

PARTIAL FRACTIONS

Answers

- 1** **a** $x = -4 \Rightarrow -12 = -6A \Rightarrow A = 2$
 $x = 2 \Rightarrow -6 = 6B \Rightarrow B = -1$
- 2** **a** $2 \equiv A(x+3) + B(x+1)$
 $x = -1 \Rightarrow 2 = 2A \Rightarrow A = 1$
 $x = -3 \Rightarrow 2 = -2B \Rightarrow B = -1$
- c** $x + 1 \equiv A(x-5) + B(x-3)$
 $x = 3 \Rightarrow 4 = -2A \Rightarrow A = -2$
 $x = 5 \Rightarrow 6 = 2B \Rightarrow B = 3$
- e** $\frac{4x-1}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1}$
 $4x - 1 \equiv A(x-1) + B(x+2)$
 $x = -2 \Rightarrow -9 = -3A \Rightarrow A = 3$
 $x = 1 \Rightarrow 3 = 3B \Rightarrow B = 1$
- 3** **a** $\frac{8}{(x-1)(x+3)} \equiv \frac{A}{x-1} + \frac{B}{x+3}$
 $8 \equiv A(x+3) + B(x-1)$
 $x = 1 \Rightarrow 8 = 4A \Rightarrow A = 2$
 $x = -3 \Rightarrow 8 = -4B \Rightarrow B = -2$
 $\therefore \frac{8}{(x-1)(x+3)} \equiv \frac{2}{x-1} - \frac{2}{x+3}$
- c** $\frac{10x}{(x+4)(x-1)} \equiv \frac{A}{x+4} + \frac{B}{x-1}$
 $10x \equiv A(x-1) + B(x+4)$
 $x = -4 \Rightarrow -40 = -5A \Rightarrow A = 8$
 $x = 1 \Rightarrow 10 = 5B \Rightarrow B = 2$
 $\therefore \frac{10x}{(x+4)(x-1)} \equiv \frac{8}{x+4} + \frac{2}{x-1}$
- e** $\frac{x+2}{(x-1)(x-4)} \equiv \frac{A}{x-1} + \frac{B}{x-4}$
 $x + 2 \equiv A(x-4) + B(x-1)$
 $x = 1 \Rightarrow 3 = -3A \Rightarrow A = -1$
 $x = 4 \Rightarrow 6 = 3B \Rightarrow B = 2$
 $\therefore \frac{x+2}{x^2-5x+4} \equiv \frac{2}{x-4} - \frac{1}{x-1}$
- g** $\frac{3x+2}{(x-6)(x+4)} \equiv \frac{A}{x-6} + \frac{B}{x+4}$
 $3x + 2 \equiv A(x+4) + B(x-6)$
 $x = 6 \Rightarrow 20 = 10A \Rightarrow A = 2$
 $x = -4 \Rightarrow -10 = -10B \Rightarrow B = 1$
 $\therefore \frac{3x+2}{x^2-2x-24} \equiv \frac{2}{x-6} + \frac{1}{x+4}$
- b** $x = -2 \Rightarrow -5 = -5A \Rightarrow A = 1$
 $x = \frac{1}{2} \Rightarrow 10 = \frac{5}{2}B \Rightarrow B = 4$
- b** $x - 3 \equiv A(x-1) + Bx$
 $x = 0 \Rightarrow -3 = -A \Rightarrow A = 3$
 $x = 1 \Rightarrow = \Rightarrow B = -2$
- d** $x + 10 \equiv A(2-x) + B(1+x)$
 $x = -1 \Rightarrow 9 = 3A \Rightarrow A = 3$
 $x = 2 \Rightarrow 12 = 3B \Rightarrow B = 4$
- f** $\frac{x-9}{(x-1)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x-3}$
 $x - 9 \equiv A(x-3) + B(x-1)$
 $x = 1 \Rightarrow -8 = -2A \Rightarrow A = 4$
 $x = 3 \Rightarrow -6 = 2B \Rightarrow B = -3$
- b** $\frac{x-1}{(x+2)(x+3)} \equiv \frac{A}{x+2} + \frac{B}{x+3}$
 $x - 1 \equiv A(x+3) + B(x+2)$
 $x = -2 \Rightarrow = \Rightarrow A = -3$
 $x = -3 \Rightarrow -4 = -B \Rightarrow B = 4$
 $\therefore \frac{x-1}{(x+2)(x+3)} \equiv \frac{4}{x+3} - \frac{3}{x+2}$
- d** $\frac{5x+7}{x(x+1)} \equiv \frac{A}{x} + \frac{B}{x+1}$
 $5x + 7 \equiv A(x+1) + Bx$
 $x = 0 \Rightarrow 7 = A \Rightarrow A = 7$
 $x = -1 \Rightarrow 2 = -B \Rightarrow B = -2$
 $\therefore \frac{5x+7}{x^2+x} \equiv \frac{7}{x} - \frac{2}{x+1}$
- f** $\frac{4x+6}{(x+3)(x-3)} \equiv \frac{A}{x+3} + \frac{B}{x-3}$
 $4x + 6 \equiv A(x-3) + B(x+3)$
 $x = -3 \Rightarrow -6 = -6A \Rightarrow A = 1$
 $x = 3 \Rightarrow 18 = 6B \Rightarrow B = 3$
 $\therefore \frac{4x+6}{x^2-9} \equiv \frac{1}{x+3} + \frac{3}{x-3}$
- h** $\frac{38-x}{(4+x)(3-x)} \equiv \frac{A}{4+x} + \frac{B}{3-x}$
 $38 - x \equiv A(3-x) + B(4+x)$
 $x = -4 \Rightarrow 42 = 7A \Rightarrow A = 6$
 $x = 3 \Rightarrow 35 = 7B \Rightarrow B = 5$
 $\therefore \frac{38-x}{12-x-x^2} \equiv \frac{6}{4+x} + \frac{5}{3-x}$

