$y = mx + c$

The gradient m of straight-line graphs can be found by taking two random points A (x_1, y_1) and B (x_2, y_2) on the line and considering the horizontal and vertical distance between these two points.

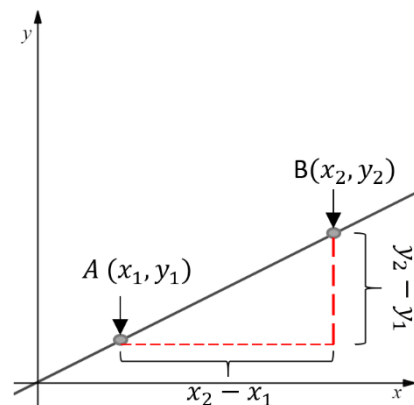The formula to calculate any given straight line's gradient is:

- $m = \frac{y_2 - y_1}{x_2 - x_1}$

Straight line equations come in the form of

- $y = mx + c$, where m is the gradient and c is the y intercept.

Or

- $ax + by + c = 0$, where a, b & c are all real numbers.

Example 1: Calculate the gradient of the following line. Point A has coordinate (4,3) and point B has coordinate (7,5). The line passes through both point A and B.

To calculate any gradient, we need know any two points which the line passes through. We are given two points, point A (4,3) and point B (7,5). The vertical distance between point A and B needs to be divided by the horizontal distance between the points.

- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{7-4} = \frac{2}{3}$
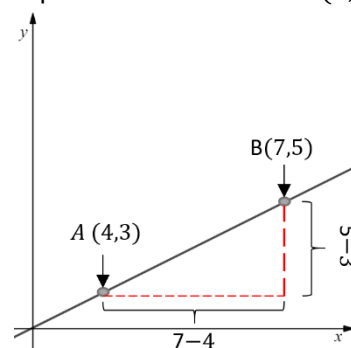
Example 2: A line l has gradient 2 and y intercept at $(0,-7)$. The line has equation $ax + by + c = 0$. Find the values of a, b and c.

Straight line equations come in the form of $y = mx + c$, where m is the gradient and c is the y intercept. In the question we are given that the gradient is 2, hence $m = 2$, and that the y intercept is at $(0,-7)$, hence $c = -7$. Substituting these values into $y = mx + c$ we get

$$y = 2x - 7$$

However, the question asks us to find the values of a, b and c on the form of $ax + by + c = 0$. Hence, we need to rearrange our equation into this form.

$$2x - y - 7 = 0$$

Therefore, the solutions to this question are $a = 2, b = -1$ and $c = -7$.

Equations of Straight Lines

Instead of relying on using $y = mx + c$ to find the equation of a line, in some cases it may be useful to find the equation of a line using the following method.

If you know one point on the line and the gradient or two distinct points on the line, we can find the equation of the line by using the formula:

- $y - y_1 = m(x - x_1)$

Example 3: Find the equation of the line that passes through the points (7,2) and (8,5).

First, we need to work out the gradient m

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{8-7} = \frac{3}{1} = 3$$

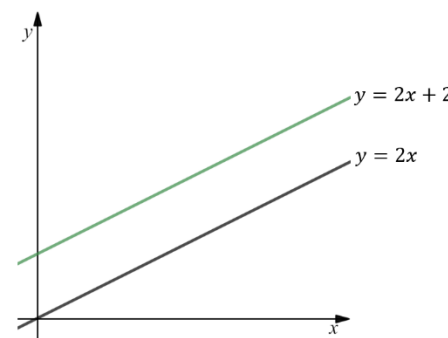We can take the point (7,2) or (8,5) to use in the formula. Let us take point (7,2):

$$y - y_1 = m(x - x_1)$$
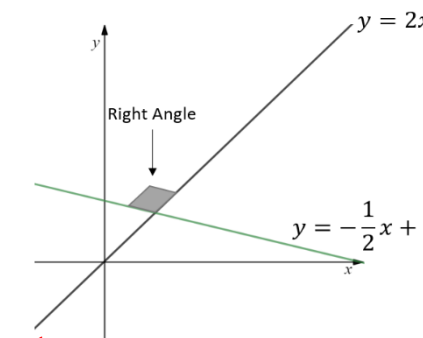$$y - 2 = 3(x - 7)$$
$$y - 2 = 3x - 21 \rightarrow y = 3x - 19$$

Parallel and Perpendicular lines

- Parallel lines have the same gradient
- Perpendicular lines are normal to each other. In other words, they make right angles when they intersect.

The lines $y = 2x + 2$ and $y = 2x$ are parallel as they have the same gradient $m = 2$.

The lines $y = -\frac{1}{2}x + 2$ and $y = 2x$ are perpendicular. If you know the gradient m_1 of one of the lines, then the gradient of the line perpendicular to it is

- $m_2 = -\frac{1}{m_1}$

Example 4: Line l_1 is perpendicular to the line $y = -\frac{2}{3}x + 4$ and passes through the point (4,6). Find the equation of line l_1.

To find the gradient of the line l_1, we need to use the rule $m_2 = -\frac{1}{m_1}$. The gradient of the line that it is perpendicular to is $m_1 = -\frac{2}{3}$

Hence, the gradient of l_1 is

$$m_2 = -\frac{1}{m_1} = -\frac{1}{-\frac{2}{3}} = \frac{3}{2}$$

The line passes through the point (4,6)

$$y - y_1 = m(x - x_1)$$
$$y - 6 = \frac{3}{2}(x - 4) \rightarrow 2y - 12 = 3x - 12$$
$$y = \frac{3}{2}x$$

Length and Area

We can calculate the distance between two points on the line by using the concept of Pythagoras theorem

To calculate the distance d between points A and B we can use the formula

- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 5: l_1 has equation $y = -\frac{1}{2}x + 2$ and l_2 has equation $y = 2x$. Lines l_1 & l_2 are perpendicular and intersect at point B. Find this intersection point and area of the triangle ABO.

The intersection point of the lines are at B

$$2x = -\frac{1}{2}x + 2 \rightarrow \frac{5}{2}x = 2 \rightarrow x = \frac{4}{5} \quad y = 2 \times \frac{4}{5} = \frac{8}{5} \quad B(\frac{4}{5}, \frac{8}{5})$$

Length AB is $AB = \sqrt{(\frac{4}{5} - 0)^2 + (\frac{8}{5} - 2)^2} = \frac{2\sqrt{5}}{5}$

Length OB is $OB = \sqrt{(\frac{4}{5} - 0)^2 + (\frac{8}{5} - 0)^2} = \frac{4\sqrt{5}}{5}$

Hence, the area of the triangle ABO is $\frac{1}{2} \times AB \times OB = \frac{1}{2} \times \frac{2\sqrt{5}}{5} \times \frac{4\sqrt{5}}{5} = \frac{4}{5}$