Questions

Q1.

A circle C with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle C also passes through the point $(10, 1)$.

(3)

The tangent to the circle C at the point $(10, 11)$ meets the y axis at the point P

and the tangent to the circle C at the point $(10, 1)$ meets the y axis at the point Q.

(b) Show that the distance PQ is 58 explaining your method clearly.

(7)

(Total for question = 10 marks)

Q2.

The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C
(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k.

(6)

(Total for question = 9 marks)

Q3.

A circle *C* has equation

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

(a) Find

(i) the coordinates of the centre of *C*,
(ii) the exact radius of *C*.

(3)

The straight line with equation $x = k$, where k is a constant, is a tangent to *C*.

(b) Find the possible values for k.

(2)

(Total for question = 5 marks)

Q4.

(i) A circle C_1 has equation

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

The line l is the tangent to C_1 at the point $P(-5, 7)$.
Find an equation of l in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where k is a constant.
Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k.

(4)

(Total for question = 9 marks)

Q5.

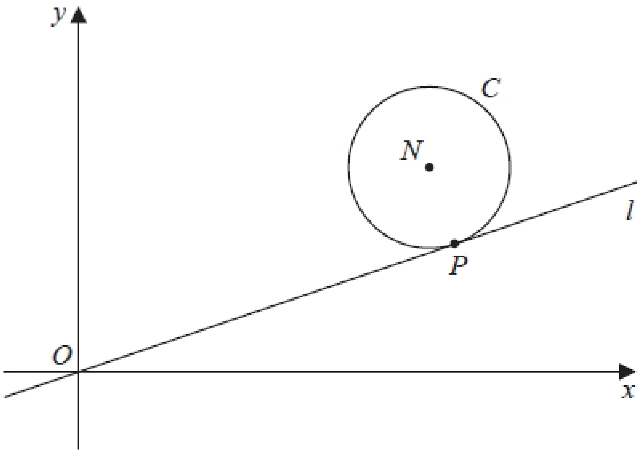


Figure 4

Figure 4 shows a sketch of a circle C with centre $N(7, 4)$

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) an equation for C.

(4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C.

(c) Find the value of k.

(3)

(Total for question = 9 marks)

Q6.

A circle C has equation

$$x^2 + y^2 - 4x + 10y = k$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

(b) State the range of possible values for k.

(2)

(Total for question = 4 marks)

Q7.

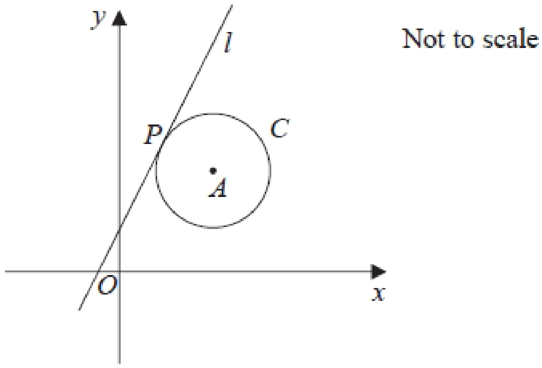


Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

(Total for question = 10 marks)

Q8.

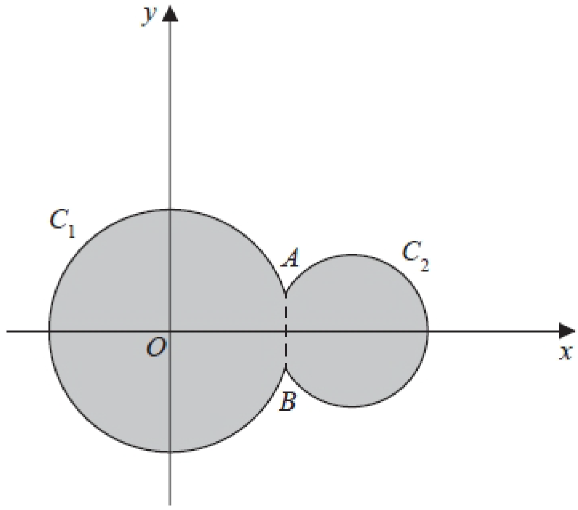


Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

(Total for question = 8 marks)

Q9.

A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

(Total for question = 7 marks)

Q10.

The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of *C*,
(ii) the exact radius of *C*, giving your answer as a simplified surd.

