FUNCTIONS

1 $f : x \rightarrow 3x - 5, \ x \in \mathbb{R}$ $g : x \rightarrow \dfrac{4}{6 - x}, \ x \in \mathbb{R}, \ x \neq 6$ $h : x \rightarrow x^2 + 4x - 1, \ x \in \mathbb{R}$

Find the value of

   a   f(3)   **b**   g(4)   **c**   h(2)   **d**   f(1)   **e**   h(−1)   **f**   g(8)

   g   g(−4)   **h**   f($\frac{2}{3}$)   **i**   h($\frac{1}{2}$)   **j**   f(−1)   **k**   h(−3)   **l**   g($1\frac{2}{3}$)

2 $f : x \rightarrow \ln (2 - 5x), \ x \in \mathbb{R}, \ x < 0.4$ $g : x \rightarrow \sin (2x + \frac{\pi}{3}), \ x \in \mathbb{R}$ $h : x \rightarrow 3 + 2e^{1-x}, \ x \in \mathbb{R}$

Find, correct to 3 significant figures where appropriate, the value of

   a   g($\frac{\pi}{3}$)   **b**   f(0)   **c**   h(1)   **d**   g($\frac{\pi}{6}$)   **e**   h(2)   **f**   f($-\frac{1}{2}$)

   g   h(−0.8)   **h**   f(0.2)   **i**   g(0.3)   **j**   h($\frac{2}{3}$)   **k**   g(−1)   **l**   f($-\frac{3}{4}$)

3 Sketch each function and state its range.

   a $f : x \rightarrow 2x + 1, \ x \in \mathbb{R}, \ 0 \leq x \leq 7$ **b** $f : x \rightarrow 3x - 2, \ x \in \mathbb{R}, \ x \geq 0$

   c $f : x \rightarrow 5 - x, \ x \in \mathbb{R}, \ -5 \leq x \leq 5$ **d** $f : x \rightarrow 4 - 7x, \ x \in \mathbb{R}$

   e $f : x \rightarrow x^2, \ x \in \mathbb{R}, \ -3 < x < 3$ **f** $f : x \rightarrow x^2 + 3, \ x \in \mathbb{R}$

   g $f : x \rightarrow x^2 - 6, \ x \in \mathbb{R}, \ x \geq 0$ **h** $f : x \rightarrow (x - 1)^2, \ x \in \mathbb{R}, \ -2 \leq x \leq 4$

   i $f : x \rightarrow (x + 2)^2, \ x \in \mathbb{R}$ **j** $f : x \rightarrow 4 - x^2, \ x \in \mathbb{R}$

   k $f : x \rightarrow x^3, \ x \in \mathbb{R}, \ -10 < x \leq 10$ **l** $f : x \rightarrow -x^3, \ x \in \mathbb{R}$

4 Sketch each function and state its range.

   a $f : x \rightarrow x^2 + 2x - 8, \ x \in \mathbb{R}$ **b** $f : x \rightarrow \dfrac{1}{x}, \ x \in \mathbb{R}, \ x \neq 0$

   c $f : x \rightarrow \dfrac{1}{x^2}, \ x \in \mathbb{R}, \ x \neq 0$ **d** $f : x \rightarrow \cos x, \ x \in \mathbb{R}, \ 0 \leq x \leq 2\pi$

   e $f : x \rightarrow 5^x, \ x \in \mathbb{R}$ **f** $f : x \rightarrow \tan x, \ x \in \mathbb{R}, \ -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

5 Find the domain of each function given its range.

   a $f : x \rightarrow x - 1, \ f(x) \in \mathbb{R}, \ -1 \leq f(x) < 6$ **b** $f : x \rightarrow 4 - 3x, \ f(x) \in \mathbb{R}, \ f(x) \leq 4$

   c $f : x \rightarrow x^3, \ f(x) \in \mathbb{R}, \ 0 \leq f(x) \leq 125$ **d** $f : x \rightarrow \dfrac{1}{x}, \ f(x) \in \mathbb{R}, \ 2 < f(x) < 10$

6 Given that for $x \in \mathbb{R}$, $f(x) \equiv 4x + 3$, $g(x) \equiv x^2 - 7$ and $h(x) \equiv \dfrac{9}{x + 2}$, $x \neq -2$, solve the equations

   a $f(x) = 9$ **b** $g(x) = 18$ **c** $h(x) = 6$

   d $f(x) = h(x)$ **e** $g(x) - \dfrac{1}{h(x)} = -6\frac{1}{3}$ **f** $f(x) + g(x) = 0$

7 Express each function in the form indicated and hence, state its range.

   a $f : x \rightarrow x^2 + 4x + 11, \ x \in \mathbb{R}$ in the form $(x + a)^2 + b$

   b $f : x \rightarrow x^2 - 2x - 6, \ x \in \mathbb{R}$ in the form $(x + a)^2 + b$

   c $f : x \rightarrow 4x^2 + 12x + 3, \ x \in \mathbb{R}$ in the form $(ax + b)^2 + c$

   d $f : x \rightarrow 9x^2 - 6x + 16, \ x \in \mathbb{R}$ in the form $(ax + b)^2 + c$

   e $f : x \rightarrow 15 - 4x - x^2, \ x \in \mathbb{R}$ in the form $a - (x + b)^2$