FUNCTIONS

1 $\mathrm{f}: x \to 4x - 3,\ x \in \mathbb{R}$ $\mathrm{g}: x \to 2 - x,\ x \in \mathbb{R}$ $\mathrm{h}: x \to x^2 + 5,\ x \in \mathbb{R}$

Evaluate

a gf(2) **b** gh(1) **c** fg(–3) **d** hf(3)

e gg(5) **f** ff($\frac{1}{2}$) **g** hg(8) **h** fh($1\frac{1}{2}$)

2 $\mathrm{f}: x \to 5x + 2,\ x \in \mathbb{R}$ $\mathrm{g}: x \to \cos x,\ x \in \mathbb{R}$ $\mathrm{h}: x \to \ln x,\ x \in \mathbb{R},\ x > 0$

Evaluate, giving your answers to 3 significant figures

a fh(20) **b** gh(3) **c** fg(5) **d** gg(–4)

e gf($1\frac{3}{4}$) **f** hg(6.7) **g** hh(50) **h** hf(–0.3)

3 $\mathrm{f}: x \to 2x + 1,\ x \in \mathbb{R}$ $\mathrm{g}: x \to 1 - 3x,\ x \in \mathbb{R}$ $\mathrm{h}: x \to x^2 + 4,\ x \in \mathbb{R}$

Given the functions f, g and h, express the following composite functions in a similar form.

a fg **b** ff **c** fh **d** hf

e gh **f** gg **g** hg **h** gf

4 $\mathrm{f}: x \to 4 - x,\ x \in \mathbb{R}$ $\mathrm{g}: x \to \mathrm{e}^x,\ x \in \mathbb{R}$ $\mathrm{h}: x \to 2x^2 + 7,\ x \in \mathbb{R}$

Given the functions f, g and h, express the following composite functions in a similar form.

a gf **b** hg **c** fh **d** gg

e gh **f** ff **g** fg **h** hf

5 $\mathrm{f}: x \to 5x - 3,\ x \in \mathbb{R}$ $\mathrm{g}: x \to 3x^2 + 1,\ x \in \mathbb{R}$ $\mathrm{h}: x \to \frac{1}{x-2},\ x \in \mathbb{R},\ x \neq 2$

Solve

a ff(x) = –8 **b** hf(x) = 2 **c** gf(x) = 28 **d** hg(x) = $\frac{1}{2}$

e fh(x) = 7 **f** fg(x) = 32 **g** gh(x) = 4 **h** hh(x) = –2

6 $\mathrm{f}: x \to \ln x,\ x \in \mathbb{R},\ x > 0$ $\mathrm{g}: x \to 3 + 2x,\ x \in \mathbb{R}$ $\mathrm{h}: x \to \mathrm{e}^x,\ x \in \mathbb{R}$

Solve, giving your answers to 2 decimal places,

a gh(x) = 9 **b** fg(x) = 3.6 **c** hg(x) = 4 **d** gf(x) = 10.4

7 The functions f and g are defined by

$$\mathrm{f}: x \to \frac{x+1}{5},\ x \in \mathbb{R} \qquad \mathrm{g}: x \to \mathrm{e}^x,\ x \in \mathbb{R}$$

a State the range of g.

b Solve fg(x) = 17.

8 The functions f and g are defined by

$$\mathrm{f}(x) \equiv 4x - 9,\ x \in \mathbb{R} \qquad \mathrm{g}(x) \equiv x^2,\ x \in \mathbb{R}$$

a Evaluate ff($3\frac{1}{4}$).

b Solve gf(x) = 25.

c Sketch the graph of y = fg(x), showing the coordinates of any points of intersection with the coordinate axes.

9 $\mathrm{f} : x \rightarrow \tan x,\ x \in \mathbb{R}$ $\qquad$ $\mathrm{g} : x \rightarrow 4 + \ln x,\ x \in \mathbb{R}^+$ $\qquad$ $\mathrm{h} : x \rightarrow \mathrm{e}^{2x-1},\ x \in \mathbb{R}$

Evaluate

a $\mathrm{gf}(\frac{\pi}{4})$ **b** $\mathrm{hg}(\mathrm{e}^{-2})$ **c** $\mathrm{gh}(-1)$ **d** $\mathrm{ff}(1)$

e $\mathrm{hf}(0.2)$ **f** $\mathrm{fg}(7)$ **g** $\mathrm{hh}(\frac{1}{4})$ **h** $\mathrm{fg}(\mathrm{e}^{\mathrm{e}})$

