1. Express $1 < x < 3$ in the form $|x - a| < b$, where a and b are to be determined.

[2]

2. Solve the equation $|3 - 2x| = 4|x|$.

[4]

3. Fig. 1 shows the graphs of $y = |x|$ and $y = a|x + b|$, where a and b are constants. The intercepts of $y = a|x + b|$ with the x- and y-axes are $(-1, 0)$ and $\left(0, \frac{1}{2}\right)$ respectively.

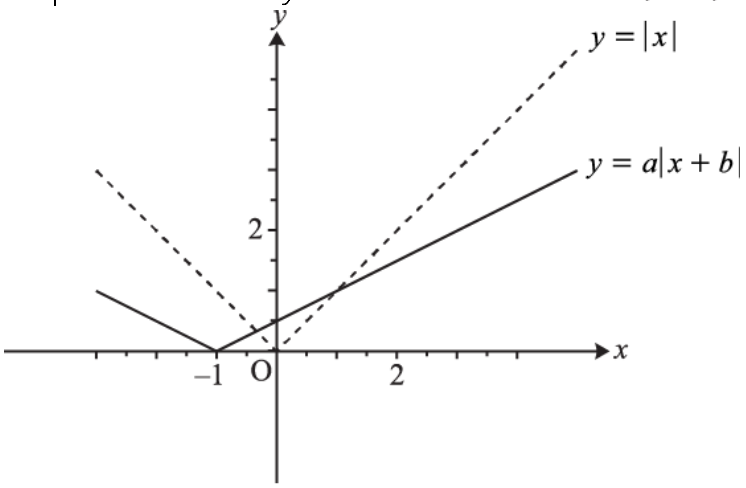


Fig. 1

i. Find a and b.

[2]

ii. Find the coordinates of the two points of intersection of the graphs.

[4]

4. You are given that $gf(x) = |3x - 1|$, for $x \in \mathbb{R}$.

(a) Given that $f(x) = 3x - 1$, express $g(x)$ in terms of x. [1]

(b) State the range of $gf(x)$. [1]

(c) Solve the inequality $|3x - 1| > 1$. [4]

5. Solve the inequality $|2x - 1| \geq 4$. [4]

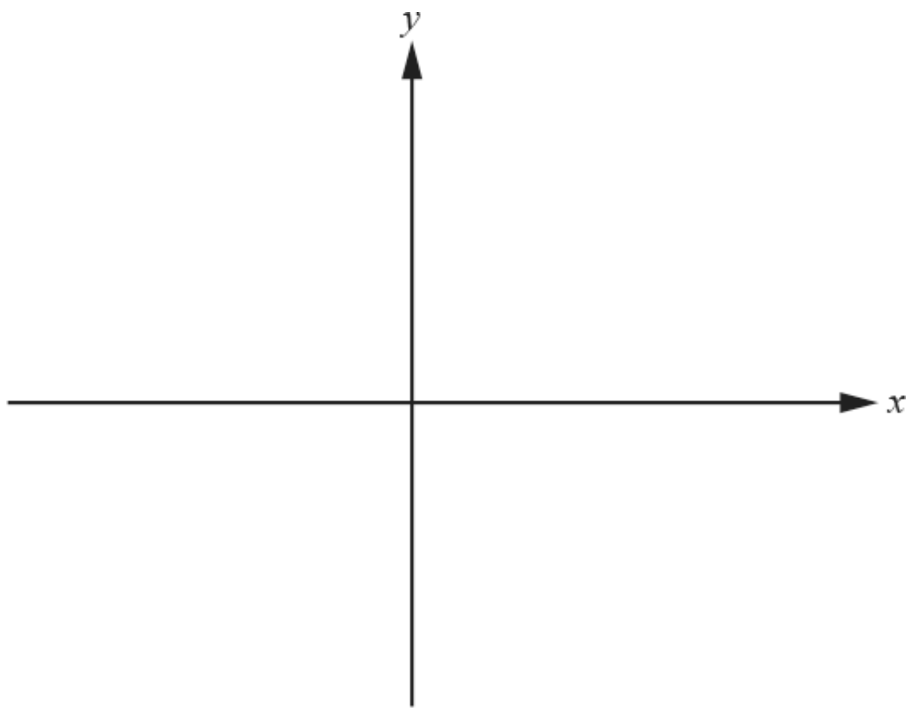
6. By sketching the graphs of $y = |2x + 1|$ and $y = -x$ on the same axes, show that the equation $|2x + 1| = -x$ has two roots. Find these roots.

