Coordinate Geometry Questions

2 The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x + 4y = 7$.

(a) (i) Show that $k = -2$. *(1 mark)*

(ii) Hence find the coordinates of the mid-point of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The line AC is perpendicular to the line AB.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the line AC. *(1 mark)*

(iii) Given that the point C lies on the x-axis, find its x-coordinate. *(2 marks)*

(b) The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$.

(i) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2 - 2x - 3 = 0$$ *(1 mark)*

(ii) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

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5 A circle with centre C has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) The point O has coordinates $(0, 0)$.

(i) Find the length of CO. *(2 marks)*

(ii) Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer. *(2 marks)*

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1 The point A has coordinates (1, 7) and the point B has coordinates (5, 1).

(a) (i) Find the gradient of the line AB. *(2 marks)*

(ii) Hence, or otherwise, show that the line AB has equation $3x + 2y = 17$. *(2 marks)*

(b) The line AB intersects the line with equation $x - 4y = 8$ at the point C. Find the coordinates of C. *(3 marks)*

(c) Find an equation of the line through A which is perpendicular to AB. *(3 marks)*

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7 A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

(a) Find:

(i) the coordinates of the centre of the circle; *(3 marks)*

(ii) the radius of the circle in the form $p\sqrt{2}$, where p is an integer. *(3 marks)*

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. *(3 marks)*

(c) A line has equation $y = 2k - x$, where k is a constant.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$ *(3 marks)*

(ii) Find the values of k for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (c)(ii). *(1 mark)*

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2 The line AB has equation $3x + 5y = 8$ and the point A has coordinates $(6, -2)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence find an equation of the straight line which is perpendicular to AB and which passes through A. *(3 marks)*

(b) The line AB intersects the line with equation $2x + 3y = 3$ at the point B. Find the coordinates of B. *(3 marks)*

(c) The point C has coordinates $(2, k)$ and the distance from A to C is 5. Find the **two** possible values of the constant k. *(3 marks)*

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4 A circle with centre C has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) Show that the circle does **not** intersect the x-axis. *(2 marks)*

(d) The line with equation $x + y = 4$ intersects the circle at the points P and Q.

(i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 + 3x - 10 = 0$$ *(3 marks)*

(ii) Given that P has coordinates $(2, 2)$, find the coordinates of Q. *(2 marks)*

(iii) Hence find the coordinates of the midpoint of PQ. *(2 marks)*

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1 The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively.

(a) (i) Show that the gradient of AB is $-\frac{3}{2}$. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(2 marks)*

(b) (i) Find an equation of the line which passes through B and which is perpendicular to the line AB. *(2 marks)*

(ii) The point C has coordinates $(k, 7)$ and angle ABC is a right angle.

Find the value of the constant k. *(2 marks)*

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5 A circle with centre C has equation $(x+3)^2 + (y-2)^2 = 25$.

(a) Write down:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

(iii) Find an equation of the normal to the circle at the point N. *(3 marks)*

(c) The point P has coordinates (2, 6).

(i) Find the distance PC, leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from P to the circle. *(3 marks)*

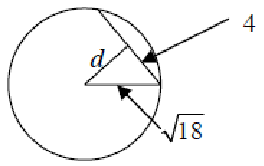
Coordinate Geometry Answers

2(a)(i)	$15+4k=7 \quad \Rightarrow 4k=-8 \quad \Rightarrow k=-2$	B1	1	**AG** (condone verification or $y=-2$)
(ii)	$\frac{1}{2}(x_1+x_2)$ or $\frac{1}{2}(y_1+y_2)$	M1		
	Midpoint coordinates $\left(3,-\frac{1}{2}\right)$	A1	2	One coordinate correct implies M1
(b)	Attempt at $\Delta y/\Delta x$ or $y=-\frac{3}{4}x+\frac{7}{4}$	M1		(Not x over y)(may use M instead of A/B)
	Gradient $AB=-\frac{3}{4}$	A1	2	-0.75 etc any correct equivalent
(c)(i)	$m_1m_2=-1$ used or stated	1		
	Hence gradient $AC=\frac{4}{3}$	A1√	2	Follow through their gradient of AB from part (b)
(ii)	$y-1=\frac{4}{3}(x-1)$ or $3y=4x-1$ etc	B1√	1	Follow through their gradient of AC from part (c) (i) must be **normal** & (1,1) used
(iii)	$y=0 \qquad \Rightarrow x-1=-\frac{3}{4}$	M1		Putting $y=0$ in their AC equation and attempting to find x
	$x=\frac{1}{4}$	A1	2	**CSO.** C has coordinates $\left(\frac{1}{4},0\right)$
	Total		**10**	

(b)(i)	$12-2x=x^2-4x+9$ $\Rightarrow x^2-2x-3=0$	B1	1	Or $x^2-4x+9+2x=12$ **AG** (be convinced) (must have = 0)
(ii)	$(x-3)(x+1)=0$	M1		Attempt at factors or quadratic formula or one value spotted
	$x=3, \quad -1$	A1		Both values correct & simplified
	Substitute one value of x to find y	M1		May substitute into equation for L or C
	Points are (3, 6) and (−1, 14)	A1	4	y-coordinates correct linked to x values

