RATIONAL EXPRESSIONS   *Answers*

1   **a** $= \dfrac{3x-1}{6(3x-1)} = \dfrac{1}{6}$ **b** $= \dfrac{3(2x+5)}{4(2x+5)} = \dfrac{3}{4}$ **c** $= \dfrac{3(y+1)}{(y+6)(y+1)}$ **d** $= \dfrac{(x+5)(x-5)}{(x-2)(x-5)}$

                            $= \dfrac{3}{y+6}$ $= \dfrac{x+5}{x-2}$

   e $= \dfrac{(a+2)(a-3)}{(a+6)(a-3)}$ **f** $= \dfrac{x(x+3)}{(2x-1)(x+3)}$ **g** $= \dfrac{(3t+1)(t-4)}{(t+4)(t-4)}$ **h** $= \dfrac{(3x-2)(2x-3)}{(4x+3)(3x-2)}$

       $= \dfrac{a+2}{a+6}$ $= \dfrac{x}{2x-1}$ $= \dfrac{3t+1}{t+4}$ $= \dfrac{2x-3}{4x+3}$

2   **a** $= \dfrac{3x^2}{9(x-1)} \times \dfrac{4(x-1)}{2x} = \dfrac{2x}{3}$ **b** $= \dfrac{(x+6)(x-6)}{(x+2)(x+5)} \times \dfrac{x+2}{x-6} = \dfrac{x+6}{x+5}$

   c $= \dfrac{n(n+2)}{(n+4)(n+2)} \times \dfrac{n+4}{n^2} = \dfrac{1}{n}$ **d** $= \dfrac{4(x-3)}{(x+2)(x-2)} \times \dfrac{x(x+2)}{(x+1)(x-3)} = \dfrac{4x}{(x-2)(x+1)}$

   e $= \dfrac{4y^2}{y(2y+1)} \times \dfrac{(2y+1)(y+5)}{(y+5)(y-3)} = \dfrac{4y}{y-3}$ **f** $= \dfrac{(x+1)(x-1)}{(2x-1)(x+4)} \times \dfrac{(3x-1)(2x-1)}{(3x-1)(x-1)} = \dfrac{x+1}{x+4}$

   g $= \dfrac{10(x-1)}{5(x+3)} \times \dfrac{(x+3)(x+4)}{(4+x)(1-x)} = -2$ **h** $= \dfrac{a^2(a-3)}{4a(2a-1)} \times \dfrac{(2a-1)(a+3)}{(a+3)(a-3)} = \dfrac{a}{4}$

3   **a** $= \dfrac{2(y+4)+7y}{y(y+4)}$ **b** $= \dfrac{2x(x+3)-(x-5)}{(x-5)(x+3)}$ **c** $= \dfrac{7-3x^2}{x(x+2)}$

      $= \dfrac{9y+8}{y(y+4)}$ $= \dfrac{2x^2+5x+5}{(x-5)(x+3)}$

   d $= \dfrac{2x+5(x-3)}{2(x-3)(x-1)}$ **e** $= \dfrac{2}{q(q+3)} + \dfrac{5q}{4(q+3)}$ **f** $= \dfrac{4}{3(x-1)} + \dfrac{x+2}{x(x-1)}$

      $= \dfrac{7x-15}{2(x-3)(x-1)}$ $= \dfrac{8+5q^2}{4q(q+3)}$ $= \dfrac{4x+3(x+2)}{3x(x-1)} = \dfrac{7x+6}{3x(x-1)}$

   g $= \dfrac{4}{x+5} + \dfrac{x}{(x+3)(x+5)}$ **h** $= \dfrac{6x}{(x+2)(x-2)} - \dfrac{3}{x+2}$ **i** $= \dfrac{5t+12}{(2t+1)(t+3)} - \dfrac{4}{2t+1}$

      $= \dfrac{4(x+3)+x}{(x+3)(x+5)}$ $= \dfrac{6x-3(x-2)}{(x+2)(x-2)}$ $= \dfrac{5t+12-4(t+3)}{(2t+1)(t+3)}$

      $= \dfrac{5x+12}{(x+3)(x+5)}$ $= \dfrac{3x+6}{(x+2)(x-2)}$ $= \dfrac{t}{(2t+1)(t+3)}$

                                  $= \dfrac{3(x+2)}{(x+2)(x-2)} = \dfrac{3}{x-2}$

4   **a** $= \dfrac{x(x-5)}{6(x-5)} = \dfrac{x}{6}$ **b** $= \dfrac{(4+x)(4-x)}{(x+4)(x-2)}$ **c** $= \dfrac{2(x-3)(x+1)}{3(x-1)(x-3)}$ **d** $= \dfrac{x(x+1)(x-1)}{(2x+1)(x-1)}$