(4)

The line *l* has equation $y = 3x + k$ where *k* is a constant.

Given that *l* is a tangent to *C*,

(b) find the possible values of *k*, giving your answers as simplified surds.

(5)

(Total for question = 9 marks)

Mark Scheme

Q1.

Question	Scheme		Marks	AOs
(a)	Way 1: Finds circle equation $(x \pm 2)^2 + (y \mp 6)^2 = (10 \pm (-2))^2 + (11 \mp 6)^2$	Way 2: Finds distance between $(-2,6)$ and $(10, 11)$	M1	3.1a
	Checks whether $(10,1)$ satisfies their circle equation	Finds distance between $(-2,6)$ and $(10, 1)$	M1	1.1b
	Obtains $(x+2)^2 + (y-6)^2 = 13^2$ and checks that $(10+2)^2 + (1-6)^2 = 13^2$ so states that $(10,1)$ lies on C *	Concludes that as distance is the same $(10, 1)$ lies on the circle C *	A1*	2.1
			(3)	
(b)	Finds radius gradient $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)		M1	3.1a
	Finds gradient perpendicular to their radius using $-\frac{1}{m}$		M1	1.1b
	Finds (equation and) y intercept of tangent (see note below)		M1	1.1b
	Obtains a correct value for y intercept of their tangent i.e. 35 or – 23		A1	1.1b
	Way 1: Deduces gradient of second tangent	Way 2: Deduces midpoint of PQ from symmetry ((0,6))	M1	1.1b
	Finds (equation and) y intercept of second tangent	Uses this to find other intercept	M1	1.1b
	So obtains distance PQ=35+23=58*		A1*	1.1b
			(7)	
				(10 marks)

Notes

(a) Way 1 and Way 2:

M1 : Starts to use information in question to find equation of circle or radius of circle

M1 : Completes method for checking that (10, 1) lies on circle

A1*: Completely correct explanation with no errors concluding with statement that circle passes through (10, 1)

(b) M1: Calculates $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)

M1: Finds $-\frac{1}{m}$ (correct answer is $-\frac{12}{5}$ or $\frac{12}{5}$) This is referred to as m' in the next note.

M1: Attempts $y - 11 = their\left(-\frac{12}{5}\right)(x-10)$ or $y - 1 = their\left(\frac{12}{5}\right)(x-10)$ and puts $x = 0$, or uses vectors to find intercept e.g. $\frac{y-11}{10} = -m'$

A1: One correct intercept 35 or – 23

Qu 17(b) continued Way 1: M1: Uses the negative of their previous tangent gradient or uses a correct $-\frac{12}{5}$ or $\frac{12}{5}$ M1: Attempts the second tangent equation and puts $x = 0$ or uses vectors to find intercept e.g. $\frac{11-y}{10} = m'$ Way 2: M1: Finds midpoint of PQ from symmetry. (This is at (0,6)) M1: Uses this midpoint to find second intercept or to find difference between midpoint and first intercept. e.g. 35 – 6 = 29 then 6 – 29 = –23 so second intercept is at (–23, 0) Ways 1 and 2: A1*: Obtain 58 correctly from a valid method.

Q2.

Question	Scheme	Marks	AOs
(a)	Attempts to complete the square $(x\pm3)^2+(y\pm5)^2=...$	M1	1.1b
	(i) Centre $(3,-5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Substitute $y=kx$ in $x^2+y^2-6x+10y+9=0$	M1	3.1a
	Collects terms to form correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$	A1	1.1b
	Attempts $b^2-4ac...0$ for their a, b and c leading to values for k $"(10k-6)^2-36(1+k^2)...0" \rightarrow k=...,...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2-4ac>0$ and chooses the outside region (see note) for their critical values (Both a and b must have been expressions in k)	dM1	3.1a
	Deduces $k<0, k>\frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

Notes

(a)

M1: Attempts $(x\pm3)^2+(y\pm5)^2=..$

This mark may be implied by candidates writing down a centre of $(\pm3,\pm5)$ or $r^2=25$

(i) A1: Centre $(3,-5)$

(ii) A1: Radius 5. Do not accept $\sqrt{25}$

Answers only (no working) scores all three marks

(b)

B1: Uses a sketch or their subsequent quadratic to deduce that $k=0$ is a critical value.
You may award for the correct $k<0$ but award if $k\leqslant0$ or even with greater than symbols

M1: Substitutes $y=kx$ in $x^2+y^2-6x+10y+9=0$ or their $(x\pm3)^2+(y\pm5)^2=...$ to form an equation in just x and k. It is possible to substitute $x=\frac{y}{k}$ into their circle equation to form an equation in just y and k.