$$\begin{aligned} \text{i} \quad \frac{4x-5}{(2x+1)(x-3)} &\equiv \frac{A}{2x+1} + \frac{B}{x-3} \\ 4x-5 &\equiv A(x-3) + B(2x+1) \\ x = -\frac{1}{2} &\Rightarrow -7 = -\frac{7}{2}A \Rightarrow A = 2 \\ x = 3 &\Rightarrow 7 = 7B \Rightarrow B = 1 \\ \therefore \frac{4x-5}{(2x+1)(x-3)} &\equiv \frac{2}{2x+1} + \frac{1}{x-3} \end{aligned}$$

$$\begin{aligned} \text{k} \quad \frac{x+1}{x(1-3x)} &\equiv \frac{A}{x} + \frac{B}{1-3x} \\ x+1 &\equiv A(1-3x) + Bx \\ x = 0 &\Rightarrow A = 1 \\ x = \frac{1}{3} &\Rightarrow \frac{4}{3} = \frac{1}{3}B \Rightarrow B = 4 \\ \therefore \frac{x+1}{x-3x^2} &\equiv \frac{1}{x} + \frac{4}{1-3x} \end{aligned}$$

$$\begin{aligned} \text{m} \quad \frac{2x+10}{(4x-1)(2x+3)} &\equiv \frac{A}{4x-1} + \frac{B}{2x+3} \\ 2x+10 &\equiv A(2x+3) + B(4x-1) \\ x = \frac{1}{4} &\Rightarrow \frac{21}{2} = \frac{7}{2}A \Rightarrow A = 3 \\ x = -\frac{3}{2} &\Rightarrow 7 = -7B \Rightarrow B = -1 \\ \therefore \frac{2(x+5)}{8x^2+10x-3} &\equiv \frac{3}{4x-1} - \frac{1}{2x+3} \end{aligned}$$

$$\begin{aligned} \text{o} \quad \frac{1-3x}{(1+x)(1-2x)} &\equiv \frac{A}{1+x} + \frac{B}{1-2x} \\ 1-3x &\equiv A(1-2x) + B(1+x) \\ x = -1 &\Rightarrow 4 = 3A \Rightarrow A = \frac{4}{3} \\ x = \frac{1}{2} &\Rightarrow -\frac{1}{2} = \frac{3}{2}B \Rightarrow B = -\frac{1}{3} \\ \therefore \frac{1-3x}{1-x-2x^2} &\equiv \frac{4}{3(1+x)} - \frac{1}{3(1-2x)} \end{aligned}$$

$$\begin{aligned} \text{4 a} \quad x = 4 &\Rightarrow 84 = 21A \Rightarrow A = 4 \\ x = -3 &\Rightarrow -56 = 28B \Rightarrow B = -2 \\ x = 1 &\Rightarrow -12 = -12C \Rightarrow C = 1 \end{aligned}$$

$$\begin{aligned} \text{b} \quad x = \frac{1}{3} &\Rightarrow \frac{20}{3} = -\frac{20}{9}A \Rightarrow A = -3 \\ x = 2 &\Rightarrow 30 = 15B \Rightarrow B = 2 \\ x = -1 &\Rightarrow -12 = 12C \Rightarrow C = -1 \end{aligned}$$

