RATIONAL EXPRESSIONS — *Answers*

1 **a**

$$\begin{array}{r} 3x^2 + 2x - 1 \\ x-4 \overline{)\, 3x^3 - 10x^2 - 9x + 15} \\ \underline{3x^3 - 12x^2} \\ 2x^2 - 9x \\ \underline{2x^2 - 8x} \\ -x + 15 \\ \underline{-x + 4} \\ 11 \end{array}$$

quotient: $3x^2 + 2x - 1$, remainder: 11

b

$$\begin{array}{r} x^2 - 5x - 3 \\ 2x-1 \overline{)\, 2x^3 - 11x^2 - x + 3} \\ \underline{2x^3 - x^2} \\ -10x^2 - x \\ \underline{-10x^2 + 5x} \\ -6x + 3 \\ \underline{-6x + 3} \\ 0 \end{array}$$

quotient: $x^2 - 5x - 3$, remainder: 0

c

$$\begin{array}{r} 2x^2 - x + 6 \\ 2x+5 \overline{)\, 4x^3 + 8x^2 + 7x + 32} \\ \underline{4x^3 + 10x^2} \\ -2x^2 + 7x \\ \underline{-2x^2 - 5x} \\ 12x + 32 \\ \underline{12x + 30} \\ 2 \end{array}$$

quotient: $2x^2 - x + 6$, remainder: 2

d

$$\begin{array}{r} -2x^2 - 6x + 4 \\ 3x+2 \overline{)\, -6x^3 - 22x^2 + 0x + 1} \\ \underline{-6x^3 - 4x^2} \\ -18x^2 + 0x \\ \underline{-18x^2 - 12x} \\ 12x + 1 \\ \underline{12x + 8} \\ -7 \end{array}$$

quotient: $-2x^2 - 6x + 4$, remainder -7

2 **a** let $f(x) = x^3 + 4x^2 + x - 6$

$f(-2) = -8 + 16 - 2 - 6 = 0$

$\therefore$ $(x + 2)$ is a factor

b

$$\begin{array}{r} x^2 + 2x - 3 \\ x+2 \overline{)\, x^3 + 4x^2 + x - 6} \\ \underline{x^3 + 2x^2} \\ 2x^2 + x \\ \underline{2x^2 + 4x} \\ -3x - 6 \\ \underline{-3x - 6} \\ 0 \end{array}$$

$\therefore$ $x^3 + 4x^2 + x - 6$

$= (x + 2)(x^2 + 2x - 3)$

$= (x + 2)(x + 3)(x - 1)$

c $= \dfrac{(x+2)(x+3)(x-1)}{(x+3)(x-3)}$

$= \dfrac{(x+2)(x-1)}{x-3}$

3 **a** let $f(x) = 2x^3 - 5x^2 + 13x - 15$

$f(\frac{3}{2}) = \frac{27}{4} - \frac{45}{4} + \frac{39}{2} - 15 = 0$

$\therefore$ $(2x - 3)$ is a factor

b

$$\begin{array}{r} x^2 - x + 5 \\ 2x-3 \overline{)\ 2x^3 - 5x^2 + 13x - 15} \\ \underline{2x^3 - 3x^2} \\ -2x^2 + 13x \\ \underline{-2x^2 + 3x} \\ 10x - 15 \\ \underline{10x - 15} \end{array}$$

$\therefore$ $2x^3 - 5x^2 + 13x - 15$

$= (2x - 3)(x^2 - x + 5)$

$\therefore$ $\dfrac{2x^3 - 5x^2 + 13x - 15}{2x^2 - 7x + 6}$

$= \dfrac{(2x-3)(x^2 - x + 5)}{(2x-3)(x-2)}$

$= \dfrac{x^2 - x + 5}{x - 2}$

4 **a** $x - 1$

b

$$\begin{array}{r} x^2 + x + 1 \\ x-1 \overline{)\ x^3 + 0x^2 + 0x - 1} \\ \underline{x^3 - x^2} \\ x^2 + 0x \\ \underline{x^2 - x} \\ x - 1 \\ \underline{x - 1} \end{array}$$

$\therefore$ $x^3 - 1 = (x - 1)(x^2 + x + 1)$

$\therefore$ $\dfrac{x^3 - 1}{x^2 + x - 2} = \dfrac{(x-1)(x^2 + x + 1)}{(x-1)(x+2)}$

