac{5}{2}\right)$ oe www	B2	allow unsimplified B1 for one coordinate correct	if working shown, should come from $\left(\frac{3+-2}{2}, \frac{4+1}{2}\right)$ oe NB B0 for *x* coord. $= \frac{5}{2}$, (obtained from subtraction instead of addition)
		i	grad $AB = \frac{4-1}{3-(-2)}$ oe	M1	must be obtained independently of given line; accept 3 and 5 correctly shown eg in a sketch, followed by 3/5 M1 for rise/run = 3/5 etc M0 for just 3/5 with no evidence	for those who find eqn of AB first, M0 for just $\frac{y-4}{1-4} = \frac{x-3}{-2-3}$ oe, but M1 for $y-4 = \frac{1-4}{-2-3}(x-3)$ oe ignore their going on to find the eqn of AB after finding grad AB
		i	using gradient of AB to obtain grad perp bisector	M1	for use of $m_1m_2 = -1$ soi or ft their gradient AB M0 for just $\frac{-5}{3}$ without AB grad found	this second M1 available for starting with given line $= \frac{-5}{3}$ and obtaining grad. of AB from it
		i	$y - 2.5 = \frac{-5}{3}(x - 0.5)$ oe	M1	eg M1 for $y = \frac{-5}{3}x + c$ and subst of midpt;	no ft for gradient of AB used

		i	completion to given answer $3y + 5x = 10$, showing at least one interim step	M1	ft their gradient of perp bisector and midpt; M0 for just rearranging given equation condone a slight slip if they recover quickly and general steps are correct (eg sometimes a slip in working with the c in $y = \frac{-5}{3}x + c$ - condone $3y = -5x + c$ followed by substitution and consistent working) M0 if clearly 'fudging' Examiner's Comments This part was usually done well. Most candidates were confident finding the gradient of AB, although a few failed to show their working. Almost all were then able to find the perpendicular gradient. A minority were unaware that the perpendicular bisector would pass through the midpoint of AB. Most who realised this were able to calculate the midpoint accurately. Once all the information was combined into a straight line equation, a significant minority struggled to rearrange the equation correctly because the arithmetic involved fractions. Pleasingly almost all the candidates managed to work towards the given equation, rather than trying to use the given equation to get back to a common form with their answer. Some wasted time finding the equation of AB first.	NB answer given; mark process not answer; annotate if full marks not earned eg with a tick for each mark earned scores such as B2M0M0M1M1 are possible after B2, allow full marks for complete method of showing given line has gradient perp to AB (grad AB must be found independently at some stage) and passes through midpt of AB
		ii	$3y + 5(4y - 21) = 10$	M1	or other valid strategy for eliminating one variable attempted eg $\frac{-5}{3}x + \frac{10}{3} = \frac{x}{4} + \frac{21}{4}$; condone one error	or eg $20y = 5x + 105$ and subtraction of two eqns attempted no ft from wrong perp bisector eqn, since given allow M1 for candidates who reach $y = 115/23$ and

						then make a worse attempt, thinking they have gone wrong
		ii	(– 1, 5) or $y = 5$, $x = -1$ isw	A2	A1 for each value; if AO allow SC1 for both values correct but unsimplified fractions, eg $\left(\frac{-23}{23}, \frac{115}{23}\right)$ **Examiner's Comments** Some wasted time finding the equation of CD, which was given. Many solved the simultaneous equations correctly, but sometimes using less efficient methods, giving themselves complicated fractions to work with. A few who eliminated x struggled with simplifying y = 115/23. A significant minority used the implication in part (iii) that E was the midpoint of CD to obtain a solution, gaining no marks for this.	NB M0A0 in this part for finding E using info from (iii) that implies E is midpt of CD
		iii	$(x - a)^2 + (y - b)^2 = r^2$ seen or used	M1	or for $(x + 1)^2 + (y - 5)^2 = k$, or ft their E, where $k > 0$	
		iii	$1^2 + 4^2$ oe (may be unsimplified), from clear use of A or B	M1	for calculating AE or BE or their squares, or for subst coords of A or B into circle eqn to find r or r^2, ft their E;	this M not earned for use of CE or DE or ½ CD NB some cands finding $AB^2 = 34$ then obtaining 17 erroneously so M0
		iii	$(x + 1)^2 + (y - 5)^2 = 17$	A1	for eqn of circle centre E, through A and B; allow A1 for $r^2 = 17$ found after $(x + 1)^2 + (y - 5)^2 = r^2$ stated and second M1 clearly earned	
		iii			if $(x + 1)^2 + (y - 5)^2 = 17$ appears without clear evidence of using A or B, allow the first M1 then M0 SC1	SC also earned if circle comes from C or D and E, but may recover and earn the second M1 later by using A or B
		iii	showing midpt of CD = (– 1, 5)	M1		

