

FUNCTIONS

Answers

- 1** **a** = $g(5) = -3$ **b** = $g(6) = -4$ **c** = $f(5) = 17$ **d** = $h(9) = 86$
e = $g(-3) = 5$ **f** = $f(-1) = -7$ **g** = $h(-6) = 41$ **h** = $f(\frac{29}{4}) = 26$
- 2** **a** = $f(\ln 20) = 17.0$ **b** = $g(\ln 3) = 0.455$ **c** = $f(\cos 5) = 3.42$ **d** = $g(\cos(-4)) = 0.794$
e = $g(\frac{43}{4}) = -0.243$ **f** = $h(\cos 6.7) = -0.0895$ **g** = $h(\ln 50) = 1.36$ **h** = $h(0.5) = -0.693$
- 3** **a** = $f(1 - 3x)$ **b** = $f(2x + 1)$ **c** = $f(x^2 + 4)$ **d** = $h(2x + 1)$
= $2(1 - 3x) + 1$ = $2(2x + 1) + 1$ = $2(x^2 + 4) + 1$ = $(2x + 1)^2 + 4$
fg : $x \rightarrow 3 - 6x$, **ff** : $x \rightarrow 4x + 3$, **fh** : $x \rightarrow 2x^2 + 9$, **hf** : $x \rightarrow 4x^2 + 4x + 5$,
 $x \in \mathbb{R}$ $x \in \mathbb{R}$ $x \in \mathbb{R}$ $x \in \mathbb{R}$
- e** = $g(x^2 + 4)$ **f** = $g(1 - 3x)$ **g** = $h(1 - 3x)$ **h** = $g(2x + 1)$
= $1 - 3(x^2 + 4)$ = $1 - 3(1 - 3x)$ = $(1 - 3x)^2 + 4$ = $1 - 3(2x + 1)$
gh : $x \rightarrow -3x^2 - 11$, **gg** : $x \rightarrow 9x - 2$, **hg** : $x \rightarrow 9x^2 - 6x + 5$, **gf** : $x \rightarrow -6x - 2$,
 $x \in \mathbb{R}$ $x \in \mathbb{R}$ $x \in \mathbb{R}$ $x \in \mathbb{R}$
- 4** **a** = $g(4 - x)$ **b** = $h(e^x)$ **c** = $f(2x^2 + 7)$ **d** = $g(e^x)$
= e^{4-x} = $2(e^x)^2 + 7$ = $4 - (2x^2 + 7)$ = e^{e^x}
gf : $x \rightarrow e^{4-x}$, **hg** : $x \rightarrow 2e^{2x} + 7$, **fh** : $x \rightarrow -2x^2 - 3$, **gg** : $x \rightarrow e^{e^x}$,
 $x \in \mathbb{R}$ $x \in \mathbb{R}$ $x \in \mathbb{R}$ $x \in \mathbb{R}$
- e** = $g(2x^2 + 7)$ **f** = $f(4 - x)$ **g** = $f(e^x)$ **h** = $h(4 - x)$
= e^{2x^2+7} = $4 - (4 - x)$ = $4 - e^x$ = $2(4 - x)^2 + 7$
gh : $x \rightarrow e^{2x^2+7}$, **ff** : $x \rightarrow x$, **fg** : $x \rightarrow 4 - e^x$, **hf** : $x \rightarrow 2x^2 - 16x + 39$,
 $x \in \mathbb{R}$ $x \in \mathbb{R}$ $x \in \mathbb{R}$ $x \in \mathbb{R}$
- 5** **a** **ff**(x) = $f(5x - 3)$ **b** **hf**(x) = $h(5x - 3)$ **c** **gf**(x) = $g(5x - 3)$ **d** **hg**(x) = $h(3x^2 + 1)$
= $5(5x - 3) - 3$ = $\frac{1}{5x - 3 - 2}$ = $3(5x - 3)^2 + 1$ = $\frac{1}{3x^2 + 1 - 2}$
= $25x - 18$ = $\frac{1}{5x - 5}$ = $75x^2 - 90x + 28$ = $\frac{1}{3x^2 - 1}$
 $25x - 18 = -8$ $\frac{1}{5x - 5} = 2$ $75x^2 - 90x + 28 = 28$ $\frac{1}{3x^2 - 1} = \frac{1}{2}$
 $x = \frac{2}{5}$ $1 = 10x - 10$ $15x(5x - 6) = 0$ $3x^2 - 1 = 2$
 $x = \frac{11}{10}$ $x = \frac{6}{5}$ $x^2 = 1$
 $x = \pm 1$
- e** **fh**(x) = $f(\frac{1}{x-2})$ **f** **fg**(x) = $f(3x^2 + 1)$ **g** **gh**(x) = $g(\frac{1}{x-2})$ **h** **hh**(x) = $h(\frac{1}{x-2})$
= $\frac{5}{x-2} - 3$ = $5(3x^2 + 1) - 3$ = $\frac{3}{(x-2)^2} + 1$ = $\frac{1}{\frac{1}{x-2} - 2} = \frac{x-2}{1-2(x-2)}$
 $\frac{5}{x-2} - 3 = 7$ = $15x^2 + 2$ $\frac{3}{(x-2)^2} + 1 = 4$ = $\frac{x-2}{5-2x}$
 $\frac{5}{x-2} = 10$ $15x^2 + 2 = 32$ $(x-2)^2 = 1$ $\frac{x-2}{5-2x} = -2$
 $5 = 10x - 20$ $x^2 = 2$ $x - 2 = \pm 1$ $x - 2 = -10 + 4x$
 $x = \frac{5}{2}$ $x = \pm\sqrt{2}$ $x = 1, 3$ $x = \frac{8}{3}$