[4]

7. The function $f(x)$ is defined by $f(x) = |x|$, for $-1 \leq x \leq 1$.

Sketch the graph of $y = g(x)$, where $g(x) = 2 - 2f(x)$. [3]

8. (a) On the same axes, sketch the graphs of $y = x$ and $y = |2x - 1|$. [2]



(b) **In this question you must show detailed reasoning.**

Solve the inequality $|2x - 1| > x$. [4]

9. Solve the equation $|2x + 1| < 5$. [3]

10. Solve the equation $|4x - 5| = 3$. [3]

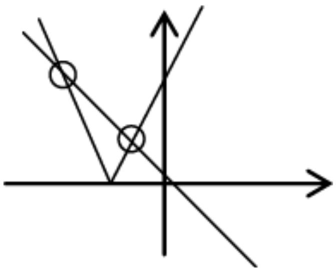
END OF QUESTION paper

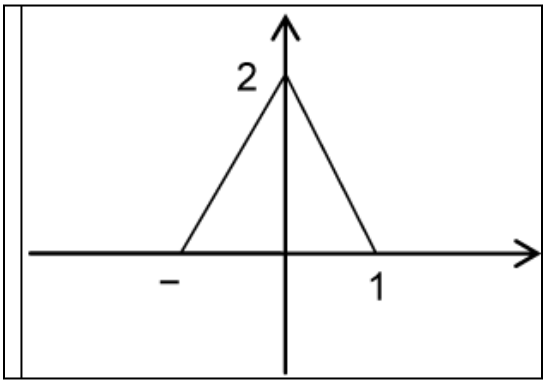
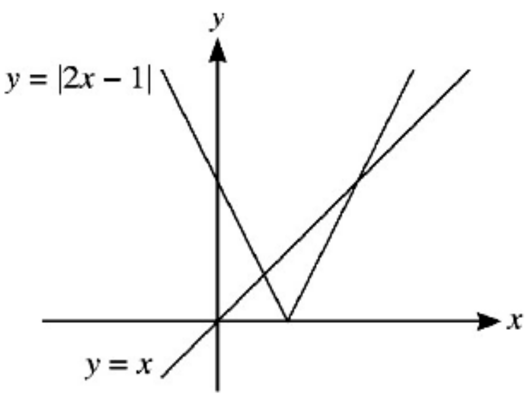
Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			$1 < x < 3 \Rightarrow \quad -1 < x\ 2 < 1$ $\Rightarrow \lvert x - 2\rvert > 1$	B1 B1	oe [or $a = 2$ and $b = 1$] **Examiner's Comments** The non-standard nature of the question made this one of the harder section A questions. Some candidates were able to write the answer down while others used an algebraic approach.	
			Total	2		
2			$\lvert 3 - 2x\rvert = 4\lvert x\rvert$ $\Rightarrow 3 - 2x = 4x,\ x = ½$ or $\quad 3 - 2x = -4x,\ x = -1 ½$ *or* $(3 - 2x)^2 = 16x^2$ $\Rightarrow 12x^2 + 12x - 9$ [= 0] $\Rightarrow x = ½, -1 ½$	M1A1 M1A1 M1 A1 A1 A1	not 3/(3 − 2) squaring both sides correct quadratic o.e. but with single x^2 term **Examiner's Comments** Although plenty of candidates scored full marks with apparent ease, there were all sorts of errors as well. Some clearly do not understand the modulus function; many duplicate work by solving 4 equations from ±(3 – 2x) = ± 4x, and in the process produced additional solutions due to poor algebra. A surprisingly common error was to write 3 = 6x ⇒ x = 2! Some even discounted the solution x = -3/2 on the grounds that answers to a modulus question need to be positive! Squaring both sides was seen occasionally, and although this method is somewhat long-winded, it does avoid conceptual errors such as |3 – 2x | = 3 + 2x.	If 3 or more final answers offered, – 1 for each incorrect additional answer – 1 for final ans written as an inequality $(3 - 2x)^2 = 4x^2$ is M0

