

FUNCTIONS

1 The domain of each of the following functions is $x \in \mathbb{R}$. For each function, find its inverse $f^{-1}(x)$.

a $f: x \rightarrow 10x + 3$ **b** $f: x \rightarrow 9 + 2x$ **c** $f: x \rightarrow 5 - 6x$
d $f: x \rightarrow \frac{x+3}{4}$ **e** $f: x \rightarrow \frac{1}{3}(2x - 5)$ **f** $f: x \rightarrow 8 - \frac{3}{5}x$

2 For each function, find $f^{-1}(x)$ and state its domain.

a $f(x) \equiv \ln x, x \in \mathbb{R}, x > 0$ **b** $f(x) \equiv \frac{1}{x}, x \in \mathbb{R}, x \neq 0$
c $f(x) \equiv \sqrt[4]{x}, x \in \mathbb{R}, x > 0$ **d** $f(x) \equiv 3x - 4, x \in \mathbb{R}, 0 \leq x < 3$
e $f(x) \equiv \frac{1}{x-5}, x \in \mathbb{R}, x \neq 5$ **f** $f(x) \equiv 2 + \frac{1}{x}, x \in \mathbb{R}, x \neq 0$

3 For each of the following functions,

- i** find, in the form $f^{-1}: x \rightarrow \dots$, the inverse function of f and state its domain,
ii sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

a $f: x \rightarrow 2x + 1, x \in \mathbb{R}$ **b** $f: x \rightarrow \frac{1-x}{5}, x \in \mathbb{R}$ **c** $f: x \rightarrow \frac{10}{x}, x \in \mathbb{R}, x \neq 0$
d $f: x \rightarrow x^2, x \in \mathbb{R}, x > 0$ **e** $f: x \rightarrow e^x, x \in \mathbb{R}$ **f** $f: x \rightarrow x^3, x \in \mathbb{R}$

4 For each of the following, solve the equation $f^{-1}(x) = g(x)$.

a $f: x \rightarrow 5x + 1, x \in \mathbb{R}$ $g: x \rightarrow 2, x \in \mathbb{R}$
b $f: x \rightarrow \frac{2x-4}{3}, x \in \mathbb{R}$ $g: x \rightarrow 7 - x, x \in \mathbb{R}$
c $f: x \rightarrow e^x + 2, x \in \mathbb{R}$ $g: x \rightarrow \ln(3x - 8), x \in \mathbb{R}, x > \frac{8}{3}$
d $f: x \rightarrow \sqrt{x+2}, x \in \mathbb{R}, x \geq -2$ $g: x \rightarrow 3x - 4, x \in \mathbb{R}$
e $f: x \rightarrow \frac{4}{x+3}, x \in \mathbb{R}, x \neq -3$ $g: x \rightarrow 5(x+1), x \in \mathbb{R}$

5 The function f is defined by $f: x \rightarrow 4 - 2x, x \in \mathbb{R}$.

- a** Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.
b Find the coordinates of the point where the lines $y = f(x)$ and $y = f^{-1}(x)$ intersect.

6 The functions f and g are defined by

$$f: x \rightarrow 3 - 2x, x \in \mathbb{R} \qquad g: x \rightarrow \frac{1}{2x+4}, x \in \mathbb{R}, x \neq -2$$

- a** Find $g^{-1}(x)$ and state its domain and range.
b Express gf in terms of x and state its domain.
c Solve the equation $gf(x) = f^{-1}(x)$.

7 The functions f and g are defined by

$$f: x \rightarrow 5x + 2, x \in \mathbb{R} \qquad g: x \rightarrow \frac{1}{x}, x \in \mathbb{R}, x \neq 0$$

- a** Find the following functions, stating the domain in each case.
i f^{-1} **ii** fg **iii** $(fg)^{-1}$
b Solve the equation $f^{-1}(x) = fg(x)$, giving your answers correct to 2 decimal places.

FUNCTIONS

continued

8 For each of the following functions, find the inverse function in the form $f^{-1}: x \rightarrow \dots$ and state its domain.

a $f: x \rightarrow \frac{1}{2} \ln(4x - 9), x \in \mathbb{R}, x > 2\frac{1}{4}$

b $f: x \rightarrow \frac{x-2}{x+5}, x \in \mathbb{R}, x \neq -5$

c $f: x \rightarrow e^{0.4x-2}, x \in \mathbb{R}$

d $f: x \rightarrow \sqrt[3]{x^5 - 3}, x \in \mathbb{R}$

e $f: x \rightarrow \log_{10}(2 - 7x), x \in \mathbb{R}, x < \frac{2}{7}$

f $f: x \rightarrow \frac{4-x}{3x+2}, x \in \mathbb{R}, x \neq -\frac{2}{3}$

9 For each of the following functions,

i find, in the form $f^{-1}: x \rightarrow \dots$, the inverse function of f and state its domain,

ii sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

a $f: x \rightarrow e^{2x}, x \in \mathbb{R}$

b $f: x \rightarrow x^2 + 4, x \in \mathbb{R}, x > 0$

c $f: x \rightarrow \ln(x - 3), x \in \mathbb{R}, x > 3$

d $f: x \rightarrow x^2 + 6x + 9, x \in \mathbb{R}, x > -3$

10 For each of the following functions,

i find the range of f ,

ii find $f^{-1}(x)$, stating its domain.

a $f(x) \equiv x^2 + 6x + 3, x \in \mathbb{R}, x < -3$

b $f(x) \equiv x^2 - 4x + 5, x \in \mathbb{R}, x \geq 2$

c $f(x) \equiv x^2 + 5x - 2, x \in \mathbb{R}, x < -2\frac{1}{2}$

d $f(x) \equiv x^2 - 3x + 5, x \in \mathbb{R}, 2 < x < 4$

e $f(x) \equiv (2 - x)(4 + x), x \in \mathbb{R}, x \geq -1$

f $f(x) \equiv 20x - 5x^2, x \in \mathbb{R}, x > 2$

11 For each of the following, solve the equation $f^{-1}(x) = g(x)$.

a $f: x \rightarrow \frac{1}{3}(2x - 5), x \in \mathbb{R}$

g $g: x \rightarrow \frac{4}{2-x}, x \in \mathbb{R}, x \neq 2$

b $f: x \rightarrow \ln \frac{x+3}{5}, x \in \mathbb{R}, x > -3$

g $g: x \rightarrow 10 - 6e^{-x}, x \in \mathbb{R}$

c $f: x \rightarrow x^2 - 4, x \in \mathbb{R}, x > 0$

g $g: x \rightarrow \frac{x+6}{3}, x \in \mathbb{R}$

12 The function f is defined by

$$f: x \rightarrow \frac{x+b}{x+a}, x \in \mathbb{R}, x \neq 2.$$

a State the value of the constant a .

Given that $f(6) = 4$,

b find the value of the constant b ,

c find $f^{-1}(x)$ and state its domain.

13 The functions f and g are defined by

$$f: x \rightarrow x^2 - 3x, x \in \mathbb{R}, x \geq 1\frac{1}{2},$$

$$g: x \rightarrow 2x + 3, x \in \mathbb{R}.$$

a Find, in the form $f^{-1}: x \rightarrow \dots$, the inverse function of f and state its domain.

b On the same set of axes, sketch $y = f(x)$ and $y = f^{-1}(x)$.

Given that $f^{-1}g^{-1}(12) = a(1 + \sqrt{3})$,

c show that $a = 1\frac{1}{2}$.