A1: Correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$ with the terms in x collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as $x^2+k^2x^2+(10k-6)x+9=0$ so long as the correct values of a, b and c are used in $b^2-4ac...0$.

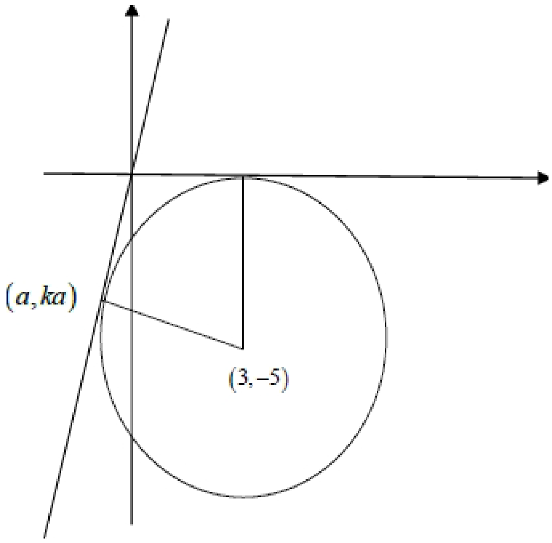
FYI The equation in y and k is $(1+k^2)y^2+(10k^2-6k)y+9k^2=0$ oe

M1: Attempts to find two critical values for k using $b^2-4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.

dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both a and b must have been expressions in k.
Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg.
$b^2-4ac=(k-6)^2-4\times(1+k^2)\times9>0\Rightarrow-35k^2-12k>0\Rightarrow k(35k+12)<0$

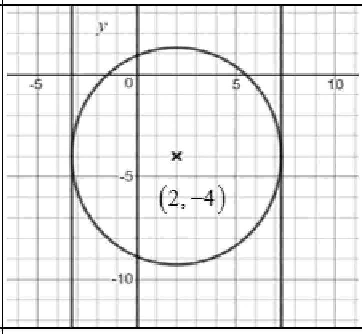
A1: Deduces $k<0, k>\frac{15}{8}$. This must be in terms of k.
Allow exact equivalents such as $k<0\cup k>1.875$
but not allow $0>k>\frac{15}{8}$ or the above with AND, & or $\cap$ between the two inequalities

Alternative using a geometric approach with a triangle with vertices at (0,0), and (3,–5)



Alt (b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Distance from (a, ka) to $(0, 0)$ is $3 \Rightarrow a^2(1 + k^2) = 9$	M1	3.1a
	Tangent and radius are perpendicular $\Rightarrow k \times \frac{ka+5}{a-3} = -1 \Rightarrow a(1+k^2) = 3 - 5k$	M1	3.1a
	Solve simultaneously, (dependent upon both M's)	dM1	1.1b
	$k = \frac{15}{8}$	A1	1.1b
	Deduces $k < 0, k > \frac{15}{8}$	A1	2.2a
		(6)	

Q3.

Question	Scheme	Marks	AOs
(a)	$x^2+y^2-4x+8y-8=0$		
	Attempts $(x-2)^2+(y+4)^2-4-16-8=0$	M1	1.1b
	(i) Centre $(2,-4)$	A1	1.1b
	(ii) Radius $\sqrt{28}$ oe Eg $2\sqrt{7}$	A1	1.1b
		(3)	
(b)	 Attempts to add/subtract 'r' from '2' $k=2\pm\sqrt{28}$	M1	3.1a
		A1ft	1.1b
		(2)	
			(5 marks)

(a)

M1: Attempts to complete the square. Look for $(x\pm 2)^2+(y\pm 4)^2 ...$

If a candidate attempts to use $x^2+y^2+2gx+2fy+c=0$ then it may be awarded for a centre of $(\pm 2,\pm 4)$ Condone $a=\pm 2, b=\pm 4$

A1: Centre $(2,-4)$ This may be written separately as $x=2, y=-4$ BUT $a=2, b=-4$ is A0

A1: Radius $\sqrt{28}$ or $2\sqrt{7}$ isw after a correct answer

(b)

M1: Attempts to add **or** subtract their radius from their 2.