10 $\mathrm{f} : x \rightarrow 3\mathrm{e}^x + 2,\ x \in \mathbb{R}$ $\qquad$ $\mathrm{g} : x \rightarrow 4x + 1,\ x \in \mathbb{R}$ $\qquad$ $\mathrm{h} : x \rightarrow \frac{1}{x+1},\ x \in \mathbb{R},\ x \neq -1$

Express the following composite functions in a similar form, stating the domain in each case.

a fg **b** gf **c** hf **d** gg

e hg **f** gh **g** hh **h** ggg

11 $\mathrm{f} : x \rightarrow \sqrt{x+4},\ x \in \mathbb{R},\ x > -4$ $\qquad$ $\mathrm{g} : x \rightarrow \mathrm{e}^{1+2x},\ x \in \mathbb{R}$ $\qquad$ $\mathrm{h} : x \rightarrow \frac{x+1}{3},\ x \in \mathbb{R}$

Solve

a $\mathrm{fh}(x) = 3$ **b** $\mathrm{fg}(x) = 7$ **c** $\mathrm{gh}(x) = 11$ **d** $\mathrm{hh}(x) = \frac{2}{3}$

e $\mathrm{hg}(x) = 1.2$ **f** $\mathrm{hf}(x) = \frac{1}{2}$ **g** $\mathrm{ff}(x) = 3$ **h** $\mathrm{ghh}(x) = \frac{1}{2}$

12 $\mathrm{f}(x) \equiv x^3,\ x \in \mathbb{R}$ $\qquad$ $\mathrm{g}(x) \equiv x + 2,\ x \in \mathbb{R}$

Find the composition of the functions f and g that corresponds to the function h, where

a $\mathrm{h}(x) \equiv (x+2)^3,\ x \in \mathbb{R}$ **b** $\mathrm{h}(x) \equiv x^3 + 2,\ x \in \mathbb{R}$ **c** $\mathrm{h}(x) \equiv x + 4,\ x \in \mathbb{R}$

d $\mathrm{h}(x) \equiv x^9,\ x \in \mathbb{R}$ **e** $\mathrm{h}(x) \equiv x^9 + 2,\ x \in \mathbb{R}$ **f** $\mathrm{h}(x) \equiv (x+2)^3 + 2,\ x \in \mathbb{R}$

13 $\mathrm{f}(x) \equiv x - 4,\ x \in \mathbb{R}$ $\qquad$ $\mathrm{g}(x) \equiv 3x^2,\ x \in \mathbb{R}$ $\qquad$ $\mathrm{h}(x) \equiv \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

Find the composition of the functions f, g and h that corresponds to the function j, where

a $\mathrm{j}(x) \equiv 3x^2 - 4,\ x \in \mathbb{R}$ **b** $\mathrm{j}(x) \equiv \frac{1}{x-4},\ x \in \mathbb{R},\ x \neq 4$

c $\mathrm{j}(x) \equiv \frac{3}{x^2},\ x \in \mathbb{R},\ x \neq 0$ **d** $\mathrm{j}(x) \equiv 27x^4,\ x \in \mathbb{R}$

e $\mathrm{j}(x) \equiv \frac{1}{3x^2} - 4,\ x \in \mathbb{R},\ x \neq 0$ **f** $\mathrm{j}(x) \equiv \frac{1}{3x^2 - 4},\ x \in \mathbb{R},\ x \neq \pm\frac{2}{\sqrt{3}}$

14 The functions f and g are defined by

$$\mathrm{f} : x \rightarrow 5^x - 7,\ x \in \mathbb{R} \qquad \mathrm{g} : x \rightarrow 2x + 3,\ x \in \mathbb{R}$$

a Find and simplify an expression for gf, stating its domain.

b Solve the equation $\mathrm{gf}(x) = 10$.

15 The functions f and g are defined by

$$\mathrm{f} : x \rightarrow 2(x+1),\ x \in \mathbb{R} \qquad \mathrm{g} : x \rightarrow x^2 - 9,\ x \in \mathbb{R}$$

a Express gf in terms of x and state its domain and range.

b Sketch the graph of $y = \mathrm{gf}(x)$, showing the coordinates of any points of intersection with the coordinate axes.

The equation $\mathrm{gf}(x) - 2\mathrm{f}(x) = a$, where a is a constant, has no real roots.

c Show that $a < -10$.