			Total	4		
3		i	$a = ½$	B1	or 0.5 **Examiner's Comments** Some candidates were able to write down the correct values of *a* and *b*. Those who chose to use transformation arguments sometimes confused the stretch (1/2 or 2) and the translation (+1 or −1). Others chose to substitute the coordinates of specific points, with variable success.	
		i	$b = 1$	B1		
		ii	$½\|x + 1\| = \|x\|$ $\Rightarrow\ ½(x + 1) = x,$	M1	o.e. ft their a ($\neq 0$), b (but allow recovery to correct values) or verified by subst $x = 1$, $y = 1$ into $y = ½\|x + 1\|$ and $y = \|x\|$	
		ii	$\Rightarrow x = 1$, $y = 1$	A1	unsupported answers M0A0	
		ii	or $½(x + 1) = -x$,	M1	o.e., ft their *a*. *b*; or verified by subst (−1/3, 1/3) into $y = ½\|x + 1\|$ and $y = \|x\|$	
		ii	$\Rightarrow x = -1/3$, $y = 1/3$	A1	or 0.33, −0.33 or better unsupported answers M0A0	
		ii	*or* $¼(x + 1)^2 = x^2$	M1	ft their *a* and *b*	
		ii	$\Rightarrow 3x^2 - 2x - 1 = 0$	M1ft	obtaining a quadratic = 0, ft their previous line, but must have an x^2 term	
		ii	$\Rightarrow x = -1/3$ or 1	A1	SC3 for (1, 1) (−1/3, 1/3) and one or more additional points	
		ii	$y = 1/3$ or 1	A1	**Examiner's Comments** Most candidates, who knew what they were doing here either used ½ (x + 1) = ± x or squared both sides to find a quadratic in x. In the latter approach, some forgot to square the ½ and got the wrong quadratic. Examiners followed through their values for *a* and *b*. Some candidates omitted the *y*-coordinates. Candidates who found (1, 1) without showing a valid method got no marks, and there was evidence of the usual mistakes in using modulus, such as |x + 1|= |x| + 1, etc.	
			Total	6		

4		a	[g(x) =] $\lvert x \rvert$	B1(AO 1.1) [1]		
		b	All non-negative real numbers	B1(AO 1.1) [1]	Allow any reasonable notation, e.g. $y \geq 0$	
		c	**Either** $\lvert 3x - 1 \rvert > 1$ $\Rightarrow 3x - 1 > 1$ or $3x - 1 < -1$ **Or** $(3x - 1)^2 > 1$ $9x^2 - 6x > 0 \Rightarrow 3x(3x - 2) > 0$ Hence $x > \frac{2}{3}, x < 0$ $\{x : x < 0\} \cup \left\{x : x > \frac{2}{3}\right\}$	M1(AO 1.1) M1(AO 1.1) M1 M1 A1(AO 1.1) A1(AO 2.5) [4]	Allow for sketch of $y = (3x - 1)^2$ and $y = 1$ Allow for critical values $x = 0, \frac{2}{3}$ For either of these inequalities Final answer must be in this set notation form or else stated as '$x < 0$ or $x > \frac{2}{3}$'	
			Total	6		
5			**Either** $\lvert 2x - 1 \rvert \geq 4$ $\Rightarrow$ $2x - 1 \geq 4$ or $2x - 1 \leq -4$ **Or** $(2x - 1)^2 \geq 16$ $4x^2 - 4x - 15 \geq 0$	M1(AO1.1) M1(AO1.1) M1(AO1.1) M1(AO1.1) A1(AO1.1) A1(AO2.5)	M1 for sketch graph of $y = (2x - 1)^2$	