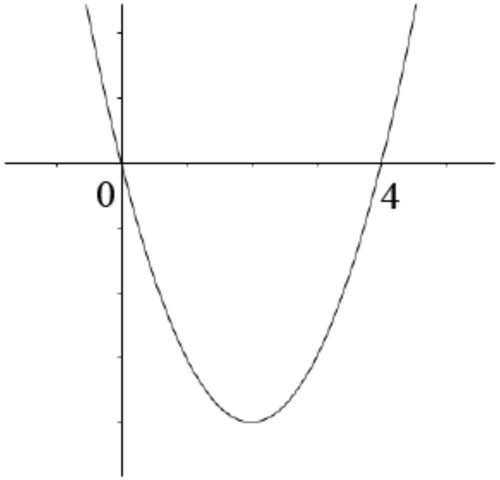
		iii	showing CE or DE = $\sqrt{17}$ oe or showing one of C and D on circle	M1	alt M1 for showing $CD^2 = 68$ oe allow to be earned earlier as an invalid attempt to find r showing that both C and D are on circle and commenting that E is on CD is enough for last M1M1; similarly showing $CD^2 = 68$ and both C and D are on circle oe earns last M1M1 **Examiner's Comments** Most knew the form for the equation of the circle, although some used r or $\sqrt{r}$ instead of r^2. Some used C or D or the length of CD to calculate the radius, instead of using A or B. Others assumed that AB was a diameter. Very few produced enough to show that CD is a diameter, with many thinking that showing that CD is twice the radius was enough. Some stated that E was the midpoint of CD without any working to support it. This meant that the full 5 marks on this question were rarely awarded, though a significant number obtained 4 marks.	other methods exist, eg: may find eqn of circle with centre E and through C or D and then show that A and B and other of C/D are on this circle – the marks are then earned in a different order; award M1 for first fact shown and then final M1 for completing the argument; if part-marks earned, annotate with a tick for each mark earned beside where earned
			Total	14		
2		i	[radius =] $\sqrt{20}$ or $2\sqrt{5}$ isw	B1	B0 for $\pm\sqrt{20}$ oe **Examiner's Comments** Almost all candidates obtained both marks for this part. Some gave 20 for the radius. (–3, –2) was only very occasionally seen.	
		i	[centre =] (3, 2)	B1		condone lack of brackets with coordinates, here and in other questions

		ii	substitution of $x = 0$ or $y = 0$ into circle equation	M1	or use of Pythagoras with radius and a coordinate of the centre eg $20 - 2^2$ or $h^2 + 3^2 = 20$ ft their centre and / or radius	equation may be expanded first, and may include an error bod intent allow M1 for $(x - 3)^2 = 20$ and / or $(y - 2)^2 = 20$
		ii	$(x - 7)(x + 1)$ [=0]	M1	no ft from wrong quadratic; for factors giving two terms correct, or formula or completing square used with at most one error	completing square attempt must reach at least $(x - a)^2 = b$ following use of Pythagoras allow M1 for attempt to add 3 to [±]4
		ii	(7, 0) and (−1, 0) isw	A1	accept $x = 7$ or −1 (both required)	
		ii	$[y =]\dfrac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times (-7)}}{2}$ oe	M1	no ft from wrong quadratic; for formula or completing square used with at most one error accept $y = \dfrac{4 \pm \sqrt{44}}{2}$ oe isw	completing square attempt must reach at least $(y - a)^2 = b$ following use of Pythagoras allow M1 for attempt to add 2 to [±] $\sqrt{11}$
		ii	$\left(0, 2 \pm \sqrt{11}\right)$ or $\left(0, \dfrac{4 \pm \sqrt{44}}{2}\right)$ isw	A1	**Examiner's Comments** Finding the intersections of the circle with the axes was often well done. Almost all candidates obtained the first mark for substituting $y = 0$ or $x = 0$ in the circle equation, although some then omitted the $(-2)^2$ and/or $(-3)^2$. Some, having correctly found the x-intersections, substituted those values instead of starting again by substituting 0 to find the y values. Since the correct y equation did not factorise, there was distinctly less success in finding the y values than the x values. Some good solutions using completing the square were seen, after reaching $(y - 2)^2 = 11$, for	annotation is required if part marks are earned in this part: putting a tick for each mark earned is sufficient