                       $= \dfrac{4-x}{x-2}$ $= \dfrac{2(x+1)}{3(x-1)}$ $= \dfrac{x(x+1)}{2x+1}$

   e $= \dfrac{x(3-x)}{2(x+3)(x-3)}$ **f** $= \dfrac{x(x+2)(x-1)}{(3x-2)(x+2)}$ **g** $= \dfrac{(2-x)(1+3x)}{(2x+5)(x-2)}$ **h** $= \dfrac{(x^2-1)(x^2-4)}{(x+1)(x-2)}$

      $= -\dfrac{x}{2(x+3)}$ $= \dfrac{x(x-1)}{3x-2}$ $= -\dfrac{3x+1}{2x+5}$ $= \dfrac{(x+1)(x-1)(x+2)(x-2)}{(x+1)(x-2)}$

                                               $= (x-1)(x+2)$

5 **a** $= \dfrac{10(x+1)(x-1)}{5(x+2)} \times \dfrac{(x+2)(x+4)}{(x+1)(x+4)} = 2(x-1)$ **b** $= \dfrac{t(t-2)}{(2t+3)(t-2)} \times \dfrac{(3t+2)(2t+3)}{(3t+2)(3t-2)} = \dfrac{t}{3t-2}$

c $= \dfrac{2(x+1)(x+5)}{(4x-3)(x-1)} \times \dfrac{x(4x-3)}{4x(x+5)} = \dfrac{x+1}{2(x-1)}$ **d** $= \dfrac{(4x-3)(2x+3)}{(2x+3)^2} \times \dfrac{x(2x+3)}{2(3-4x)} = -\dfrac{x}{2}$

e $= \dfrac{(x^2+1)(x^2+5)}{(x+3)(x-3)} \times \dfrac{2x(x-3)}{4(x^2+1)} = \dfrac{x(x^2+5)}{2(x+3)}$ **f** $= \dfrac{(y^2+4)(y+2)(y-2)}{(5y-1)(y+2)} \times \dfrac{(5y-1)^2}{y^2+4} = (y-2)(5y-1)$