$$\begin{aligned} \text{c} \quad x = -5 &\Rightarrow 32 = 16A \Rightarrow A = 2 \\ x = -1 &\Rightarrow 12 = 4C \Rightarrow C = 3 \\ \text{coeffs of } x^2 &\Rightarrow 1 = A + B \Rightarrow B = -1 \end{aligned}$$

$$\begin{aligned} \text{d} \quad x = 3 &\Rightarrow 196 = 49A \Rightarrow A = 4 \\ x = -\frac{1}{2} &\Rightarrow 21 = -\frac{7}{2}C \Rightarrow C = -6 \\ \text{coeffs of } x^2 &\Rightarrow 20 = 4A + 2B \Rightarrow B = 2 \end{aligned}$$

$$\begin{aligned} \text{j} \quad \frac{1-3x}{(3x+4)(2x+1)} &\equiv \frac{A}{3x+4} + \frac{B}{2x+1} \\ 1-3x &\equiv A(2x+1) + B(3x+4) \\ x = -\frac{4}{3} &\Rightarrow 5 = -\frac{5}{3}A \Rightarrow A = -3 \\ x = -\frac{1}{2} &\Rightarrow \frac{5}{2} = \frac{5}{2}B \Rightarrow B = 1 \\ \therefore \frac{1-3x}{(3x+4)(2x+1)} &\equiv \frac{1}{2x+1} - \frac{3}{3x+4} \end{aligned}$$

$$\begin{aligned} \text{l} \quad \frac{5}{(2x-1)(x+2)} &\equiv \frac{A}{2x-1} + \frac{B}{x+2} \\ 5 &\equiv A(x+2) + B(2x-1) \\ x = \frac{1}{2} &\Rightarrow 5 = \frac{5}{2}A \Rightarrow A = 2 \\ x = -2 &\Rightarrow 5 = -5B \Rightarrow B = -1 \\ \therefore \frac{5}{2x^2+3x-2} &\equiv \frac{2}{2x-1} - \frac{1}{x+2} \end{aligned}$$

$$\begin{aligned} \text{n} \quad \frac{3x-7}{(x+1)(x-3)} &\equiv \frac{A}{x+1} + \frac{B}{x-3} \\ 3x-7 &\equiv A(x-3) + B(x+1) \\ x = -1 &\Rightarrow -10 = -4A \Rightarrow A = \frac{5}{2} \\ x = 3 &\Rightarrow 2 = 4B \Rightarrow B = \frac{1}{2} \\ \therefore \frac{3x-7}{x^2-2x-3} &\equiv \frac{5}{2(x+1)} + \frac{1}{2(x-3)} \end{aligned}$$

5 a $8x + 14 \equiv A(x+1)(x+3) + B(x-2)(x+3) + C(x-2)(x+1)$

$$x = 2 \quad \Rightarrow \quad 30 = 15A \quad \Rightarrow \quad A = 2$$

$$x = -1 \quad \Rightarrow \quad 6 = -6B \quad \Rightarrow \quad B = -1$$

$$x = -3 \quad \Rightarrow \quad -10 = 10C \quad \Rightarrow \quad C = -1$$

b $2x^2 - 6x + 20 \equiv A(x+2)(x-6) + B(x+1)(x-6) + C(x+1)(x+2)$

$$x = -1 \quad \Rightarrow \quad 28 = -7A \quad \Rightarrow \quad A = -4$$

$$x = -2 \quad \Rightarrow \quad 40 = 8B \quad \Rightarrow \quad B = 5$$

$$x = 6 \quad \Rightarrow \quad 56 = 56C \quad \Rightarrow \quad C = 1$$

c $9x - 14 \equiv A(x-1)^2 + B(x+4)(x-1) + C(x+4)$

$$x = -4 \quad \Rightarrow \quad -50 = 25A \quad \Rightarrow \quad A = -2$$

$$x = 1 \quad \Rightarrow \quad -5 = 5C \quad \Rightarrow \quad C = -1$$

$$\text{coeffs of } x^2 \Rightarrow 0 = A + B \quad \Rightarrow \quad B = 2$$

d $3x^2 - 7x - 4 \equiv A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

$$x = 3 \quad \Rightarrow \quad A = 2$$

$$x = 2 \quad \Rightarrow \quad -6 = -C \quad \Rightarrow \quad C = 6$$

$$\text{coeffs of } x^2 \Rightarrow 3 = A + B \quad \Rightarrow \quad B = 1$$

6 a $\frac{2x^2 + 4}{x(x-1)(x-4)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-4}$

