

RATIONAL EXPRESSIONS

1 Find the quotient and remainder obtained in dividing

a $(3x^3 - 10x^2 - 9x + 15)$ by $(x - 4)$

b $(2x^3 - 11x^2 - x + 3)$ by $(2x - 1)$

c $(4x^3 + 8x^2 + 7x + 32)$ by $(2x + 5)$

d $(1 - 22x^2 - 6x^3)$ by $(3x + 2)$

2 **a** Show that $(x + 2)$ is a factor of $(x^3 + 4x^2 + x - 6)$.

b Fully factorise $x^3 + 4x^2 + x - 6$.

c Simplify $\frac{x^3 + 4x^2 + x - 6}{x^2 - 9}$.

3 **a** Show that $(2x - 3)$ is a factor of $(2x^3 - 5x^2 + 13x - 15)$.

b Simplify $\frac{2x^3 - 5x^2 + 13x - 15}{2x^2 - 7x + 6}$.

4 **a** State a linear factor of $x^3 - 1$.

b Simplify $\frac{x^3 - 1}{x^2 + x - 2}$.

5 Find the integers A and B such that

$$\frac{2x+5}{x+3} \equiv A + \frac{B}{x+3}.$$

6 Express each of the following in the form $A + \frac{B}{f(x)}$, where $f(x)$ is linear.

a $\frac{x+2}{x+1}$

b $\frac{x+3}{x-2}$

c $\frac{x}{1-x}$

d $\frac{2x+1}{x+2}$

e $\frac{x-1}{2x-1}$

f $\frac{1-4x}{3+2x}$

7 Find the quotient and remainder obtained in dividing

a $(x^2 + 3x + 5)$ by $(x^2 + x + 2)$

b $(2x^2 + 3x - 8)$ by $(x^2 - x - 4)$

c $(x^2 + 7)$ by $(x^2 + 3x - 1)$

d $(3x^2 - x - 4)$ by $(x^2 + 2)$

e $(x^3 - 2x^2 - 5x + 8)$ by $(x^2 + x - 2)$

f $(2x^3 - 7x^2 + 1)$ by $(x^2 - 5x + 1)$

g $(3x^3 + 6x^2 - 2x + 5)$ by $(3x^2 + 4)$

h $(6x^3 - x^2 - 44x - 6)$ by $(2x^2 - 5x - 2)$

8 **a** Divide $(x^3 + 5x^2 + 7x - 13)$ by $(x^2 + 3x - 4)$.

b Hence show that

$$\frac{x^3 + 5x^2 + 7x - 13}{x^2 + 3x - 4} \equiv x + 2 + \frac{5}{x + 4}.$$

9 $f(x) = \frac{x^3 - 2x^2 - 21x + 70}{x^2 + 2x - 15}$, $x \neq 3$.

a Express $f(x)$ in the form $Ax + B + \frac{C}{g(x)}$, where $g(x)$ is linear.

b Hence, or otherwise, solve the equation $f(x) = \frac{3x-7}{x-3}$.