$= \dfrac{x^2 + x + 1}{x + 2}$

5 $\dfrac{2x+5}{x+3} = \dfrac{2(x+3)-1}{x+3} = 2 - \dfrac{1}{x+3}$

$\therefore$ $A = 2$, $B = -1$

6 **a** $= \dfrac{(x+1)+1}{x+1} = 1 + \dfrac{1}{x+1}$

b $= \dfrac{(x-2)+5}{x-2} = 1 + \dfrac{5}{x-2}$

c $= \dfrac{-(1-x)+1}{1-x} = -1 + \dfrac{1}{1-x}$

d $= \dfrac{2(x+2)-3}{x+2} = 2 - \dfrac{3}{x+2}$

e $= \dfrac{\frac{1}{2}(2x-1)-\frac{1}{2}}{2x-1} = \frac{1}{2} - \dfrac{1}{2(2x-1)}$

f $= \dfrac{-2(3+2x)+7}{3+2x} = -2 + \dfrac{7}{3+2x}$

7 **a**

$$\begin{array}{r} 1 \\ x^2+x+2 \overline{)\ x^2+3x+5} \\ \underline{x^2+x+2} \\ 2x+3 \end{array}$$

quotient: 1, remainder: $2x + 3$

b

$$\begin{array}{r} 2 \\ x^2-x-4 \overline{)\ 2x^2+3x-8} \\ \underline{2x^2-2x-8} \\ 5x \end{array}$$

quotient: 2, remainder: $5x$

c

$$\begin{array}{r} 1 \\ x^2+3x-1 \overline{)\ x^2+0x+7} \\ \underline{x^2+3x-1} \\ -3x+8 \end{array}$$

quotient: 1, remainder: $-3x + 8$

d

$$\begin{array}{r} 3 \\ x^2+2 \overline{)\ 3x^2-x-4} \\ \underline{3x^2+0x+6} \\ -x-10 \end{array}$$

quotient: 3, remainder: $-x - 10$

e

$$\begin{array}{r} x-3 \\ x^2+x-2 \overline{)\ x^3-2x^2-5x+8} \\ \underline{x^3+x^2-2x} \\ -3x^2-3x+8 \\ \underline{-3x^2-3x+6} \\ 2 \end{array}$$

quotient: $x - 3$, remainder: 2

f

$$\begin{array}{r} 2x+3 \\ x^2-5x+1 \overline{)\ 2x^3-7x^2+0x+1} \\ \underline{2x^3-10x^2+2x} \\ 3x^2-2x+1 \\ \underline{3x^2-15x+3} \\ 13x-2 \end{array}$$

quotient: $2x + 3$, remainder: $13x - 2$

g

$$\begin{array}{r} x+2 \\ 3x^2+4 \overline{)\ 3x^3+6x^2-2x+5} \\ \underline{3x^3+0x^2+4x} \\ 6x^2-6x+5 \\ \underline{6x^2+0x+8} \\ -6x-3 \end{array}$$

quotient: $x + 2$, remainder: $-6x - 3$

h

$$\begin{array}{r} 3x+7 \\ 2x^2-5x-2 \overline{)\ 6x^3-x^2-44x-6} \\ \underline{6x^3-15x^2-6x} \\ 14x^2-38x-6 \\ \underline{14x^2-35x-14} \\ -3x+8 \end{array}$$

quotient: $3x + 7$, remainder: $-3x + 8$

8 **a**

$$\begin{array}{r} x+2 \\ x^2+3x-4 \overline{)\ x^3+5x^2+7x-13} \\ \underline{x^3+3x^2-4x} \\ 2x^2+11x-13 \\ \underline{2x^2+6x-8} \\ 5x-5 \end{array}$$

$\therefore$ $= x + 2$ remainder $5x - 5$

b $\text{LHS} = x + 2 + \dfrac{5x-5}{x^2+3x-4}$

$= x + 2 + \dfrac{5(x-1)}{(x-1)(x+4)}$

$= x + 2 + \dfrac{5}{x+4}$

9 **a**

$$\begin{array}{r} x-4 \\ x^2+2x-15 \overline{)\ x^3-2x^2-21x+70} \\ \underline{x^3+2x^2-15x} \\ -4x^2-6x+70 \\ \underline{-4x^2-8x+60} \\ 2x+10 \end{array}$$

$\therefore$ $f(x) = x - 4 + \dfrac{2x+10}{x^2+2x-15}$

$= x - 4 + \dfrac{2(x+5)}{(x+5)(x-3)}$

$= x - 4 + \dfrac{2}{x-3}$

b $\dfrac{3x-7}{x-3} = \dfrac{3(x-3)+2}{x-3} = 3 + \dfrac{2}{x-3}$

$\therefore$ $f(x) = \dfrac{3x-7}{x-3}$

$\Rightarrow \quad x - 4 + \dfrac{2}{x-3} = 3 + \dfrac{2}{x-3}$

$x - 4 = 3$

$x = 7$