Alternatively substitutes $y=-4$ into circle equation and finds x/k by solving the quadratic equation by a suitable method.

A third (and more difficult) method would be to substitute $x=k$ into the equation to form a quadratic eqn in $y \Rightarrow y^2+8y+k^2-4k-8=0$ and use the fact that this would have one root. E.g. $b^2-4ac=0 \Rightarrow 64-4\left(k^2-4k-8\right)=0 \Rightarrow k=..$ Condone slips but the method must be sound.

A1ft: $k=2+\sqrt{28}$ and $k=2-\sqrt{28}$ Follow through on their 2 and their $\sqrt{28}$

If decimals are used the values must be calculated. Eg $k=2\pm 5.29 \rightarrow k=7.29, k=-3.29$

Accept just $2\pm\sqrt{28}$ or equivalent such as $2\pm 2\sqrt{7}$

Condone $x=2+\sqrt{28}$ and $x=2-\sqrt{28}$ but not $y=2+\sqrt{28}$ and $y=2-\sqrt{28}$

Q4.

Question	Scheme	Marks	AOs
(i)	$x^2+y^2+18x-2y+30=0 \Rightarrow (x+9)^2+(y-1)^2=...$	M1	1.1b
	Centre $(-9,1)$	A1	1.1b
	Gradient of line from $P(-5,7)$ to "$(-9,1)$" $=\frac{7-1}{-5+9}=\left(\frac{3}{2}\right)$	M1	1.1b
	Equation of tangent is $y-7=-\frac{2}{3}(x+5)$	dM1	3.1a
	$3y-21=-2x-10 \Rightarrow 2x+3y-11=0$	A1	1.1b
		(5)	
(ii)	$x^2+y^2-8x+12y+k=0 \Rightarrow (x-4)^2+(y+6)^2=52-k$	M1	1.1b
	Lies in Quadrant 4 if radius $<4 \Rightarrow "52-k"<4^2$	M1	3.1a
	$\Rightarrow k>36$	A1	1.1b
	Deduces $52-k>0 \Rightarrow$ Full solution $36<k<52$	A1	3.2a
		(4)	
			(9 marks)

Notes

(i)

M1: Attempts $(x\pm9)^2 ....(y\pm1)^2=...$ It is implied by a centre of $(\pm9,\pm1)$

A1: States or uses the centre of C is $(-9,1)$

M1: A correct attempt to find the gradient of the radius using their $(-9,1)$ and P. E.g. $\frac{7-"1"}{-5-"-9"}$

dM1: For the complete strategy of using perpendicular gradients and finding the equation of the tangent to the circle. It is dependent upon both previous M's. $y-7=-\frac{1}{\text{gradient } CP}(x+5)$

Condone a sign slip on one of the -7 or the 5.

A1: $2x+3y-11=0$ oe such as $k(2x+3y-11)=0, k\in\mathbb{Z}$

Attempt via implicit differentiation. The first three marks are awarded

M1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow ...x+...y\frac{dy}{dx}+18-2\frac{dy}{dx}....=0$

A1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow 2x+2y\frac{dy}{dx}+18-2\frac{dy}{dx}=0$

M1: Substitutes $P(-5,7)$ into their equation involving $\frac{dy}{dx}$

(ii)

M1: For reaching $(x \pm 4)^2 + (y \pm 6)^2 = P - k$ where P is a positive constant. Seen or implied by centre coordinates $(\mp 4, \mp 6)$ and a radius of $\sqrt{P-k}$

M1: Applying the strategy that it lies entirely within quadrant if "their radius" < 4 and proceeding to obtain an inequality in k only (See scheme). Condone $\ldots \leqslant 4$ for this mark.

A1: Deduces that $k > 36$

A1: A rigorous argument leading to a full solution. In the context of the question the circle exists so that as well as $k > 36$ $52 - k > 0 \Rightarrow 36 < k < 52$ Allow $36 < k \leqslant 52$

Q5.

Question	Scheme	Marks	AOs
(a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=...$	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	

Notes:
(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of *PN* is -3 with point $(7,4)$ to find the equation of line *PN*
So sight of $y-4=-3(x-7)$ would score this mark
If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1: Achieves $y=-3x+25$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving their $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates.

