FUNCTIONS   *Answers*

1   **a** 4   **b** 2   **c** 11   **d** −2   **e** −4   **f** −2

   g $\frac{2}{5}$ **h** −3   **i** $\frac{5}{4}$ **j** −8   **k** −4   **l** $\frac{12}{13}$

2   **a** $= \sin \pi$ **b** $= \ln 2$ **c** $= 5$ **d** $= \sin \frac{2\pi}{3}$ **e** $= 3 + 2e^{-1}$ **f** $= \ln \frac{9}{2}$

   $= 0$ $= 0.693$ $= \frac{\sqrt{3}}{2}$ or 0.866 $= 3.74$ $= 1.50$

   g $= 3 + 2e^{1.8}$ **h** $= \ln 1$ **i** $= \sin(0.6 + \frac{\pi}{3})$ **j** $= 3 + 2e^{\frac{1}{3}}$ **k** $= \sin(\frac{\pi}{3} - 2)$ **l** $= \ln \frac{23}{4}$

   $= 15.1$ $= 0$ $= 0.997$ $= 5.79$ $= -0.815$ $= 1.75$

3   **a**
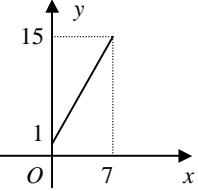
range:
$1 \leq f(x) \leq 15$

   b

range:
$f(x) \geq -2$

   c
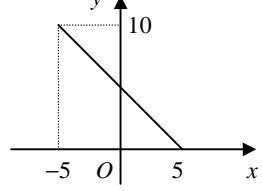
range:
$0 \leq f(x) \leq 10$

   d
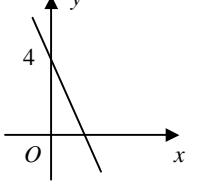
range:
$f(x) \in \mathbb{R}$

   e
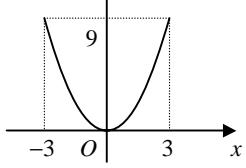
range:
$0 \leq f(x) < 9$

   f
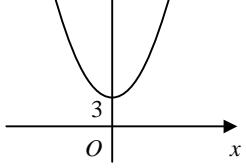
range:
$f(x) \geq 3$

   g
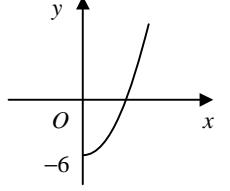
range:
$f(x) \geq -6$

   h
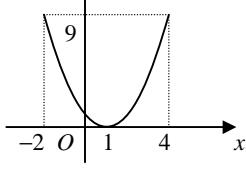
range:
$0 \leq f(x) \leq 9$

   i
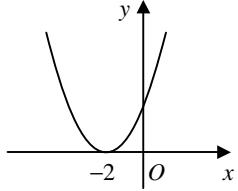
range:
$f(x) \geq 0$

   j
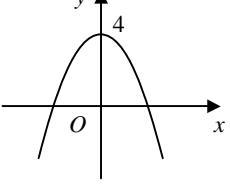
range:
$f(x) \leq 4$

   k
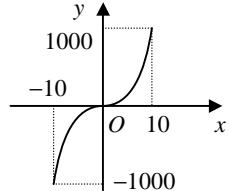
range:
$-1000 < f(x) \leq 1000$

   l
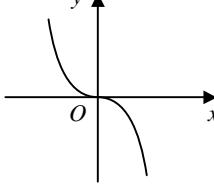
range:
$f(x) \in \mathbb{R}$

4   **a** $f(x) = (x + 1)^2 - 9$ $\therefore$ $(-1, -9)$ **b**
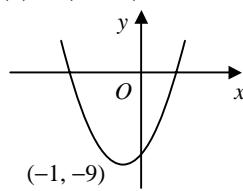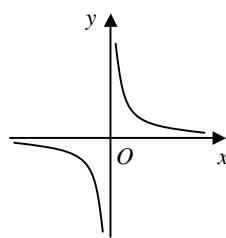

range:
$f(x) \geq -9$

range:
$f(x) \in \mathbb{R}, \; f(x) \neq 0$

   c
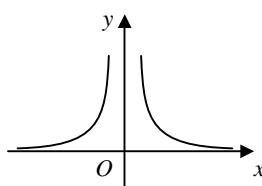
range:
$f(x) > 0$

   d
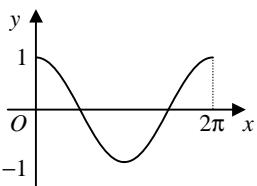
range:
$-1 \leq f(x) \leq 1$

   e
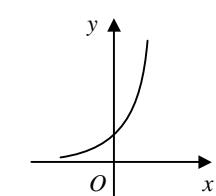

range:
$f(x) > 0$

   f
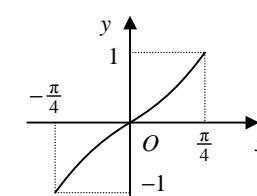
range:
$-1 \leq f(x) \leq 1$

5   **a** $f(0) = -1$, $f(7) = 6$
      $\therefore \; 0 \leq x < 7$
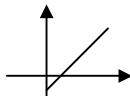
   b $f(0) = 4$
      $\therefore \; x \geq 0$
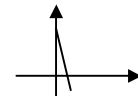
   c $f(0) = 0$, $f(5) = 125$
      $\therefore \; 0 \leq x \leq 5$
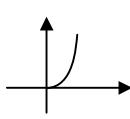
   d $f(\frac{1}{2}) = 2$, $f(\frac{1}{10}) = 10$
      $\therefore \; \frac{1}{10} < x < \frac{1}{2}$
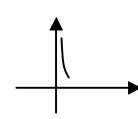
6   **a** $4x + 3 = 9$

      $x = \frac{3}{2}$

   b $x^2 - 7 = 18$

      $x^2 = 25$

      $x = \pm 5$

   c $\dfrac{9}{x + 2} = 6$

      $6x + 12 = 9$

      $x = -\frac{1}{2}$

   d $4x + 3 = \dfrac{9}{x + 2}$

      $(4x + 3)(x + 2) = 9$

      $4x^2 + 11x - 3 = 0$

      $(4x - 1)(x + 3) = 0$

      $x = -3, \frac{1}{4}$

   e $x^2 - 7 - \dfrac{x + 2}{9} = -\dfrac{19}{3}$

      $9x^2 - 63 - x - 2 = -57$

      $9x^2 - x - 8 = 0$

      $(9x + 8)(x - 1) = 0$

      $x = -\frac{8}{9}, 1$

   f $4x + 3 + x^2 - 7 = 0$

      $x^2 + 4x - 4 = 0$

      $x = \dfrac{-4 \pm \sqrt{16 + 16}}{2}$

      $x = -2 \pm 2\sqrt{2}$

      or $-4.83, 0.828$ (3sf)

7   **a** $f(x) = (x + 2)^2 - 4 + 11 = (x + 2)^2 + 7$ range: $f(x) \geq 7$

   b $f(x) = (x - 1)^2 - 1 - 6 = (x - 1)^2 - 7$ range: $f(x) \geq -7$

   c $f(x) = (2x + 3)^2 - 9 + 3 = (2x + 3)^2 - 6$ range: $f(x) \geq -6$

   d $f(x) = (3x - 1)^2 - 1 + 16 = (3x - 1)^2 + 15$ range: $f(x) \geq 15$

   e $f(x) = 15 - [x^2 + 4x] = 15 - [(x + 2)^2 - 4] = 19 - (x + 2)^2$ range: $f(x) \leq 19$