5(a)	$(x-4)^2+(y+3)^2$	B2		B1 for one term correct
	$(11+16+9=36)$ RHS $=6^2$	B1	3	Condone 36
(b)(i)	Centre $(4, -3)$	B1√	1	Ft their a and b from part (a)
(ii)	Radius = 6	B1√	1	Ft their r from part (a)
(c)(i)	$CO^2=(-4)^2+3^2$	M1		Accept + or – with numbers but must add
	$CO=5$	A1√	2	Full marks for answer only
(ii)	Considering CO and radius	M1		
	$CO < r \Rightarrow O$ is **inside** the circle	A1√	2	Ft outside circle when 'their CO' > r or on the circle when 'their CO' = r SC B1√ *if no explanation given*
	Total		**9**	

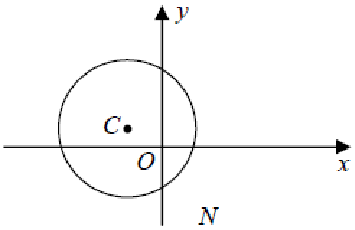
1(a)(i)	Gradient $AB=\frac{1-7}{5-1}$	M1		Must be y on top and subtr'n of cords
	$=-\frac{6}{4}=-\frac{3}{2}=-1.5$	A1	2	Any correct equivalent
(ii)	$y-7=m(x-1)$ or $y-1=m(x-5)$	M1		Verifying 2 points or $y=-\frac{3}{2}x+c$
	leading to $3x+2y=17$	A1	2	AG (or grad & 1 point verified)
(b)	Attempt to eliminate x or y : $7x=42$ etc	M1		Solving $x-4y=8$; $3x+2y=17$
	$x=6$	A1		
	$y=-\frac{1}{2}$	A1	3	C is point $(6, -\frac{1}{2})$
(c)	Grad of perp = – 1 / *their* gradient AB	M1		Or $m_1m_2=-1$ used or stated
	$=\frac{2}{3}$	A1√		ft their gradient AB
	$y-7=\frac{2}{3}(x-1)$ or $3y-2x=19$	A1	3	CSO Any correct form of equation
	Total		**10**	

7(a)(i)	$(x-2)^2$ centre has x-coordinate = 2 and y-coordinate = 0	M1 A1 B1	3	Attempt to complete square for x M1 implied if value correct or –2 Centre (2,0)
(ii)	RHS = 18 Radius = $\sqrt{18}$ Radius = $3\sqrt{2}$	B1 M1 A1	3	Withhold if circle equation RHS incorrect Square root of RHS of equation (if > 0)
(b)	Perpendicular bisects chord so need to use Length of 4 d^2 = (radius)$^2 - 4^2$ $d^2 = 18 - 16$ so perpendicular distance = $\sqrt{2}$	 B1 M1 A1	3	
(c)(i)	$x^2 + (2k-x)^2 - 4x - 14 = 0$ $(2k-x)^2 = 4k^2 - 4kx + x^2$ $\Rightarrow 2x^2 + 4k^2 - 4kx - 4x - 14 = 0$ $(\Rightarrow x^2 + 2k^2 - 2kx - 2x - 7 = 0)$ $\Rightarrow x^2 - 2(k+1)x + 2k^2 - 7 = 0$	M1 B1 A1	3	AG (be convinced about algebra and = 0)
(ii)	$4(k+1)^2 - 4(2k^2 - 7)$ $4k^2 - 8k - 32 = 0$ or $k^2 - 2k - 8 = 0$	M1 A1		"$b^2 - 4ac$" in terms of k (either term correct) $b^2 - 4ac = 0$ correct quadratic equation in k
	$(k-4)(k+2) = 0$ $k = -2$, $k = 4$	m1 A1	4	Attempt to factorise, solve equation SC B1, B1 for –2 , 4 (if M0 scored)
(iii)	Line is a tangent to the circle	E1	1	Line touches circle at one point etc
	Total		**17**	

2(a)(i)	$y = -\frac{3}{5}x + \ldots$; Gradient $AB = -\frac{3}{5}$	M1 A1	2	**Attempt** to find $y =$ or $\Delta y / \Delta x$ or $\frac{3}{5}$ or $3x/5$ Gradient correct – condone slip in $y = \ldots$
(ii)	$m_1 m_2 = -1$ Gradient of perpendicular = $\frac{5}{3}$ $\Rightarrow y + 2 = \frac{5}{3}(x-6)$	M1 A1√ A1	3	Stated or used correctly **ft** gradient of AB **CSO** Any correct form eg $y = \frac{5}{3}x - 12$, $5x - 3y = 36$ etc
(b)	Eliminating x or y (unsimplified) $x = -9$ $y = 7$	M1 A1 A1	3	Must use $3x + 5y = 8$; $2x + 3y = 3$ B (–9,7)
(c)	$4^2 + (k+2)^2$ (= 25) or $16 + d^2 = 25$ $k = 1$ or $k = -5$	M1 A1 A1	3	Diagram with 3,4, 5 triangle Condone slip in one term (or $k + 2 = 3$) **SC1** with no working for spotting one correct value of k. Full marks if both values spotted with no contradictory work
	Total		**11**	

