FUNCTIONS — *Answers*

1 **a** $3 + \ln(x + 2) \geq 3$

$\ln(x + 2) \geq 0$

$x + 2 \geq 1$

$x \geq -1$

$\therefore\ k = -1$

b $y = 3 + \ln(x + 2)$

swap $x = 3 + \ln(y + 2)$

$y + 2 = e^{x-3}$

$y = e^{x-3} - 2$

$f^{-1}(x) = e^{x-3} - 2,\ x \in \mathbb{R},\ x \geq 3$

c $3 + \ln(x + 2) = 4 + \ln(x - 1)$

$\ln(x + 2) - \ln(x - 1) = 1$

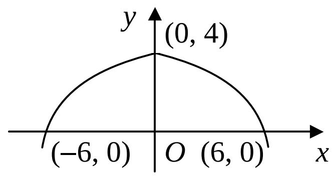
$\frac{x+2}{x-1} = e$

$x + 2 = e(x - 1)$

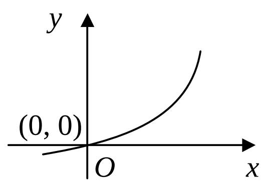
$x(e - 1) = e + 2$

$x = \frac{e+2}{e-1}$

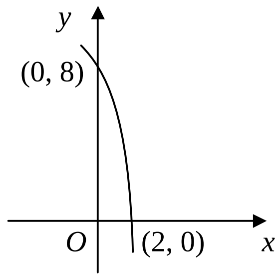
2 **a**



b



c



3 **a** $fg(x) = f(\frac{3}{x}) = \frac{\frac{3}{x}}{\frac{3}{x} + 2} = \frac{3}{3 + 2x}$

$\therefore\ \frac{3}{3+2x} = 4$

$3 + 2x = \frac{3}{4}$

$x = -\frac{9}{8}$

b $y = \frac{x}{x+2}$

swap $x = \frac{y}{y+2}$

$x(y + 2) = y$

$2x = y(1 - x)$

$y = \frac{2x}{1-x}$

$f^{-1}(x) = \frac{2x}{1-x},\ x \in \mathbb{R},\ x \neq 1$

c $\frac{x}{x+2} = \frac{2x}{1-x}$

$x(1 - x) = 2x(x + 2)$

$3x^2 + 3x = 0$

$3x(x + 1) = 0$

$x = -1, 0$

4 **a** $f(x) = (x - 1)^2 - 1 - 9 = (x - 1)^2 - 10$

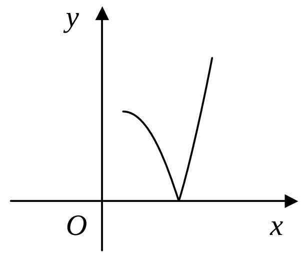
for f^{-1} to exist, f must be one-one

$\therefore$ min value of $k = 1$

b $f^{-1}(x) = 4 \Rightarrow x = f(4)$

$\therefore\ x = 16 - 8 - 9 = -1$

c



d $x^2 - 2x - 9 = 6$

$x^2 - 2x - 15 = 0$

$(x + 3)(x - 5) = 0$

$x \geq 1\ \therefore\ x = 5$

$-(x^2 - 2x - 9) = 6$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x \geq 1\ \therefore\ x = 3$

$\therefore\ x = 3, 5$

5 **a** $f(1) = 2 - 3 = -1$
$ff(1) = f(-1) = 2 + 3 = 5$

b $y = 2 - \frac{3}{x}$ swap $x = 2 - \frac{3}{y}$

$\frac{3}{y} = 2 - x$

$y = \frac{3}{2-x}$

$f^{-1}(x) = \frac{3}{2-x},\ x \in \mathbb{R},\ x \neq 2$

c $gf(x) = 1 \Rightarrow (2 - \frac{3}{x})^2 = 1$

$2 - \frac{3}{x} = \pm 1$

$\frac{3}{x} = 1, 3$

$x = 1, 3$

6 **a** $f(\ln 9) = f(2 \ln 3) = e^{\ln 3} - 2 = 3 - 2 = 1$

b $f(x) > -2$

c $y = e^{\frac{1}{2}x} - 2$ swap $x = e^{\frac{1}{2}y} - 2$

$\frac{1}{2}y = \ln(x + 2)$

$y = 2\ln(x + 2)$

$f^{-1}(x) = 2\ln(x + 2),\ x \in \mathbb{R},\ x > -2$

d $gf(x) = (e^{\frac{1}{2}x} - 2)^2 + 4(e^{\frac{1}{2}x} - 2)$

$= e^x - 4e^{\frac{1}{2}x} + 4 + 4e^{\frac{1}{2}x} - 8$

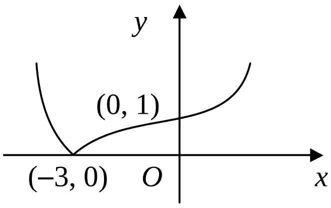
$gf(x) = e^x - 4$

e $e^x - 4 + 1 = 0$
$e^x = 3$
$x = \ln 3$

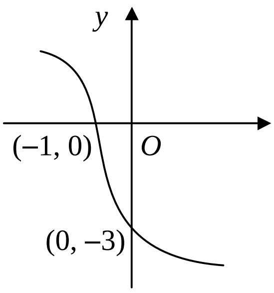
7 **a** each value of f(x) corresponds

to a unique value of x

b **i**



ii



8 **a** $f(x) = \frac{5 + (2x-3)}{(x+1)(2x-3)} = \frac{2x+2}{(x+1)(2x-3)}$

$= \frac{2(x+1)}{(x+1)(2x-3)} = \frac{2}{2x-3}$

b $f(2) = 2$
$\therefore$ range: $0 < f(x) \leq 2$

c $y = \frac{2}{2x-3}$ swap $x = \frac{2}{2y-3}$

$2y - 3 = \frac{2}{x}$

$f^{-1}(x) = \frac{1}{x} + \frac{3}{2},\ x \in \mathbb{R},\ 0 < x \leq 2$

d $fg(x) = \frac{2}{2(\frac{1}{x-2}) - 3} = \frac{2(x-2)}{2 - 3(x-2)} = \frac{2x-4}{8-3x}$

$\therefore \frac{2x-4}{8-3x} = \frac{2}{3}$

$6x - 12 = 16 - 6x$

$x = \frac{7}{3}$