$$2x^2 + 4 \equiv A(x-1)(x-4) + Bx(x-4) + Cx(x-1)$$

$$x = 0 \quad \Rightarrow \quad 4 = 4A \quad \Rightarrow \quad A = 1$$

$$x = 1 \quad \Rightarrow \quad 6 = -3B \quad \Rightarrow \quad B = -2$$

$$x = 4 \quad \Rightarrow \quad 36 = 12C \quad \Rightarrow \quad C = 3 \quad \therefore \frac{2x^2 + 4}{x(x-1)(x-4)} \equiv \frac{1}{x} - \frac{2}{x-1} + \frac{3}{x-4}$$

b $\frac{9}{(x-2)(x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$$9 \equiv A(x+1)^2 + B(x-2)(x+1) + C(x-2)$$

$$x = 2 \quad \Rightarrow \quad 9 = 9A \quad \Rightarrow \quad A = 1$$

$$x = -1 \quad \Rightarrow \quad 9 = -3C \quad \Rightarrow \quad C = -3$$

$$\text{coeffs of } x^2 \Rightarrow 0 = A + B \quad \Rightarrow \quad B = -1 \quad \therefore \frac{9}{(x-2)(x+1)^2} \equiv \frac{1}{x-2} - \frac{1}{x+1} - \frac{3}{(x+1)^2}$$

c $\frac{x^2 + 11x - 21}{(2x+1)(x-2)(x-3)} \equiv \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{x-3}$

$$x^2 + 11x - 21 \equiv A(x-2)(x-3) + B(2x+1)(x-3) + C(2x+1)(x-2)$$

$$x = -\frac{1}{2} \quad \Rightarrow \quad -\frac{105}{4} = \frac{35}{4}A \quad \Rightarrow \quad A = -3$$

$$x = 2 \quad \Rightarrow \quad 5 = -5B \quad \Rightarrow \quad B = -1$$

$$x = 3 \quad \Rightarrow \quad 21 = 7C \quad \Rightarrow \quad C = 3 \quad \therefore \frac{x^2 + 11x - 21}{(2x+1)(x-2)(x-3)} \equiv \frac{3}{x-3} - \frac{3}{2x+1} - \frac{1}{x-2}$$

d $\frac{10x+9}{(x-4)(x+3)^2} \equiv \frac{A}{x-4} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

$$10x + 9 \equiv A(x+3)^2 + B(x-4)(x+3) + C(x-4)$$

$$x = 4 \quad \Rightarrow \quad 49 = 49A \quad \Rightarrow \quad A = 1$$

$$x = -3 \quad \Rightarrow \quad -21 = -7C \quad \Rightarrow \quad C = 3$$

$$\text{coeffs of } x^2 \Rightarrow 0 = A + B \quad \Rightarrow \quad B = -1 \quad \therefore \frac{10x+9}{(x-4)(x+3)^2} \equiv \frac{1}{x-4} - \frac{1}{x+3} + \frac{3}{(x+3)^2}$$