4(a)	$(x+1)^2+(y-6)^2$	B2		B1 for one term correct or missing + sign
	$(1+36-12=25)$ RHS $=5^2$	B1	3	Condone 25
(b)(i)	Centre $(-1, 6)$	B1√	1	**FT** their a and b from part (a) or correct
(ii)	Radius = 5	B1√	1	**FT** their r from part (a) RHS must be > 0
(c)	Attempt to solve "their" $x^2+2x+12=0$	M1		Or comparing "their" $y_c=6$ and their $r=5$ may use a diagram with values shown
	(all working correct) so no real roots or statement that does not intersect	A1	2	$r<y_c$ so does not intersect condone ± 1or ± 6in centre for A1
(d)(i)	$(4-x)^2=16-8x+x^2$	B1		Or $(-2-x)^2=4+4x+x^2$
	$x^2+(4-x)^2+2x-12(4-x)+12=0$ or $(x+1)^2+(-2-x)^2=25$	M1		Sub $y=4-x$ in circle eqn (condone slip) or "their" circle equation
	$\Rightarrow 2x^2+6x-20=0 \quad \Rightarrow x^2+3x-10=0$	A1	3	**AG CSO** (must have = 0)
(ii)	$(x+5)(x-2)=0 \quad \Rightarrow x=-5, x=2$	M1		Correct factors or unsimplified solution to quadratic
	Q has coordinates $(-5, 9)$	A1	2	(give credit if factorised in part (i)) SC2 if Q correct. Allow x = –5 $y=9$
(iii)	Mid point of 'their' $(-5, 9)$ and $(2,2)$	M1		Arithmetic mean of **either** x or y coords
	$\left(-1\frac{1}{2}, 5\frac{1}{2}\right)$	A1	2	Must follow from correct value in (ii)
	Total		**14**	

1(a)(i)	Gradient $AB=\frac{-1-5}{6-2}$ or $\frac{5--1}{2-6}$	M1		$\pm\frac{6}{4}$ implies M1
	$=\frac{-6}{4}=-\frac{3}{2}$	A1	2	AG
(ii)	$\left.\begin{matrix}y-5\\y+1\end{matrix}\right\}=-\frac{3}{2}\left\{\begin{matrix}(x-2)\\(x-6)\end{matrix}\right.$	M1		or $y=-\frac{3}{2}x+c$ and attempt to find c
	$\Rightarrow 3x+2y=16$	A1	2	OE; must have integer coefficients
(b)(i)	Gradient of perpendicular $=\frac{2}{3}$	M1		or use of $m_1m_2=-1$
	$\Rightarrow y-5=\frac{2}{3}(x-2)$	A1	2	$3y-2x=11$ (no misreads permitted)
(ii)	Substitute $x=k,\ y=7$ into their (b)(i)	M1		or grads $\frac{7-5}{k-2}\times\frac{-3}{2}=-1$
	$\Rightarrow 2=\frac{2}{3}(k-2) \Rightarrow k=5$	A1	2	or Pythagoras $(k-2)^2=(k-6)^2+8$
	Total		**8**	

5(a)(i)	Centre $(-3, 2)$	M1 A1	2	± 3 or ± 2 correct
(ii)	Radius = 5	B1	1	accept $\sqrt{25}$ but not $\pm\sqrt{25}$
(b)(i)	$3^2 + (-4)^2 = 9 + 16 = 25$ $\Rightarrow N$ lies on circle	B1	1	must have $9 + 16 = 25$ or a statement
(ii)		M1 A1	2	must draw axes; ft their centre in correct quadrant correct (reasonable freehand circle enclosing origin)
(iii)	Attempt at gradient of CN grad $CN = -\frac{4}{3}$ $y = -\frac{4}{3}x - 2$ (or equivalent)	M1 A1 A1√	3	withhold if subsequently finds tangent CSO ft their grad CN
(c)(i)	$P(2, 6)$ Hence $PC^2 = 5^2 + 4^2$ $\Rightarrow PC = \sqrt{41}$	M1 A1	2	"their" PC^2
(ii)	Use of Pythagoras correctly $PT^2 = PC^2 - r^2 = 41 - 25$, where T is a point of contact of tangent $\Rightarrow PT = 4$	M1 A1√ A1	3	ft their PC^2 and r^2 **Alternative** sketch with vertical tangent M1 showing that tangent touches circle at point (2, 2) A1 hence $PT = 4$ A1
	Total		**14**	