PARTIAL FRACTIONS

1   Find the values of the constants A and B in each identity.

    a $x - 8 \equiv A(x - 2) + B(x + 4)$ **b** $6x + 7 \equiv A(2x - 1) + B(x + 2)$

2   Find the values of the constants A and B in each identity.

    a $\dfrac{2}{(x+1)(x+3)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x+3}$ **b** $\dfrac{x-3}{x(x-1)} \equiv \dfrac{A}{x} + \dfrac{B}{x-1}$

    c $\dfrac{x+1}{(x-3)(x-5)} \equiv \dfrac{A}{x-3} + \dfrac{B}{x-5}$ **d** $\dfrac{x+10}{(1+x)(2-x)} \equiv \dfrac{A}{1+x} + \dfrac{B}{2-x}$

    e $\dfrac{4x-1}{x^2+x-2} \equiv \dfrac{A}{x+2} + \dfrac{B}{x-1}$ **f** $\dfrac{x-9}{x^2-4x+3} \equiv \dfrac{A}{x-1} + \dfrac{B}{x-3}$

3   Express in partial fractions

    a $\dfrac{8}{(x-1)(x+3)}$ **b** $\dfrac{x-1}{(x+2)(x+3)}$ **c** $\dfrac{10x}{(x+4)(x-1)}$

    d $\dfrac{5x+7}{x^2+x}$ **e** $\dfrac{x+2}{x^2-5x+4}$ **f** $\dfrac{4x+6}{x^2-9}$

    g $\dfrac{3x+2}{x^2-2x-24}$ **h** $\dfrac{38-x}{12-x-x^2}$ **i** $\dfrac{4x-5}{(2x+1)(x-3)}$

    j $\dfrac{1-3x}{(3x+4)(2x+1)}$ **k** $\dfrac{x+1}{x-3x^2}$ **l** $\dfrac{5}{2x^2+3x-2}$

    m $\dfrac{2(x+5)}{8x^2+10x-3}$ **n** $\dfrac{3x-7}{x^2-2x-3}$ **o** $\dfrac{1-3x}{1-x-2x^2}$

4   Find the values of the constants A, B and C in each identity.

    a $3x^2 + 17x - 32 \equiv A(x - 1)(x + 3) + B(x - 1)(x - 4) + C(x + 3)(x - 4)$

    b $14x + 2 \equiv A(x + 1)(x - 2) + B(x + 1)(3x - 1) + C(x - 2)(3x - 1)$

    c $x^2 + x + 12 \equiv A(x + 1)^2 + B(x + 1)(x + 5) + C(x + 5)$

    d $4(5x^2 + 4) \equiv A(2x + 1)^2 + B(2x + 1)(x - 3) + C(x - 3)$

5   Find the values of the constants A, B and C in each identity.

    a $\dfrac{8x+14}{(x-2)(x+1)(x+3)} \equiv \dfrac{A}{x-2} + \dfrac{B}{x+1} + \dfrac{C}{x+3}$

    b $\dfrac{2x^2-6x+20}{(x+1)(x+2)(x-6)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{x-6}$

    c $\dfrac{9x-14}{(x+4)(x-1)^2} \equiv \dfrac{A}{x+4} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

    d $\dfrac{3x^2-7x-4}{(x-3)(x-2)^2} \equiv \dfrac{A}{x-3} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$

6   Express in partial fractions

    a $\dfrac{2x^2+4}{x(x-1)(x-4)}$ **b** $\dfrac{9}{(x-2)(x+1)^2}$ **c** $\dfrac{x^2+11x-21}{(2x+1)(x-2)(x-3)}$

    d $\dfrac{10x+9}{(x-4)(x+3)^2}$ **e** $\dfrac{x^2+4x+5}{(x+1)(x+2)^2}$ **f** $\dfrac{16-2x}{(x-3)(x^2-4)}$

    g $\dfrac{2-9x}{(x-3)(2x-1)^2}$ **h** $\dfrac{3+24x-4x^2}{(x+1)(x-4)^2}$ **i** $\dfrac{9x^2-2x-12}{x^3+x^2-6x}$

    j $\dfrac{5x^2+3x-20}{x^3+4x^2}$ **k** $\dfrac{13-3x^2}{(2x+3)(x-1)^2}$ **l** $\dfrac{26-x-x^2}{(x-1)(x+3)(x+5)}$

7   Find the values of the constants A, B and C in each identity.

    a $\dfrac{x^2}{(x-2)(x-6)} \equiv A + \dfrac{B}{x-2} + \dfrac{C}{x-6}$

    b $\dfrac{x^2+2x+9}{x^2+4x-5} \equiv A + \dfrac{B}{x-1} + \dfrac{C}{x+5}$

8   **a** Find the quotient and remainder obtained in dividing (x^3+4x^2-2) by (x^2+x-2).

    b Hence, express $\dfrac{x^3+4x^2-2}{x^2+x-2}$ in partial fractions.

9   Express in partial fractions

    a $\dfrac{x^2+3}{(x-3)(x+1)}$ **b** $\dfrac{x^3-3x^2-x+2}{x^2-4}$ **c** $\dfrac{2x^2+7x}{x^2+6x+8}$

    d $\dfrac{3(x+1)(x-1)}{(x-4)(x+5)}$ **e** $\dfrac{3x^3+7x^2+4}{x^2+4x+3}$ **f** $\dfrac{4x^2-7x+5}{2x^2-7x+3}$

    g $\dfrac{2x^2}{x^2-2x-3}$ **h** $\dfrac{x^3-6x^2+6x+1}{x^2-6x+5}$ **i** $\dfrac{9x^3-27x-2}{3x^2-4x-4}$

10 $\qquad\qquad f(x) = \dfrac{x+5}{(x-1)(2x+1)}$.

    a Express $f(x)$ in partial fractions.

    b Find the exact x-coordinates of the stationary points of the curve $y = f(x)$.

11 $\qquad\qquad f(x) = \dfrac{x(4x+5)}{(x-1)(x+2)^2}$.

    a Find the values of the constants A, B and C such that

            $f(x) = \dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$.

    b Show that the tangent to the curve $y = f(x)$ at the point where $x = -1$ has the equation

        $3x - 4y + 5 = 0$.