6 **a** $= \dfrac{5}{(x+1)(x-1)} - \dfrac{1}{2(x+1)}$

$= \dfrac{10-(x-1)}{2(x+1)(x-1)}$

$= \dfrac{11-x}{2(x+1)(x-1)}$

b $= \dfrac{3x}{(x+2)(x-2)} - \dfrac{4}{(2x-1)(x+2)}$

$= \dfrac{3x(2x-1)-4(x-2)}{(x+2)(x-2)(2x-1)}$

$= \dfrac{6x^2-7x+8}{(x+2)(x-2)(2x-1)}$

c $= \dfrac{4}{(x+3)(x-1)} + \dfrac{1}{(x-1)(x-2)}$

$= \dfrac{4(x-2)+(x+3)}{(x+3)(x-1)(x-2)}$

$= \dfrac{5x-5}{(x+3)(x-1)(x-2)}$

$= \dfrac{5}{(x+3)(x-2)}$

d $= \dfrac{x+1}{(x+5)(x-5)} + \dfrac{2}{x(x+5)}$

$= \dfrac{x(x+1)+2(x-5)}{x(x+5)(x-5)}$

$= \dfrac{x^2+3x-10}{x(x+5)(x-5)}$

$= \dfrac{(x+5)(x-2)}{x(x+5)(x-5)}$

$= \dfrac{x-2}{x(x-5)}$

e $= \dfrac{2x-1}{(x+2)^2} + \dfrac{x}{3(x+2)}$

$= \dfrac{3(2x-1)+x(x+2)}{3(x+2)^2}$

$= \dfrac{x^2+8x-3}{3(x+2)^2}$

f $= \dfrac{1}{x-3} + \dfrac{3}{x(x-3)} + \dfrac{x}{(x-3)^2}$

$= \dfrac{x(x-3)+3(x-3)+x^2}{x(x-3)^2}$

$= \dfrac{2x^2-9}{x(x-3)^2}$

g $= \dfrac{x}{(x+2)(x-3)} + \dfrac{2}{(2x-1)(x+2)}$

$= \dfrac{x(2x-1)+2(x-3)}{(x+2)(x-3)(2x-1)}$

$= \dfrac{2x^2+x-6}{(x+2)(x-3)(2x-1)}$

$= \dfrac{(2x-3)(x+2)}{(x+2)(x-3)(2x-1)}$

$= \dfrac{2x-3}{(x-3)(2x-1)}$

h $= \dfrac{1}{x^2} - \dfrac{1}{x(3x-2)} + \dfrac{3}{2(3x-2)}$

$= \dfrac{2(3x-2)-2x+3x^2}{2x^2(3x-2)}$

$= \dfrac{3x^2+4x-4}{2x^2(3x-2)}$

$= \dfrac{(3x-2)(x+2)}{2x^2(3x-2)}$

$= \dfrac{x+2}{2x^2}$

7 **a** $x(2x - 5) - 2(2x - 5) = 3x$

$x^2 - 6x + 5 = 0$
$(x - 1)(x - 5) = 0$
$x = 1, 5$

b $\dfrac{2}{(x+1)(x-1)} + \dfrac{3}{x+1} = 1$

$2 + 3(x - 1) = x^2 - 1$
$x^2 - 3x = 0$
$x(x - 3) = 0$
$x = 0, 3$

c $\dfrac{20}{(2x+1)(x+2)} + 1 = \dfrac{10}{2x+1}$

$20 + 2x^2 + 5x + 2 = 10(x + 2)$
$2x^2 - 5x + 2 = 0$
$(2x - 1)(x - 2) = 0$
$x = \frac{1}{2}, 2$

d $2y(y + 3) - y(y + 5) = 2(y + 5)(2y - 1)$

$2y^2 + 6y - y^2 - 5y = 4y^2 + 18y - 10$
$3y^2 + 17y - 10 = 0$
$y = \dfrac{-17 \pm \sqrt{289+120}}{6} = \dfrac{-17 \pm \sqrt{409}}{6}$
$y = -6.20, 0.537 \text{ (3sf)}$

e $5 + \dfrac{1}{(x+3)(x+2)} = \dfrac{11}{x+3}$

$5(x^2 + 5x + 6) + 1 = 11(x + 2)$
$5x^2 + 14x + 9 = 0$
$(5x + 9)(x + 1) = 0$
$x = -\frac{9}{5}, -1$

f $\dfrac{3}{(1-2x)^2} - \dfrac{10}{(1+2x)(1-2x)} = \dfrac{5}{1+2x}$

$3(1 + 2x) - 10(1 - 2x) = 5(1 - 4x + 4x^2)$
$10x^2 - 23x + 6 = 0$
$(10x - 3)(x - 2) = 0$
$x = \frac{3}{10}, 2$

8 $f(x) = \dfrac{7x - 15}{x(x-5)} - \dfrac{4}{x-5}$

$= \dfrac{7x - 15 - 4x}{x(x-5)} = \dfrac{3x - 15}{x(x-5)}$

$= \dfrac{3(x-5)}{x(x-5)} = \dfrac{3}{x}$ $[k = 3]$

9 $f(x) = \dfrac{x - 5}{(3x-1)(x+2)} + \dfrac{2}{3x-1}$

$= \dfrac{x - 5 + 2(x+2)}{(3x-1)(x+2)} = \dfrac{3x - 1}{(3x-1)(x+2)}$

$= \dfrac{1}{x+2}$

10 $f(x) - \dfrac{1}{f(x)} = \dfrac{x+2}{x-2} - \dfrac{x-2}{x+2}$

$= \dfrac{(x+2)^2 - (x-2)^2}{(x-2)(x+2)}$

$= \dfrac{x^2 + 4x + 4 - (x^2 - 4x + 4)}{x^2 - 4}$

$= \dfrac{8x}{x^2 - 4}$

11 **a** $= \dfrac{2(x+2) + 3}{(x+2)(x+5)} = \dfrac{2x + 7}{(x+2)(x+5)}$

b $\dfrac{2x + 7}{(x+2)(x+5)} = \dfrac{1}{3}$

$3(2x + 7) = (x + 2)(x + 5)$

$x^2 + x - 11 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 44}}{2} = \dfrac{-1 \pm \sqrt{45}}{2} = -3.85, 2.85$

12 $\dfrac{3}{2(2x+1)} - \dfrac{5}{(2x+1)^2} = 2$

$3(2x + 1) - 10 = 4(4x^2 + 4x + 1)$

$16x^2 + 10x + 11 = 0$

$b^2 - 4ac = 100 - 704 = -604$

$b^2 - 4ac < 0 \ \therefore \text{ no real roots}$

13 $= \dfrac{6x - (x+5)}{x(x+5)} \div \dfrac{x-1}{(x+5)(x-5)}$

$= \dfrac{5x - 5}{x(x+5)} \times \dfrac{(x+5)(x-5)}{x-1}$

$= \dfrac{5(x-1)}{x(x+5)} \times \dfrac{(x+5)(x-5)}{x-1}$

$= \dfrac{5(x-5)}{x}$