			$(2x-5)(2x+3) \geq 0$ $\Rightarrow x \geq 2½$ $\Rightarrow x \leq -1½$ $\left\{x : x \leq -1\tfrac{1}{2}\right\} \cup \left\{x : x \geq 2\tfrac{1}{2}\right\}$	[4]	and $y = 16$ M1 for $x = 2½, -1½$ OR $x \geq 2½$ or $x \leq -1½$ If final ans not in one of these forms then withhold final A1	
			Total	4		
6				M1	Sketch of $y = \lvert 2x+1 \rvert$	condone no intercept labels, but must be a 'V' shape with vertex on –ve x axis
				A1	$y = -x$ and two intersections indicated	
			$x = -1$	B1	not from ww, condone (–1, 1)	squaring: $(2x+1)^2 = x^2 \Rightarrow 3x^2 + 4x + 1 = 0$ $\Rightarrow (3x+1)(x+1) = 0$, $x = -1, -1/3$ **Examiner's Comments**
			$x = -1/3$	B1	not from ww, condone (–1/3, 1/3)	Sketches of the modulus function with $y = -x$ were generally well done, though quite a few lost a mark for neither clearly indicating the intercepts nor making a clear

						statement that there were two of them. The roots were then usually found correctly, with less evidence of faulty modulus algebra than in recent years.
			Total	4		
7			 	M1 A1 A1 [3]	inverted ‘v’ shape through (–1, 0), (1, 0) and (0, 2) correct domain ($-1 \le x \le 1$) **Examiner’s Comments** Only a third of candidates scored all three marks here. The final mark required the domain of the graph to be correct – often the ‘v’ shape extended beyond $x = -1$ to 1. Other attempts bore no relation to the correct answer.	
			Total	3		
8		a	 	B1(AO1.1) B1(AO1.1) [2]	$y = x$ straight line with positive gradient through origin $y = \|2x - 1\|$ steeper than $y = x$ and crossing $y = x$ and y-	

					axis as shown	
		b	DR $2x - 1 > x$ $-(2x - 1) > x$ Hence $x > 1,\ \frac{1}{3} > x$ $\left\{x : x < \frac{1}{3}\right\} \cup \left\{x : x > 1\right\}$	M1(AO1.1) M1(AO1.1a) A1(AO1.1) A1(AO2.5) [4]	Allow $2x - 1 = x$ Allow $-(2x - 1) = x$ For either of these inequalities Must be in this set notation form or else stated as '$x < \frac{1}{3}$ or $x > 1$'	
			Total	9		
9			$-5 < 2x + 1 < 5$ $-6 < 2x < 4$ $-3 < x < 2$	M1 (AO 2.1) A1 (AO 1.1) A1 (AO 1.1) [3]	$-(2x + 1) < 5$ oe and $2x + 1 < 5$ -3 and 2 identified $-3 < x < 2$ allow $x > -3$ **and** $x < 2$ Examiner's Comments This question proved routine for the majority of candidates.	or 2 $(2x + 1)^2 < 25$ if **M0** allow **B1** for either condition identified
			Total	3		
10			Either $4x - 5 = 3$ giving $x = 2$ Or $4x - 5 = -3$ $4x = 2$ giving $x = 0.5$	B1 (AO 1.1) M1 (AO 1.1a) A1 (AO 1.1) [3]	For answer $x = 2$ www May be implied	

			Total	3	