## PARTIAL FRACTIONS — *Answers*

**1** $22 \equiv A(x + 4) + B(2x - 3)$

$x = \frac{3}{2} \quad \Rightarrow \quad 22 = \frac{11}{2}A \quad \Rightarrow \quad A = 4$

$x = -4 \quad \Rightarrow \quad 22 = -11B \quad \Rightarrow \quad B = -2$

**2** $x + 5 \equiv A(x - 3)^2 + B(x + 1)(x - 3) + C(x + 1)$

$x = -1 \quad \Rightarrow \quad 4 = 16A \quad \Rightarrow \quad A = \frac{1}{4}$

$x = 3 \quad \Rightarrow \quad 8 = 4C \quad \Rightarrow \quad C = 2$

coeffs of $x^2 \quad \Rightarrow \quad 0 = A + B \quad \Rightarrow \quad B = -\frac{1}{4}$

**3** $4x^2 - 16x - 7 \equiv A(2x - 1)(x - 4) + B(x - 4) + C(2x - 1)$

$x = 4 \quad \Rightarrow \quad -7 = 7C \quad \Rightarrow \quad C = -1$

$x = \frac{1}{2} \quad \Rightarrow \quad -14 = -\frac{7}{2}B \quad \Rightarrow \quad B = 4$

coeffs of $x^2 \quad \Rightarrow \quad 4 = 2A \quad \Rightarrow \quad A = 2$

**4** **a** $f(1) = 3 + 11 + 8 - 4 = 18$

$f(-1) = -3 + 11 - 8 - 4 = -4$

$f(2) = 24 + 44 + 16 - 4 = 80$

$f(-2) = -24 + 44 - 16 - 4 = 0$

$\therefore$ $(x + 2)$ is a factor

$$\begin{array}{r} 3x^2 + 5x - 2 \\ x+2 \overline{)\,3x^3 + 11x^2 + 8x - 4} \\ \underline{3x^3 + 6x^2} \\ 5x^2 + 8x \\ \underline{5x^2 + 10x} \\ -2x - 4 \\ \underline{-2x - 4} \end{array}$$

$\therefore$ $f(x) = (x + 2)(3x^2 + 5x - 2)$

$= (3x - 1)(x + 2)^2$

**b** $\frac{x+16}{f(x)} \equiv \frac{A}{3x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

$x + 16 \equiv A(x + 2)^2 + B(3x - 1)(x + 2) + C(3x - 1)$

$x = \frac{1}{3} \quad \Rightarrow \quad \frac{49}{3} = \frac{49}{9}A \quad \Rightarrow \quad A = 3$

$x = -2 \quad \Rightarrow \quad 14 = -7C \quad \Rightarrow \quad C = -2$

coeffs of $x^2 \quad \Rightarrow \quad 0 = A + 3B \quad \Rightarrow \quad B = -1$

$\therefore \quad \frac{x+16}{f(x)} \equiv \frac{3}{3x-1} - \frac{1}{x+2} - \frac{2}{(x+2)^2}$

**5** $\frac{1}{x(2x-1)^2} \equiv \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$

$1 \equiv A(2x - 1)^2 + Bx(2x - 1) + Cx$

$x = 0 \quad \Rightarrow \quad A = 1$

$x = \frac{1}{2} \quad \Rightarrow \quad 1 = \frac{1}{2}C \quad \Rightarrow \quad C = 2$

coeffs of $x^2 \quad \Rightarrow \quad 0 = 4A + 2B \Rightarrow \quad B = -2$

$\therefore \quad f(x) = \frac{1}{x} - \frac{2}{2x-1} + \frac{2}{(2x-1)^2}$

**6**

$$x^2 + 7x + 10 \overline{)\; x^3 + 5x^2 - 2x - 19}$$

$$\begin{array}{r} x - 2 \\ \hline x^3 + 7x^2 + 10x \\ \hline -2x^2 - 12x - 19 \\ -2x^2 - 14x - 20 \\ \hline 2x + 1 \end{array}$$

$\therefore\ f(x) \equiv x - 2 + \dfrac{2x+1}{x^2+7x+10}$

$\dfrac{2x+1}{(x+2)(x+5)} \equiv \dfrac{B}{x+2} + \dfrac{C}{x+5}$

$2x + 1 \equiv B(x + 5) + C(x + 2)$

$x = -2 \quad \Rightarrow \quad -3 = 3B \quad \Rightarrow \quad B = -1$

$x = -5 \quad \Rightarrow \quad -9 = -3C \quad \Rightarrow \quad C = 3$

$\therefore\ f(x) \equiv x - 2 - \dfrac{1}{x+2} + \dfrac{3}{x+5}$ [ $A = -2$, $B = -1$, $C = 3$ ]

**7** **a** $\dfrac{4}{(x+1)(x-1)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x-1}$

$4 \equiv A(x - 1) + B(x + 1)$

$x = -1 \quad \Rightarrow \quad 4 = -2A \quad \Rightarrow \quad A = -2$

$x = 1 \quad \Rightarrow \quad 4 = 2B \quad \Rightarrow \quad B = 2$

$\therefore\ f(x) \equiv \dfrac{2}{x-1} - \dfrac{2}{x+1}$

**b** $\dfrac{2+5x-x^2}{(x-4)(x-2)(x-1)} \equiv \dfrac{A}{x-4} + \dfrac{B}{x-2} + \dfrac{C}{x-1}$

$2 + 5x - x^2 \equiv A(x - 2)(x - 1) + B(x - 4)(x - 1) + C(x - 4)(x - 2)$

$x = 4 \quad \Rightarrow \quad 6 = 6A \quad \Rightarrow \quad A = 1$

$x = 2 \quad \Rightarrow \quad 8 = -2B \quad \Rightarrow \quad B = -4$

$x = 1 \quad \Rightarrow \quad 6 = 3C \quad \Rightarrow \quad C = 2$

$\therefore\ g(x) \equiv \dfrac{1}{x-4} - \dfrac{4}{x-2} + \dfrac{2}{x-1}$

**c** $\dfrac{2}{x-1} - \dfrac{2}{x+1} = \dfrac{1}{x-4} - \dfrac{4}{x-2} + \dfrac{2}{x-1}$

$\dfrac{4}{x-2} - \dfrac{1}{x-4} - \dfrac{2}{x+1} = 0$

$\dfrac{4(x-4)(x+1) - (x-2)(x+1) - 2(x-2)(x-4)}{(x-2)(x-4)(x+1)} = 0$

$4(x^2 - 3x - 4) - (x^2 - x - 2) - 2(x^2 - 6x + 8) = 0$

$x^2 + x - 30 = 0$

$(x + 6)(x - 5) = 0$

$x = -6, 5$