Questions

Q1.

A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

(Total for question = 5 marks)

Q2.

In 1997 the average CO_2 emissions of new cars in the UK was 190 g/km.

In 2005 the average CO_2 emissions of new cars in the UK had fallen to 169 g/km.

Given A g/km is the average CO_2 emissions of new cars in the UK n years after 1997 and using a linear model,

(a) form an equation linking A with n.

(3)

In 2016 the average CO_2 emissions of new cars in the UK was 120 g/km.

(b) Comment on the suitability of your model in light of this information.

(3)

(Total for question = 6 marks)

Q3.

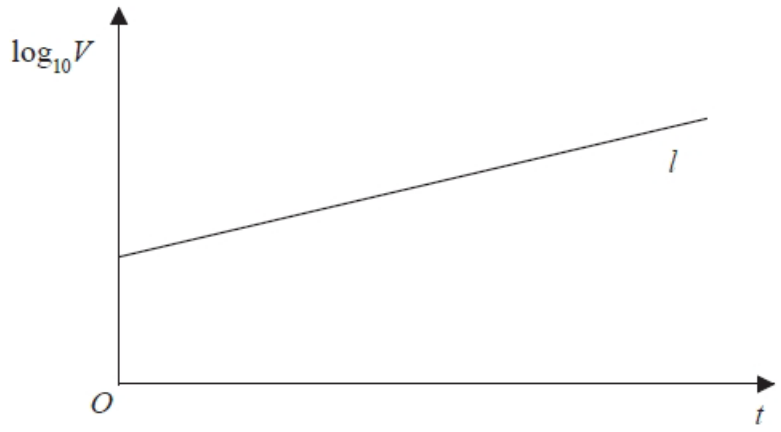

Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,
(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

(Total for question = 8 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Attempts $H = mt + c$ with both $(3, 2.35)$ and $(6, 3.28)$	M1	3.3
	Method to find both m and c	dM1	3.1a
	$H = 0.31t + 1.42$ oe	A1	1.1b
		(3)	
(b)	Uses the model and states that the initial height is their 'b'	B1ft	3.4
	Compares 140 cm with their 1.42 (m) and makes a valid comment. In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close.	B1ft	3.5a
		(2)	
			(5 marks)

Notes
Mark parts (a) and (b) as one **(a)** **M1:** For creating a linear model with both pieces of information given. Eg. Accept sight of $2.35 = 3m + c$ and $3.28 = 6m + c$ Condone slips on the 2.35 and 3.28. Allow for an attempt at the "gradient" $m = \frac{3.28 - 2.35}{6 - 3} (= 0.31)$ or the intercept. Allow for a pair of simultaneous in any variable even x and y **dM1:** A full method to find both constants. For simultaneous equations award if they arrive at values for m and c. If they attempted the gradient it would be for attempting to find "c" using $y = mx + c$ with their m and one of the points $(3, 2.35)$ or $(6, 3.28)$ **A1:** A correct model using allowable/correct variables. $H = 0.31t + 1.42$ Condone $h \leftrightarrow H, t \leftrightarrow T$ Allow equivalents such as $H = \frac{31}{100}t + \frac{142}{100}$, $t = \frac{H - 1.42}{0.31}$ but not $H = \frac{0.93}{3}t + 1.42$ Do not allow $H = 0.31t + 1.42$ m (with the units) **(b)** To score any marks in (b) the model must be of the form $H = mt + b$ where $m > 0, b > 0$ **B1ft:** States or implies that 1.42 (with or without units) or 142 cm (including the units) is the original height or the height when $t = 0$ You should allow statements such as c = 1.42 or original height = 1.42 (m) Follow through on their value of 'c' , so for $H = 0.25t + 1.60$ it is scored for stating the initial height is 1.60 (m) or 160 cm. Do not follow through if $c \leq 0$ **B1ft:** Compares 140 cm with their 1.42 (m) **and** makes a valid comment. In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close or approximately the same. Allow $1.42\text{m} \approx 1.4\text{m}$ or similar In the case of $H = 0.25t + 1.60$ it would be for stating that the fact that it does not support the use of the model as the values are too different. If they state $1.60 > 1.40$ this is insufficient. They cannot just state that they are not the same. It must be implied that there is a significant difference. As a rule of thumb use "good model" for between 135cm and 145 cm. This requires a correct calculation for their H, a correct statement with an appreciation shown for the units and a correct conclusion. **SC B1 B0** Award SC for incomplete answers which suggest the candidate knows what to do. Eg. In (b) $H = 0.31t + 1.42$ followed by in (c) It supports the model as when $t = 0$ it is approximately 140 cm

Q2.