- e** $\frac{x^2+4x+5}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$
 $x^2+4x+5 \equiv A(x+2)^2 + B(x+1)(x+2) + C(x+1)$
 $x = -1 \Rightarrow \quad \quad \quad \Rightarrow A = 2$
 $x = -2 \Rightarrow 1 = -C \Rightarrow C = -1$
 coeffs of $x^2 \Rightarrow 1 = A + B \Rightarrow B = -1 \therefore \frac{x^2+4x+5}{(x+1)(x+2)^2} \equiv \frac{2}{x+1} - \frac{1}{x+2} - \frac{1}{(x+2)^2}$
- f** $\frac{16-2x}{(x-3)(x+2)(x-2)} \equiv \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{x-2}$
 $16-2x \equiv A(x+2)(x-2) + B(x-3)(x-2) + C(x-3)(x+2)$
 $x = 3 \Rightarrow 10 = 5A \Rightarrow A = 2$
 $x = -2 \Rightarrow 20 = 20B \Rightarrow B = 1$
 $x = 2 \Rightarrow 12 = -4C \Rightarrow C = -3 \therefore \frac{16-2x}{(x-3)(x^2-4)} \equiv \frac{2}{x-3} + \frac{1}{x+2} - \frac{3}{x-2}$
- g** $\frac{2-9x}{(x-3)(2x-1)^2} \equiv \frac{A}{x-3} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$
 $2-9x \equiv A(2x-1)^2 + B(x-3)(2x-1) + C(x-3)$
 $x = 3 \Rightarrow -25 = 25A \Rightarrow A = -1$
 $x = \frac{1}{2} \Rightarrow -\frac{5}{2} = -\frac{5}{2}C \Rightarrow C = 1$
 coeffs of $x^2 \Rightarrow 0 = 4A + 2B \Rightarrow B = 2 \therefore \frac{2-9x}{(x-3)(2x-1)^2} \equiv \frac{2}{2x-1} + \frac{1}{(2x-1)^2} - \frac{1}{x-3}$
- h** $\frac{3+24x-4x^2}{(x+1)(x-4)^2} \equiv \frac{A}{x+1} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$
 $3+24x-4x^2 \equiv A(x-4)^2 + B(x+1)(x-4) + C(x+1)$
 $x = -1 \Rightarrow -25 = 25A \Rightarrow A = -1$
 $x = 4 \Rightarrow 35 = 5C \Rightarrow C = 7$
 coeffs of $x^2 \Rightarrow -4 = A + B \Rightarrow B = -3 \therefore \frac{3+24x-4x^2}{(x+1)(x-4)^2} \equiv \frac{7}{(x-4)^2} - \frac{3}{x-4} - \frac{1}{x+1}$
- i** $\frac{9x^2-2x-12}{x(x+3)(x-2)} \equiv \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2}$
 $9x^2-2x-12 \equiv A(x+3)(x-2) + Bx(x-2) + Cx(x+3)$
 $x = 0 \Rightarrow -12 = -6A \Rightarrow A = 2$
 $x = -3 \Rightarrow 75 = 15B \Rightarrow B = 5$
 $x = 2 \Rightarrow 20 = 10C \Rightarrow C = 2 \therefore \frac{9x^2-2x-12}{x^3+x^2-6x} \equiv \frac{2}{x} + \frac{5}{x+3} + \frac{2}{x-2}$
- j** $\frac{5x^2+3x-20}{x^2(x+4)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4}$
 $5x^2+3x-20 \equiv Ax(x+4) + B(x+4) + Cx^2$
 $x = 0 \Rightarrow -20 = 4B \Rightarrow B = -5$
 $x = -4 \Rightarrow 48 = 16C \Rightarrow C = 3$
 coeffs of $x^2 \Rightarrow 5 = A + C \Rightarrow A = 2 \therefore \frac{5x^2+3x-20}{x^3+4x^2} \equiv \frac{2}{x} - \frac{5}{x^2} + \frac{3}{x+4}$
- k** $\frac{13-3x^2}{(2x+3)(x-1)^2} \equiv \frac{A}{2x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$
 $13-3x^2 \equiv A(x-1)^2 + B(2x+3)(x-1) + C(2x+3)$
 $x = -\frac{3}{2} \Rightarrow \frac{25}{4} = \frac{25}{4}A \Rightarrow A = 1$
 $x = 1 \Rightarrow 10 = 5C \Rightarrow C = 2$
 coeffs of $x^2 \Rightarrow -3 = A + 2B \Rightarrow B = -2 \therefore \frac{13-3x^2}{(2x+3)(x-1)^2} \equiv \frac{1}{2x+3} - \frac{2}{x-1} + \frac{2}{(x-1)^2}$

$$\begin{aligned}
 \mathbf{1} \quad \frac{26-x-x^2}{(x-1)(x+3)(x+5)} &\equiv \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{x+5} \\
 26-x-x^2 &\equiv A(x+3)(x+5) + B(x-1)(x+5) + C(x-1)(x+3) \\
 x=1 &\Rightarrow 24 = 24A \Rightarrow A = 1 \\
 x=-3 &\Rightarrow 20 = -8B \Rightarrow B = -\frac{5}{2} \\
 x=-5 &\Rightarrow 6 = 12C \Rightarrow C = \frac{1}{2} \quad \therefore \frac{26-x-x^2}{(x-1)(x+3)(x+5)} \equiv \frac{1}{x-1} - \frac{5}{2(x+3)} + \frac{1}{2(x+5)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7} \quad \mathbf{a} \quad x^2 &\equiv A(x-2)(x-6) + B(x-6) + C(x-2) \\
 x=2 &\Rightarrow 4 = -4B \Rightarrow B = -1 \\
 x=6 &\Rightarrow 36 = 4C \Rightarrow C = 9 \\
 \text{coeffs of } x^2 &\Rightarrow A = 1
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \frac{x^2+2x+9}{(x-1)(x+5)} &\equiv A + \frac{B}{x-1} + \frac{C}{x+5} \\
 x^2+2x+9 &\equiv A(x-1)(x+5) + B(x+5) + C(x-1) \\
 x=1 &\Rightarrow 12 = 6B \Rightarrow B = 2 \\
 x=-5 &\Rightarrow 24 = -6C \Rightarrow C = -4 \\
 \text{coeffs of } x^2 &\Rightarrow A = 1
 \end{aligned}$$