					instance, although some omitted the negative square root and then gave just one value.	
		iii	show both A and B are on circle	B1	explicit substitution in circle equation and at least one stage of interim working required oe	or clear use of Pythagoras to show AC and BC each $= \sqrt{20}$
		iii	(4, 5)	B2	B1 each or M1 for $\left(\frac{7+1}{2}, \frac{6+4}{2}\right)$	
		iii	$\sqrt{10}$	B2	from correct midpoint and centre used; B1 for $\pm\sqrt{10}$ M1 for $(4-3)^2 + (5-2)^2$ or $1^2 + 3^2$ or ft their centre and / or midpoint, or for the square root of this **Examiner's Comments** Almost all candidates were able to show that A and B lie on the circle, usually by substituting the coordinates or finding the distance between each point and the centre, though some used the longer method of substituting one coordinate and solving the resultant quadratic equation. A few candidates omitted to show that B, as well as A, lies on the circle. Almost all candidates obtained the coordinates of the midpoint of AB (4, 5) successfully, with a small minority subtracting rather than adding. Most candidates realised that the distance of the chord from the centre of the circle was the distance from (4,5) to (3,2) and obtained the correct answer of √10. Some calculated the length of AB and proceeded no further; some halved it and used Pythagoras but only a minority were successful with this approach.	may be a longer method finding length of ½ AB and using Pythag. with radius; no ft if one coord of midpoint is same as that of centre so that distance formula / Pythag is not required eg centre correct and midpt (3, −1) annotation is required if part marks are earned in this part: putting a tick for each mark earned is sufficient

			Total	12		
3		a	Radius = 5 (2, −3)	B1(AO1.1) B1(AO1.1) [2]		
		b	$(y+3)^2 = 21$ or $y^2 + 6y - 12 = 0$ Roots $-3+\sqrt{21}$ and $-3-\sqrt{21}$ $\left(0,-3+\sqrt{21}\right)$ and $\left(0,-3-\sqrt{21}\right)$	M1(AO1.1a) A1(AO1.1) A1(AO1.1) [3]	Substituting $x = 0$ and rearranging For one y-value All correct	
		c	$(1-2)^2 + (2+3)^2 = 1^2 + 5^2$ E.g. This is more than 25 so outside the circle	M1(AO1.1) A1(AO1.1) [2]	Or distance of (1, 2) from their centre Or distance of (1, 2) from centre is $\sqrt{26} > 5$, so outside the circle	
		d	Gradient $\text{CP} = \dfrac{1-(-3)}{-1-2} = -\dfrac{4}{3}$ FT their C(entre)	M1(AO1.1a) M1(AO1.1)		

			Gradient of tangent $=\frac{3}{4}$ FT their grad CP Equation of tangent $y-1=\frac{3}{4}(x-(-1))$ FT their grad $4y = 3x + 7$ oe	M1(AO1.1) A1(AO1.1) [4]		
			Total	11		
4			DR $2x^2 - kx = x^2 - k$ $x^2 - kx + k = 0$ discriminant $= k^2 - 4k$ $k^2 - 4k \geq 0$	B1(AO3.1a) M1(AO2.1) B1(AO1.2) M1(AO1.1) M1(AO2.4) A1(AO2.5)	Equating the two expressions **must** be seen Condone one error in rearranging Or give table of values, oe	

			 $k \leq 0$ or $k \geq 4$	[6]	or $\{k : k \leq 0\} \cup \{k : k \geq 4\}$		
			Total	6			
5			DR $y = 4 - 2x$ $4 - 2x = x^2 + x$ $\Rightarrow x^2 + 3x - 4 = 0$ $\Rightarrow x = 1$ or $x = -4$ $y = 2$ or $y = 12$ (1,2) and (−4,12)	M1(AO2.1) M1(AO1.1) A1(AO1.1) A1(AO1.1) A1(AO2.5)	Eliminating x or y must be seen Form a quadratic equation For final A mark, corresponding	Or $y^2 - 14y + 24 = 0$ SC1 for one pair of coordinates only	

				[5]	values of x and y must be expressed as coordinates from well set out correct solution	
			Total	5		
6		a	Distance $CO=\sqrt{(-4)^2+3^2}=5$ $(x+4)^2+(y-3)^2=25$	M1(AO 1.1a) M1(AO 1.1a) A1(AO 2.1) [3]	Allow $(-4)^2+3^2$ if CO^2 clearly stated Use of circle equation with their r^2 Correct answer; isw	
		b	A is the point (0, 6) Gradient of AC is $\frac{3}{4}$ Gradient of tangent is $-\frac{4}{3}$ [Equation of tangent is $y-6=-\frac{4}{3}x$]	B1(AO 3.1a) M1(AO 1.1a) M1(AO 1.1a) M1FT(AO 1.1b) A1(AO 2.2a)		

			When $y = 0$, $x = \frac{9}{2}$	[5]	Use their point A and gradient to find B cao	
			Total	8		
7			OP = OQ = 6 Centre of circle is at (3, 3) $r^2 = 3^2 + 3^2$ $= 18$ $(x-3)^2 + (y-3)^2 = 18$	M1(AO 3.1a) M1(AO 3.1a) M1(AO 1.1) A1(AO 1.1) A1(AO 1.1) [5]	Use area of triangle to find OP and OQ Midpoint of PQ oe isw	
			Total	5		
8		a	**DR** At intersections, $x^2 - kx = 3(k+1) + kx - x^2$ $2x^2 - 2kx - 3(k+1) = 0$ For touching, '$b^2 - 4ac = 0$'	M1(AO 3.1a) M1(AO 1.1) M1(AO 2.1)	Forming quadratic	