A1: $P=\left(\frac{15}{2},\frac{5}{2}\right)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=\left(\frac{15}{2},\frac{5}{2}\right)$ and $(7,4)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: Full and careful work leading to a correct equation. Eg $(x-7)^2+(y-4)^2=\frac{5}{2}$ or its expanded form. Do not accept $(x-7)^2+(y-4)^2=\left(\sqrt{\frac{5}{2}}\right)^2$

(c)

M1: Attempts to find where $y=\frac{1}{3}x+k$ meets C using a vector approach

M1: For a full method leading to k. Scored for substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$

A1: $k=\frac{10}{3}$ only

Alternative I

M1: For solving $y=\frac{1}{3}x+k$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both** b **and** c **are dependent upon** k. The terms in x^2 and x must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$ oe

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in k.
Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=\frac{10}{3}$ only

Alternative II

M1: For solving $y=-3x+25$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$, creating a 3TQ and solving.

M1: For substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ into $y=\frac{1}{3}x+k$ and finding k

A1: $k=\frac{10}{3}$ only

Q6.

Question	Scheme	Marks	AOs
(a)	Attempts $(x-2)^2+(y+5)^2=....$	M1	1.1b
	Centre $(2, -5)$	A1	1.1b
		(2)	
(b)	Sets $k+2^2+5^2>0$	M1	2.2a
	$\Rightarrow k>-29$	A1ft	1.1b
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts to complete the square so allow $(x-2)^2+(y+5)^2=....$

A1: States the centre is at $(2, -5)$. Also allow written separately $x=2, y=-5$

$(2, -5)$ implies both marks

(b)

M1: Deduces that the right hand side of their $(x\pm...)^2+(y\pm...)^2=...$ is >0 or $\geqslant 0$

A1ft: $k>-29$ Also allow $k\geqslant -29$ Follow through on their rhs of $(x\pm...)^2+(y\pm...)^2=...$

Q7.

Question	Scheme	Marks	AOs
(a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)

(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=$...	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1*: Completes proof with no errors or omissions $2y+x=17$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving $2y+x=17$ and $y=2x+1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17-x=2x+1$ as they have set $2y=y$ but condone bracketing errors, eg $2\times 2x+1+x=17$

A1: $P=(3,7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=(3,7)$ and $(7,5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2+(y-5)^2=20$. Do not accept $(x-7)^2+(y-5)^2=(\sqrt{20})^2$

(c)

M1: Attempts to find where $y=2x+k$ meets *C*.

Awarded for using $\overrightarrow{OA}+\overrightarrow{PA}$. $(11,3)$ or one correct coordinate of $(11,3)$ is evidence of this award.

M1: For a full method leading to k. Scored for either substituting their $(11,3)$ in $y=2x+k$

or, **in the alternative**, for solving their $(4k-34)^2-4\times 5\times(k^2-10k+54)=0 \Rightarrow k=...$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

Alternative I

M1: For solving $y=2x+k$ with their $(x-7)^2+(y-5)^2=20$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $5x^2+(4k-34)x+k^2-10k+54=0$

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.

$(4k-34)^2-4\times 5\times(k^2-10k+54)=0 \Rightarrow k=...$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

Alternative II

M1: For solving $2y+x=17$ with their $(x-7)^2+(y-5)^2=20$, creating a 3TQ and solving.

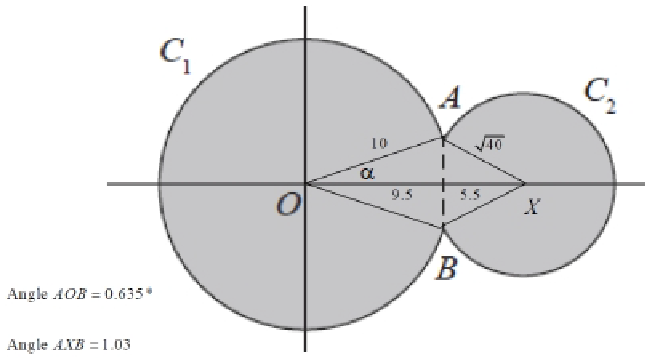
M1: For substituting their $(11,3)$ into $y=2x+k$ and finding *k*

A1: $k=-19$ only

Other method are possible using trigonometry.

Q8.