6	a	$gh(x) = g(e^x)$ $= 3 + 2e^x$ $3 + 2e^x = 9$ $e^x = 3$ $x = \ln 3$ $x = 1.10$	b	$fg(x) = f(3 + 2x)$ $= \ln(3 + 2x)$ $\ln(3 + 2x) = 3.6$ $3 + 2x = e^{3.6}$ $x = \frac{1}{2}(e^{3.6} - 3)$ $x = 16.80$	c	$hg(x) = h(3 + 2x)$ $= e^{3+2x}$ $e^{3+2x} = 4$ $3 + 2x = \ln 4$ $x = \frac{1}{2}(\ln 4 - 3)$ $x = -0.81$	d	$gf(x) = g(\ln x)$ $= 3 + 2 \ln x$ $3 + 2 \ln x = 10.4$ $\ln x = 3.7$ $x = e^{3.7}$ $x = 40.45$
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7 a $g(x) > 0$

b $fg(x) = f(e^x) = \frac{e^x + 1}{5}$

$$\frac{e^x + 1}{5} = 17$$

$$e^x + 1 = 85$$

$$e^x = 84$$

$$x = \ln 84 = 4.43 \text{ (3sf)}$$

8 a $= f(4) = 7$

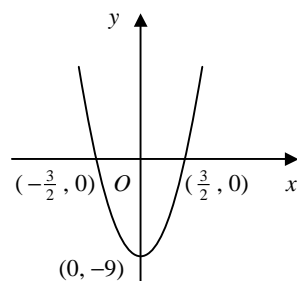
b $gf(x) = g(4x - 9) = (4x - 9)^2$

$$\therefore (4x - 9)^2 = 25$$

$$4x - 9 = \pm 5$$

$$x = \frac{1}{4}(9 \pm 5) = 1, \frac{7}{2}$$

c $fg(x) = f(x^2) = 4x^2 - 9$



9 a $= g(1) = 4$

b $= h(2) = e^3$

c $= g(e^{-3}) = 1$

d $= f(\tan 1) = 74.7 \text{ (3sf)}$

e $= h(\tan 0.2)$
 $= 0.552 \text{ (3sf)}$

f $= f(4 + \ln 7)$
 $= -0.351 \text{ (3sf)}$

g $= h(e^{-\frac{1}{2}})$
 $= 1.24 \text{ (3sf)}$

h $= f(4 + e)$
 $= 0.465 \text{ (3sf)}$

10 a $= f(4x + 1)$
 $= 3e^{4x+1} + 2$

b $= g(3e^x + 2)$
 $= 4(3e^x + 2) + 1$

c $= h(3e^x + 2)$
 $= \frac{1}{3e^x + 2 + 1}$

d $= g(4x + 1)$
 $= 4(4x + 1) + 1$

$fg : x \rightarrow 3e^{4x+1} + 2,$
 $x \in \mathbb{R}$

$gf : x \rightarrow 12e^x + 9,$
 $x \in \mathbb{R}$

$hf : x \rightarrow \frac{1}{3e^x + 3},$
 $x \in \mathbb{R}$

$gg : x \rightarrow 16x + 5,$
 $x \in \mathbb{R}$

e $= h(4x + 1)$
 $= \frac{1}{4x+1+1}$

f $= g\left(\frac{1}{x+1}\right)$
 $= \frac{4}{x+1} + 1$

g $= h\left(\frac{1}{x+1}\right)$
 $= \frac{1}{\frac{1}{x+1} + 1}$

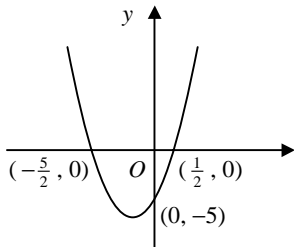
h $= g(16x + 5)$
 $= 4(16x + 5) + 1$

$hg : x \rightarrow \frac{1}{4x+2},$
 $x \in \mathbb{R}, x \neq -\frac{1}{2}$

$= \frac{4+x+1}{x+1}$
 $gh : x \rightarrow \frac{x+5}{x+1},$
 $x \in \mathbb{R}, x \neq -1$

