Rational Expressions

1 Express $\dfrac{6}{x^2-9} - \dfrac{7}{2x^2-5x-3}$ as a single fraction in its simplest form. **(6)**

2 $$\mathrm{f}(x) \equiv \frac{3}{2x+3} - \frac{x+9}{2x^2+11x+12}, \quad x > 0.$$

Show that $\mathrm{f}(x) = \dfrac{1}{x+4}$. **(5)**

3 **a** Express $\dfrac{1}{x-6} - \dfrac{2}{x^2-36}$ as a single fraction in its simplest form. **(3)**

b Hence solve the equation

$$\frac{1}{x-6} - \frac{2}{x^2-36} = \frac{1}{2},$$

giving your answers in the form $a + b\sqrt{5}$, where $a, b \in \mathbb{Z}$. **(4)**

4 $$\mathrm{f}(x) \equiv 2x^3 - 5x^2 - 23x - 10.$$

a Show that $(x - 5)$ is a factor of $\mathrm{f}(x)$. **(2)**

b Express $\dfrac{\mathrm{f}(x)}{2x^2-9x-5}$ in its simplest form. **(5)**

5 Given that the equation

$$\frac{x+6}{x^2+9x+18} + \frac{x-p}{x+7} = 0$$

has real, equal roots, find the possible values of the constant p. **(7)**

6 Express $\dfrac{1}{3x-1} - \dfrac{3x}{9x^2-6x+1} - \dfrac{1}{3x^2-x}$ as a single fraction in its simplest form. **(5)**

7 **a** Simplify

i $\dfrac{7x+14}{4-x^2}$,

ii $\dfrac{2x^2+x-28}{3x^2+12x}$. **(4)**

b Hence show that the equation $\dfrac{7x+14}{4-x^2} = \dfrac{2x^2+x-28}{3x^2+12x}$ has no real roots. **(4)**

8 The first three terms of an arithmetic series are $\dfrac{1}{t-2}$, $\dfrac{1}{2}$ and $\dfrac{4}{t^2-2t}$ respectively.

a Show that $\dfrac{4}{t^2-2t} + \dfrac{1}{t-2} = 1$. **(2)**

b Given that the common difference of the series is not zero, find the value of t and the first term of the series. **(5)**