Question	Scheme	Marks	AOs
(a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos\alpha=\dfrac{"9.5"}{10}$ to find α then $\times 2$	M1	3.1a
	Angle $AOB = 2\times\arccos\left(\dfrac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ = 56.48	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos\beta=\dfrac{15-"9.5"}{\sqrt{40}} \Rightarrow \beta=...$ (Note AXB =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	= 89.7	A1	1.1b
		(4)	
			(8 marks)
Notes:			



(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet.
Look for an attempt to set up an equation in a single variable leading to a value for x or y.

A1: $x=9.5$ (or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle *AOB* in circle C_1

E.g. Attempts $\cos\alpha = \frac{"9.5"}{10}$ to find α then $\times 2$

Alternatives include $\tan\alpha = \frac{\sqrt{100-"9.5"^2}}{"9.5"} = (0.3286...)$ to find α then $\times 2$

And $\cos AOB = \frac{10^2+10^2-\left(\sqrt{39}\right)^2}{2\times10\times10} = \frac{161}{200}$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy.
Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin\alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt $36.4°$ (without the need to shown any calculation.)
E

(b)

M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$

The formula may be embedded. You may see $\underline{\underline{2\pi10}}+2\pi\sqrt{40}\underline{\underline{-10\times0.635}}$... which is fine for this M1

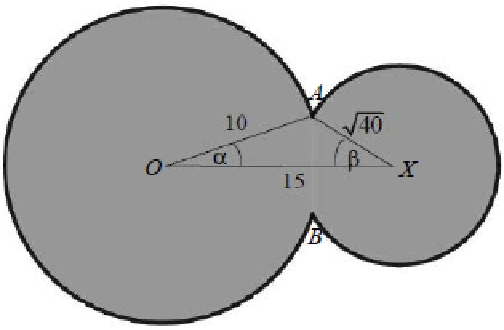
M1: Attempts to use a correct method in order to find angle *AXB* or *AXO* in circle C_2

Amongst many other methods are $\tan\beta = \frac{"3.12"}{15-9.5}$ and $\cos AXB = \frac{40+40-\left(\sqrt{39}\right)^2}{2\times\sqrt{40}\times\sqrt{40}} = \frac{41}{80}$

Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.
It is dependent upon having scored both M's.

A1: awrt 89.7



(a)

M1: For the key step in attempting to find all lengths in triangle *OAX*, condoning slips

A1: All three lengths correct

M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

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Q9.

Question	Scheme	Marks	AOs
(a)	C is $(x-r)^2+(y-r)^2=r^2$ or $x^2+y^2-2rx-2ry+r^2=0$	B1	2.2a
	$y=12-2x,\ x^2+y^2-2rx-2ry+r^2=0$ $\Rightarrow x^2+(12-2x)^2-2rx-2r(12-2x)+r^2=0$ or	M1	1.1b
	$y=12-2x,\ (x-r)^2+(y-r)^2=r^2$ $\Rightarrow (x-r)^2+(12-2x-r)^2=r^2$		
	$x^2+144-48x+4x^2-2rx-24r+4rx+r^2=0$ $\Rightarrow 5x^2+(2r-48)x+(r^2-24r+144)=0$ *	A1*	2.1
		(3)	
(b)	$b^2-4ac=0\Rightarrow(2r-48)^2-4\times5\times(r^2-24r+144)=0$	M1	3.1a
	$r^2-18r+36=0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow(r-9)^2-81+36=0\Rightarrow r=\ldots$	dM1	1.1b
	$r=9\pm3\sqrt{5}$	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)

B1: Deduces the correct equation of the circle

M1: Attempts to form an equation with terms of the form x^2, x, r^2, and xr **only** using $y=12\pm2x$ and their circle equation which must be of an appropriate form. I.e. includes or implies an x^2, y^2, r^2 such as $x^2+y^2=r^2$

If their circle equation starts off as e.g. $(x\pm a)^2+(y\pm b)^2=r^2$ then the B mark and the M mark can be awarded when the "a" and "b" are replaced by r or $-r$ as appropriate for their circle equation.

A1*: Uses correct and accurate algebra leading to the given solution.

(b)

M1: Attempts to use $b^2-4ac\ldots0$ o.e. with $a=5, b=2r-48, c=r^2-24r+144$ and where ... is "=" or any inequality

Allow minor slips when copying the a, b and c provided it does not make the work easier and allow **their** a, b and c if they are similar expressions.