$$\begin{array}{r}
 \mathbf{8} \quad \mathbf{a} \\
 x^2+x-2 \overline{) \begin{array}{r} x+3 \\ x^3+4x^2+0x-2 \\ \underline{x^3+x^2-2x} \\ 3x^2+2x-2 \\ \underline{3x^2+3x-6} \\ -x+4 \end{array}
 \end{array}$$

quotient: $x+3$ remainder: $-x+4$

$$\begin{aligned}
 \mathbf{b} \quad \frac{x^3+4x^2-2}{x^2+x-2} &\equiv x+3 + \frac{4-x}{x^2+x-2} \\
 \frac{4-x}{(x+2)(x-1)} &\equiv \frac{A}{x+2} + \frac{B}{x-1} \\
 4-x &\equiv A(x-1) + B(x+2) \\
 x=-2 &\Rightarrow 6 = -3A \Rightarrow A = -2 \\
 x=1 &\Rightarrow 3 = 3B \Rightarrow B = 1 \quad \therefore \frac{x^3+4x^2-2}{x^2+x-2} \equiv x+3 - \frac{2}{x+2} + \frac{1}{x-1}
 \end{aligned}$$

$$\mathbf{9} \quad \mathbf{a} \quad (x-3)(x+1) = x^2 - 2x - 3$$

$$\begin{aligned}
 &\frac{1}{x^2-2x-3} \overline{) \begin{array}{r} x^2+0x+3 \\ \underline{x^2-2x-3} \\ 2x+6 \end{array} \\
 \therefore \frac{x^2+3}{(x-3)(x+1)} &\equiv 1 + \frac{2x+6}{(x-3)(x+1)} \\
 \frac{2x+6}{(x-3)(x+1)} &\equiv \frac{A}{x-3} + \frac{B}{x+1} \\
 2x+6 &\equiv A(x+1) + B(x-3) \\
 x=3 &\Rightarrow 12 = 4A \Rightarrow A = 3 \\
 x=-1 &\Rightarrow 4 = -4B \Rightarrow B = -1 \quad \therefore \frac{x^2+3}{(x-3)(x+1)} \equiv 1 + \frac{3}{x-3} - \frac{1}{x+1}
 \end{aligned}$$

b

$$\begin{array}{r}
 x^2 - 4 \overline{) \begin{array}{l} x^3 - 3x^2 - x + 2 \\ x^3 + 0x^2 - 4x \\ \hline -3x^2 + 3x + 2 \\ -3x^2 + 0x + 12 \\ \hline 3x - 10 \end{array} }
 \end{array}$$

$$\therefore \frac{x^3 - 3x^2 - x + 2}{x^2 - 4} \equiv x - 3 + \frac{3x - 10}{x^2 - 4}$$

$$\frac{3x - 10}{(x + 2)(x - 2)} \equiv \frac{A}{x + 2} + \frac{B}{x - 2}$$

$$3x - 10 \equiv A(x - 2) + B(x + 2)$$

$$x = -2 \Rightarrow -16 = -4A \Rightarrow A = 4$$

$$x = 2 \Rightarrow -4 = 4B \Rightarrow B = -1$$

$$\therefore \frac{x^3 - 3x^2 - x + 2}{x^2 - 4} \equiv x - 3 + \frac{4}{x + 2} - \frac{1}{x - 2}$$

c

$$\begin{array}{r}
 x^2 + 6x + 8 \overline{) \begin{array}{l} 2x^2 + 7x + 0 \\ 2x^2 + 12x + 16 \\ \hline -5x - 16 \end{array} }
 \end{array}$$

$$\therefore \frac{2x^2 + 7x}{x^2 + 6x + 8} \equiv 2 + \frac{-5x - 16}{x^2 + 6x + 8}$$

$$\frac{-5x - 16}{(x + 2)(x + 4)} \equiv \frac{A}{x + 2} + \frac{B}{x + 4}$$

$$-5x - 16 \equiv A(x + 4) + B(x + 2)$$

$$x = -2 \Rightarrow -6 = 2A \Rightarrow A = -3$$

$$x = -4 \Rightarrow 4 = -2B \Rightarrow B = -2$$

$$\therefore \frac{2x^2 + 7x}{x^2 + 6x + 8} \equiv 2 - \frac{3}{x + 2} - \frac{2}{x + 4}$$