			$4k^2 + 24(k+1) = 0 \Rightarrow k^2 + 6k + 6 = 0$ $$k = \frac{-6 \pm \sqrt{12}}{2}$$ $$k = -3 \pm \sqrt{3}$$	A1(AO 2.2a) B1(AO 1.1) B1(AO 1.1) [6]	0 on one side of quadratic Forming quadratic from discriminant Correct quadratic in 3-term form Use of formula or completing the square	$(k+3)^2 = 3$
		b	On y-axis $x = 0$, so $y = 0^2 - 0k = 0$ If they cross on the y-axis, they cross at the origin $0 = 3(k+1) + k0 - 0^2$ $k = -1$ [so a value exists]	M1(AO 3.1a) B1(AO 2.2a) M1(AO 1.1) A1(AO 2.1) [4]	Use of $x = 0$ to find y Use of $x = 0$ and $y = 0$	
			Total	10		

9		i	[centre] (–2,3) [radius] $\sqrt{5}$	B1 B1 **[2]**	<table><tr><td>B0 for ± $\sqrt{5}$</td><td></td></tr></table> **Examiner's Comments** There were a small number of candidates who incorrectly worked out the centre of the circle, usually giving the centre with incorrect signs. A number of candidates gave the radius as either 5 or 25 but most candidates scored full marks here. The vast majority of candidates found the correct equation of the required line, with those who dropped marks usually because they chose 5 as the gradient and not –5.	
		ii	$5x + y = -7$ or $y = -5x - 7$ or $5x + y + 7 = 0$	2 **[2]**	<table><tr><td>M1 for $5x + y = k$, $k \neq 4$ or for gradient of parallel line = –5 or for answer $-5x - 7$</td><td>if wrong centre in 5(i), can earn just M1</td></tr></table> **Examiner's Comments** Some of these candidates clearly misunderstood the concept of $y = mx + c$ as they clearly believed that the coefficient of x was the gradient no matter what side of the equation the x term appeared on. Only a small minority used a gradient based on the negative reciprocal.	

			Total	4		
10		a	$(x-2)^2 + (y+3)^2$ seen $(2, -3)$	M1(AO1.1) A1(AO1.1b) [2]	ignore other working	
		b	$(x-2)^2 + (y+3)^2 - 2^2 - 3^2 - 12 = 0$ or better seen $r = 5$	M1(AO1.1b) A1 (AO 1.1b) [2]		
			Total	2		
11			$(x - -1)^2 + (y-3)^2 = r^2$ $r^2 = (1 - -1)^2 + (-1 - 3)^2$ L: $m = -2$ $y = -2x + 11$ oe substitution of their $y = -2x + 11$ in their $(x+1)^2 + (y-3)^2 = 20$ $x^2 - 6x + 9 = 0$ oe	M1 (AO2.1) M1 (AO1.1) B1 B1 (AO1.1) (AO1.1) M1 (AO1.1) A1 (AO1.1) E1 (AO2.4)	Left side correct and $= r^2$ Or find L first (B1B1), then find equation of line perp to L through $(-1, 3)$ (M1M1) then substitute (M1), solve (A1) then check (E1). soi	or line through centre which is perpendicular to L has equation $y - 3 = ½(x - -1)$ meets L at $(3, 5)$ $(3+1)^2 + (5-3)^2 = r^2 = 20$	

			$(x-3)^2 = 0$ so repeated root Hence line touches the curve and is a tangent	[7]	<table><tr><td> or $(-6)^2 -$ $4\times1\times9 = 0$</td><td>so lines meet at circumference of circle at right angles so L is a tangent</td></tr></table> **Examiner's Comments** There were a good many completely correct answers to this question. Many candidates worked out the equation of C and the equation of L and found that C and L only intersected at the point where $x = 3$ (and $y = 5$). Most of these candidates then went on to say that, since this was a repeated root, L was a tangent to C. Some of these candidates omitted to state this final essential fact, while others spent a considerable amount of time establishing it by other means. Another successful approach to this question was to find the equation of L and the equation of the line perpendicular to L that passes through the centre of C, and show that they intersect at (3, 5). It is then necessary to show that this point lies on C, either by finding the equation of C or by showing that the distance from (3, 5) to the centre of C, (–1, 3), is equal to the radius of C.	

					A small number of candidates noted that the gradient of L is the same as the gradient of the radius joining the centre of C to the point (1, −1); a few candidates tried to argue that this made L a tangent to C.	
			Total	7		