$= \frac{x+1}{1+x+1}$
 $hh : x \rightarrow \frac{x+1}{x+2},$
 $x \in \mathbb{R}, x \neq -1, -2$

$ggg : x \rightarrow 64x + 21,$
 $x \in \mathbb{R}$

- 11 a** $fh(x) = f\left(\frac{x+1}{3}\right)$
 $= \sqrt{\frac{x+1}{3} + 4}$
 $= \sqrt{\frac{x+13}{3}}$
 $\sqrt{\frac{x+13}{3}} = 3$
 $\frac{x+13}{3} = 9$
 $x + 13 = 27$
 $x = 14$
- b** $fg(x) = f(e^{1+2x})$
 $= \sqrt{e^{1+2x} + 4}$
 $\sqrt{e^{1+2x} + 4} = 7$
 $e^{1+2x} + 4 = 49$
 $e^{1+2x} = 45$
 $1 + 2x = \ln 45$
 $x = \frac{1}{2}(\ln 45 - 1)$
 $x = 1.40$ (3sf)
- c** $gh(x) = g\left(\frac{x+1}{3}\right)$
 $= e^{1 + \frac{2(x+1)}{3}}$
 $= e^{\frac{2x+5}{3}}$
 $e^{\frac{2x+5}{3}} = 11$
 $\frac{2x+5}{3} = \ln 11$
 $2x + 5 = 3 \ln 11$
 $x = \frac{1}{2}(3 \ln 11 - 5)$
 $x = 1.10$ (3sf)
- d** $hh(x) = h\left(\frac{x+1}{3}\right)$
 $= \frac{\frac{x+1}{3} + 1}{3}$
 $= \frac{x+1+3}{9}$
 $= \frac{x+4}{9}$
 $\frac{x+4}{9} = \frac{2}{3}$
 $3x + 12 = 18$
 $x = 2$
- e** $hg(x) = h(e^{1+2x})$
 $= \frac{e^{1+2x} + 1}{3}$
 $\frac{e^{1+2x} + 1}{3} = 1.2$
 $e^{1+2x} = 2.6$
 $1 + 2x = \ln 2.6$
 $x = \frac{1}{2}(\ln 2.6 - 1)$
 $x = -0.0222$ (3sf)
- f** $hf(x) = h(\sqrt{x+4})$
 $= \frac{\sqrt{x+4} + 1}{3}$
 $\frac{\sqrt{x+4} + 1}{3} = \frac{1}{2}$
 $\sqrt{x+4} = \frac{1}{2}$
 $x + 4 = \frac{1}{4}$
 $x = -3\frac{3}{4}$
- g** $ff(x) = f(\sqrt{x+4})$
 $= \sqrt{\sqrt{x+4} + 4}$
 $\sqrt{\sqrt{x+4} + 4} = 3$
 $\sqrt{x+4} = 5$
 $x + 4 = 25$
 $x = 21$
- h** $ghh(x) = g\left(\frac{x+4}{9}\right)$
 $= e^{1 + \frac{2(x+4)}{9}}$
 $= e^{\frac{2x+17}{9}}$
 $e^{\frac{2x+17}{9}} = \frac{1}{2}$
 $\frac{2x+17}{9} = \ln \frac{1}{2}$
 $x = \frac{1}{2}(9 \ln \frac{1}{2} - 17)$
 $x = -11.6$ (3sf)
- 12 a** $h(x) = fg(x)$ **b** $h(x) = gf(x)$ **c** $h(x) = gg(x)$
d $h(x) = ff(x)$ **e** $h(x) = gff(x)$ **f** $h(x) = gfg(x)$
- 13 a** $j(x) = fg(x)$ **b** $j(x) = hf(x)$ **c** $j(x) = gh(x)$
d $j(x) = gg(x)$ **e** $j(x) = fhg(x)$ **f** $j(x) = hfg(x)$
- 14 a** $gf(x) = g(5^x - 7)$
 $= 2(5^x - 7) + 3$
 $= 2(5^x) - 11$
 $gf : x \rightarrow 2(5^x) - 11, x \in \mathbb{R}$
b $2(5^x) - 11 = 10$
 $5^x = \frac{21}{2}$
 $x = \frac{\ln \frac{21}{2}}{\ln 5} = 1.46$ (3sf)
- 15 a** $gf(x) = g[2(x+1)] = [2(x+1)]^2 - 9$
 $gf : x \rightarrow 4x^2 + 8x - 5, x \in \mathbb{R}$
range: $gf(x) \geq -9$
b $gf(x) = (2x+5)(2x-1)$
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- c** $gf(x) - 2f(x) = a$
 $4x^2 + 8x - 5 - 2[2(x+1)] = a$
 $4x^2 + 4x - (a+9) = 0$
no real roots $\therefore b^2 - 4ac < 0$
 $16 + 16(a+9) < 0$
 $1 + a + 9 < 0$
 $a < -10$