Question	Scheme	Marks	AOs
(a)	Attempts $A = mn + c$ with either $(0,190)$ or $(8,169)$ Or attempts gradient eg $m = \pm\frac{190-169}{8}\left(=-2.625\right)$	M1	3.3
	Full method to find a linear equation linking A with n E.g. Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously	dM1	3.1b
	$A = -2.625n + 190$	A1	1.1b
		(3)	
(b)	Attempts $A = -2.625 \times 19 + 190 = \ldots$	M1	3.4
	$A = 140.125$ g km^{-1}	A1	1.1b
	It is predicting a much higher value and so is not suitable	B1ft	3.5a
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts $A = mn + c$ with either $(0,190)$ or $(8,169)$ considered.
Eg Accept sight of $190 = 0n + c$ or $169 = 8m + c$ or $A - 169 = m(n-8)$ or $A = 190 + mn$ where m could be a value.
Also accept an attempt to find the gradient $\pm\frac{190-169}{8}$ or sight of ± 2.625 or $\pm\frac{21}{8}$ oe

dM1: A full method to find both constants of a linear equation
Method 1: Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously
Method 2: Uses gradient and a point Eg $m = \pm\frac{190-169}{8}\left(=-2.625\right)$ and $c = 190$
Condone different variables for this mark. Eg. y in terms of x.

A1: $A = -2.625n + 190$ or $A = -\frac{21}{8}n + 190$ oe

(b)

M1: Attempts to substitute "n" $= 19$ into their linear model to find A. They may call it $x = 19$
Alternatively substitutes $A = 120$ into their linear model to find n.

A1: $A = 140.125$ from $n = 19$ Allow $A = 140$
or $n = 26 / 27$ following $A = 120$

B1ft: Requires a correct calculation for their model, a correct statement and a conclusion
E.g For correct (a) $A = 140$ is (much) higher than 120 so the model is not suitable/appropriate.
Follow through on a correct statement for their equation. As a guide allow anything within [114,126] to be regarded as suitable. Anything less than 108 or more than 132 should be justified as unsuitable.

Note B0 Recorded value is not the same as/does not equal/does not match the value predicted

Q3.

Question	Scheme	Marks	AOs
(a)	For a correct equation in p or q $p=10^{4.8}$ or $q=10^{0.05}$	M1	1.1b
	For $p=$ awrt 63100 or $q=$ awrt 1.122	A1	1.1b
	For correct equations in p and q $p=10^{4.8}$ and $q=10^{0.05}$	dM1	3.1a
	For $p=$ awrt 63100 and $q=$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$	M1	3.4
	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a)

M1: For a correct equation in p or q This is usually $p = 10^{4.8}$ or $q = 10^{0.05}$ but may be $\log q = 0.05$ or $\log p = 4.8$

A1: For $p =$ awrt 63100 or $q =$ awrt 1.122

M1: For linking the two equations and forming correct equations in p and q. This is usually $p = 10^{4.8}$ and $q = 10^{0.05}$ but may be $\log q = 0.05$ and $\log p = 4.8$

A1: For $p =$ awrt 63100 and $q =$ awrt 1.122 Both these values implies M1 M1

.....

ALT I(a)

M1: Substitutes $t = 0$ and states that $\log p = 4.8$

A1: $p =$ awrt 63100

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p =$ awrt 63100 and $q =$ awrt 1.122

.....

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled " p is..... " and "q is ....."

(c)

M1: For substituting $t = 30$ into $V = pq^t$ using their values for p and q or substituting t =30 into $\log_{10} V = 0.05t + 4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign. Remember to isw after a correct answer