d $3(x + 1)(x - 1) = 3x^2 - 3$

$(x - 4)(x + 5) = x^2 + x - 20$

$$\begin{array}{r}
 x^2 + x - 20 \overline{) \begin{array}{l} 3x^2 + 0x - 3 \\ 3x^2 + 3x - 60 \\ \hline -3x + 57 \end{array} }
 \end{array}$$

$$\therefore \frac{3(x + 1)(x - 1)}{(x - 4)(x + 5)} \equiv 3 + \frac{57 - 3x}{(x - 4)(x + 5)}$$

$$\frac{57 - 3x}{(x - 4)(x + 5)} \equiv \frac{A}{x - 4} + \frac{B}{x + 5}$$

$$57 - 3x \equiv A(x + 5) + B(x - 4)$$

$$x = 4 \Rightarrow 45 = 9A \Rightarrow A = 5$$

$$x = -5 \Rightarrow 72 = -9B \Rightarrow B = -8$$

$$\therefore \frac{3(x + 1)(x - 1)}{(x - 4)(x + 5)} \equiv 3 + \frac{5}{x - 4} - \frac{8}{x + 5}$$

e

$$\begin{array}{r}
 3x - 5 \\
 x^2 + 4x + 3 \overline{) 3x^3 + 7x^2 + 0x + 4} \\
 \underline{3x^3 + 12x^2 + 9x} \\
 -5x^2 - 9x + 4 \\
 \underline{-5x^2 - 20x - 15} \\
 11x + 19
 \end{array}$$

$$\therefore \frac{3x^3 + 7x^2 + 4}{x^2 + 4x + 3} \equiv 3x - 5 + \frac{11x + 19}{x^2 + 4x + 3}$$

$$\frac{11x + 19}{(x+1)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+3}$$

$$11x + 19 \equiv A(x+3) + B(x+1)$$

$$x = -1 \Rightarrow 8 = 2A \Rightarrow A = 4$$

$$x = -3 \Rightarrow -14 = -2B \Rightarrow B = 7$$

$$\therefore \frac{3x^3 + 7x^2 + 4}{x^2 + 4x + 3} \equiv 3x - 5 + \frac{4}{x+1} + \frac{7}{x+3}$$

f

$$\begin{array}{r}
 2 \\
 2x^2 - 7x + 3 \overline{) 4x^2 - 7x + 5} \\
 \underline{2x^2 - 14x + 6} \\
 7x - 1
 \end{array}$$

$$\therefore \frac{4x^2 - 7x + 5}{2x^2 - 7x + 3} \equiv 2 + \frac{7x - 1}{2x^2 - 7x + 3}$$

$$\frac{7x - 1}{(2x-1)(x-3)} \equiv \frac{A}{2x-1} + \frac{B}{x-3}$$

$$7x - 1 \equiv A(x-3) + B(2x-1)$$

$$x = \frac{1}{2} \Rightarrow \frac{5}{2} = -\frac{5}{2}A \Rightarrow A = -1$$

$$x = 3 \Rightarrow 20 = 5B \Rightarrow B = 4$$

$$\therefore \frac{4x^2 - 7x + 5}{2x^2 - 7x + 3} \equiv 2 - \frac{1}{2x-1} + \frac{4}{x-3}$$

g

$$\begin{array}{r}
 2 \\
 x^2 - 2x - 3 \overline{) 2x^2 + 0x + 0} \\
 \underline{2x^2 - 4x - 6} \\
 4x + 6
 \end{array}$$

$$\therefore \frac{2x^2}{x^2 - 2x - 3} \equiv 2 + \frac{4x + 6}{x^2 - 2x - 3}$$

$$\frac{4x + 6}{(x+1)(x-3)} \equiv \frac{A}{x+1} + \frac{B}{x-3}$$

$$4x + 6 \equiv A(x-3) + B(x+1)$$

$$x = -1 \Rightarrow 2 = -4A \Rightarrow A = -\frac{1}{2}$$

$$x = 3 \Rightarrow 18 = 4B \Rightarrow B = \frac{9}{2}$$

$$\therefore \frac{2x^2}{x^2 - 2x - 3} \equiv 2 - \frac{1}{2(x+1)} + \frac{9}{2(x-3)}$$

