

# FUNCTIONS

- 1 The function  $f$  is defined by

$$f : x \rightarrow 3 + \ln(x + 2), \quad x \in \mathbb{R}, \quad x \geq k,$$

where  $k$  is a constant.

Given that the range of  $f$  is  $f(x) \geq 3$ ,

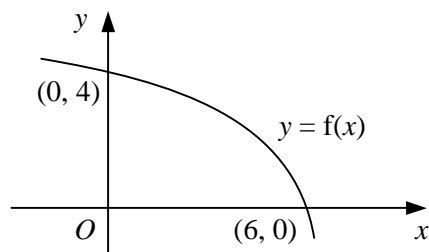
- a** find the value of  $k$ , (3)  
**b** find  $f^{-1}(x)$ , stating its domain clearly. (4)

The function  $g$  is defined by

$$g : x \rightarrow 4 + \ln(x - 1), \quad x \in \mathbb{R}, \quad x > 1.$$

- c** Find, in terms of  $e$ , the value of  $x$  such that  $f(x) = g(x)$ . (4)

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The diagram shows the curve with equation  $y = f(x)$  which crosses the coordinate axes at the points  $(0, 4)$  and  $(6, 0)$ .

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the curves

- a**  $y = f(|x|)$ , (2)  
**b**  $y = 4 - f(x)$ , (2)  
**c**  $y = 2f(3x)$ . (3)
- 3 The functions  $f$  and  $g$  are given by

$$f(x) \equiv \frac{x}{x+2}, \quad x \in \mathbb{R}, \quad x \neq -2,$$

$$g(x) \equiv \frac{3}{x}, \quad x \in \mathbb{R}, \quad x \neq 0$$

- a** Solve the equation  $fg(x) = 4$ . (4)  
**b** Find  $f^{-1}(x)$ , stating its domain clearly. (4)  
**c** Solve the equation  $f(x) = f^{-1}(x)$ . (3)

- 4 The function  $f$  is defined by

$$f(x) \equiv x^2 - 2x - 9, \quad x \in \mathbb{R}, \quad x \geq k.$$

- a** Find the minimum value of the constant  $k$  for which  $f^{-1}(x)$  exists. (3)  
 Given that  $k$  takes the value found in part **a**,
- b** solve the equation  $f^{-1}(x) = 4$ , (2)  
**c** sketch the curve  $y = |f(x)|$ , (3)  
**d** find the values of  $x$  for which  $|f(x)| = 6$ . (5)

## FUNCTIONS

continued

- 5 The function  $f$  is defined by

$$f : x \rightarrow 2 - \frac{3}{x}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

- a Find the value of  $ff(1)$ . (2)  
 b Find  $f^{-1}(x)$  and state its domain. (4)

The function  $g$  is defined by

$$g : x \rightarrow x^2, \quad x \in \mathbb{R}.$$

- c Solve the equation  $gf(x) = 1$ . (4)

- 6 The function  $f$  is defined by

$$f : x \rightarrow e^{\frac{1}{2}x} - 2, \quad x \in \mathbb{R}.$$

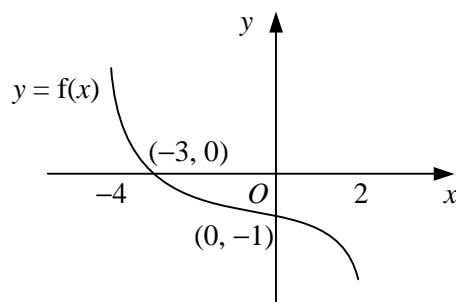
- a Evaluate  $f(\ln 9)$ . (2)  
 b State the range of  $f$ . (1)  
 c Find  $f^{-1}(x)$  and state its domain. (4)

The function  $g$  is defined by

$$g : x \rightarrow x^2 + 4x, \quad x \in \mathbb{R}.$$

- d Find and simplify an expression for  $gf(x)$ . (3)  
 e Solve the equation  $gf(x) + 1 = 0$ . (2)

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The diagram shows the curve  $y = f(x)$ . The domain of  $f$  is  $-4 \leq x \leq 2$  and the curve intersects the coordinate axes at the points  $(-3, 0)$  and  $(0, -1)$ .

- a Explain how the graph shows that  $f$  is one-one. (1)  
 b Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of  
 i  $y = |f(x)|$ ,  
 ii  $y = f^{-1}(x)$ . (5)

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$$f(x) \equiv \frac{5}{(x+1)(2x-3)} + \frac{1}{x+1}, \quad x \in \mathbb{R}, \quad x \geq 2.$$

- a Show that  $f(x) = \frac{2}{2x-3}$ . (4)  
 b Find the range of  $f$ . (2)  
 c Find an expression for  $f^{-1}(x)$ . (3)

$$g(x) \equiv \frac{1}{x-2}, \quad x \in \mathbb{R}, \quad x \neq 2.$$

- d Solve the equation  $fg(x) = \frac{2}{3}$ . (4)