FYI $(2r-48)^2-4\times5\times(r^2-24r+144)=4r^2-192r+2304-20r^2+480r-2880=-16r^2+288r-576$

A1: Correct quadratic **equation** in r (or inequality). Terms need not be all one side but must be collected.

E.g. allow $r^2-18r=-36$ and allow any multiple of this equation (or inequality).

dM1: Correct attempt to solve their 3TQ in r. Dependent upon previous M

A1: Careful and accurate work leading to both answers in the required form (must be simplified surds)

Q10.

Question	Scheme	Marks	AOs
(a)(i)	$(x-5)^2+(y+2)^2=...$	M1	1.1b
	$(5,-2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k \Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0 \Rightarrow (6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0 \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
			(9 marks)

Notes

(a)(i)

M1: Attempts to complete the square on by halving both x and y terms.

Award for sight of $(x\pm5)^2, (y\pm2)^2=...$ This mark can be implied by a centre of (±5, ±2).

A1: Correct coordinates. (Allow $x = 5$, $y = -2$)

(a)(ii)

M1: Correct strategy for the radius or radius^2. For example award for $r=\sqrt{"\pm5"^2+"\pm2"^2-11}$

or an attempt such as $(x-a)^2-a^2+(y-b)^2-b^2+11=0 \Rightarrow (x-a)^2+(y-b)^2=k \Rightarrow r^2=k$

A1: $r=3\sqrt{2}$. Do not accept for the A1 either $r=\pm3\sqrt{2}$ or $\sqrt{18}$

The A1 can be awarded following sign slips on $(5, -2)$ so following $r^2="\pm5"^2+"\pm2"^2-11$

(b) Main method seen

M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of = 0

A1: Correct 3 term quadratic equation.

The terms must be collected but this can be implied by correct a, b and c

M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both b and c are expressions in k. It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation

M1: Solves 3TQ in k. See General Principles.

The 3TQ in k must have been found as a result of attempt at $b^2 - 4ac$ ... 0

A1: Correct simplified values

Look carefully at the method used. It is possible to attempt this using gradients

(b) Alt 1	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for C $\Rightarrow 5x^2-50x+44=0$ or $5y^2+20y+11=0$ $\Rightarrow x=\dots$ or $y=\dots$	M1	3.1a
	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x \Rightarrow k=\dots$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms

A1: Correct differentiation.

M1: Sets $\frac{dy}{dx}=3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y.

M1: Uses at least one pair of coordinates and l to find at least one value for k. It is dependent upon having attempted both M's

A1: Correct simplified values

(b) Alt 2	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for l $y=3x+k,\ x+3y=1$ $\Rightarrow x=\dots$ and $y=\dots$ in terms of k	M1	3.1a
	$x=\frac{-3k-1}{10},\ y=\frac{k-3}{10},\ x^2+y^2-10x+4y+11=0 \Rightarrow k=\dots$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3

M1 A1: Correct differentiation (See alt 1)

M1: Sets $\frac{dy}{dx}=3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k

M1: Substitutes for x and y into C and solves resulting 3TQ in k

A1: Correct simplified values

(b) Alt 3	$y=3x+k \Rightarrow m=3 \Rightarrow m_r=-\frac{1}{3}$	M1
	$y+2=-\frac{1}{3}(x-5)$	A1
	$(x-5)^2+(y+2)^2=18,\ y+2=-\frac{1}{3}(x-5)$ $\Rightarrow \frac{10}{9}(x-5)^2=18 \Rightarrow x=\dots$ or $\Rightarrow 10(y+2)^2=18 \Rightarrow y=\dots$	M1
	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x \Rightarrow k=\dots$	M1
	$k=-17\pm6\sqrt{5}$	A1

M1: Applies negative reciprocal rule to obtain gradient of radius
A1: Correct equation of radial line passing through the centre of C
M1: Solves simultaneously to find x or y

Alternatively solves "$y=-\frac{1}{3}x-\frac{1}{3}$" and $y=3x+k$ to get x in terms of k which they substitute in $x^2+(3x+k)^2-10x+4(3x+k)+11=0$ to form an equation in k.

M1: Applies $k=y-3x$ with at least one pair of values to find k
A1: Correct simplified values