h

$$x^2 - 6x + 5 \overline{) \begin{array}{r} x^3 - 6x^2 + 6x + 1 \\ x^3 - 6x^2 + 5x \\ \hline x + 1 \end{array}}$$

$$\therefore \frac{x^3 - 6x^2 + 6x + 1}{x^2 - 6x + 5} \equiv x + \frac{x + 1}{x^2 - 6x + 5}$$

$$\frac{x + 1}{(x - 1)(x - 5)} \equiv \frac{A}{x - 1} + \frac{B}{x - 5}$$

$$x + 1 \equiv A(x - 5) + B(x - 1)$$

$$x = 1 \quad \Rightarrow \quad 2 = -4A \quad \Rightarrow \quad A = -\frac{1}{2}$$

$$x = 5 \quad \Rightarrow \quad 6 = 4B \quad \Rightarrow \quad B = \frac{3}{2}$$

$$\therefore \frac{x^3 - 6x^2 + 6x + 1}{x^2 - 6x + 5} \equiv x - \frac{1}{2(x - 1)} + \frac{3}{2(x - 5)}$$

i

$$3x^2 - 4x - 4 \overline{) \begin{array}{r} 9x^3 + 0x^2 - 27x - 2 \\ 9x^3 - 12x^2 - 12x \\ \hline 12x^2 - 15x - 2 \\ 12x^2 - 16x - 16 \\ \hline x + 14 \end{array}}$$

$$\therefore \frac{9x^3 - 27x - 2}{3x^2 - 4x - 4} \equiv 3x + 4 + \frac{x + 14}{3x^2 - 4x - 4}$$

$$\frac{x + 14}{(3x + 2)(x - 2)} \equiv \frac{A}{3x + 2} + \frac{B}{x - 2}$$

$$x + 14 \equiv A(x - 2) + B(3x + 2)$$

$$x = -\frac{2}{3} \quad \Rightarrow \quad \frac{40}{3} = -\frac{8}{3}A \quad \Rightarrow \quad A = -5$$

$$x = 2 \quad \Rightarrow \quad 16 = 8B \quad \Rightarrow \quad B = 2$$

$$\therefore \frac{9x^3 - 27x - 2}{3x^2 - 4x - 4} \equiv 3x + 4 - \frac{5}{3x + 2} + \frac{2}{x - 2}$$

10 a $\frac{x + 5}{(x - 1)(2x + 1)} \equiv \frac{A}{x - 1} + \frac{B}{2x + 1}$

$$x + 5 \equiv A(2x + 1) + B(x - 1)$$

$$x = 1 \quad \Rightarrow \quad 6 = 3A \quad \Rightarrow \quad A = 2$$

$$x = -\frac{1}{2} \quad \Rightarrow \quad \frac{9}{2} = -\frac{3}{2}B \quad \Rightarrow \quad B = -3$$

$$\therefore f(x) = \frac{2}{x - 1} - \frac{3}{2x + 1}$$

b $f'(x) = -2(x - 1)^{-2} + 3(2x + 1)^{-2} \times 2$

$$= \frac{6}{(2x + 1)^2} - \frac{2}{(x - 1)^2}$$

SP: $\frac{6}{(2x + 1)^2} - \frac{2}{(x - 1)^2} = 0$

$$6(x - 1)^2 - 2(2x + 1)^2 = 0$$

$$x^2 + 10x - 2 = 0$$

$$x = \frac{-10 \pm \sqrt{100 + 8}}{2}$$

$$x = -5 \pm 3\sqrt{3}$$

11 a $x(4x + 5) \equiv A(x + 2)^2 + B(x - 1)(x + 2) + C(x - 1)$

$$x = 1 \quad \Rightarrow \quad 9 = 9A \quad \Rightarrow \quad A = 1$$

$$x = -2 \quad \Rightarrow \quad 6 = -3C \quad \Rightarrow \quad C = -2$$

$$\text{coeffs } x^2 \Rightarrow 4 = A + B \quad \Rightarrow \quad B = 3$$

b $x = -1 \therefore y = \frac{1}{2}$

$$f(x) = (x - 1)^{-1} + 3(x + 2)^{-1} - 2(x + 2)^{-2}$$

$$f'(x) = -(x - 1)^{-2} - 3(x + 2)^{-2} + 4(x + 2)^{-3}$$

$$\text{grad} = -\frac{1}{4} - 3 + 4 = \frac{3}{4}$$

$$\therefore y - \frac{1}{2} = \frac{3}{4}(x + 1)$$

$$4y - 2 = 3x + 3$$

$$3x - 4y + 5 = 0$$