

1. Express $1 < x < 3$ in the form $|x - a| < b$, where a and b are to be determined. [2]

2. Solve the equation $|3 - 2x| = 4|x|$. [4]

3. Fig. 1 shows the graphs of $y = |x|$ and $y = a|x + b|$, where a and b are constants. The intercepts of $y = a|x + b|$ with the x - and y -axes are $(-1, 0)$ and $(0, \frac{1}{2})$ respectively.

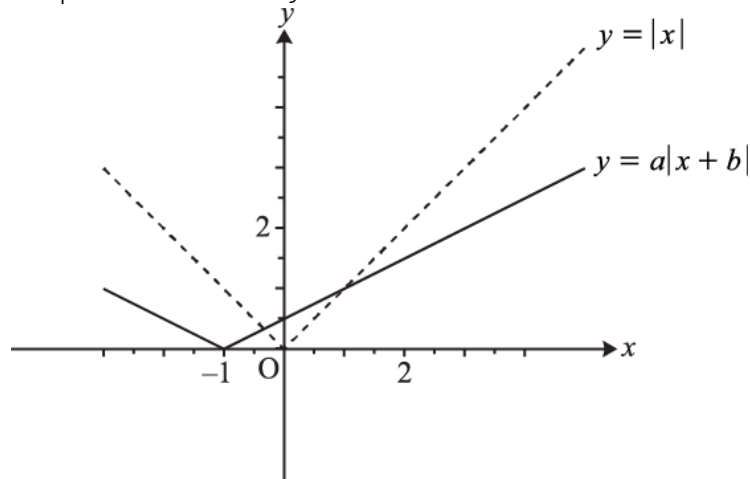


Fig. 1

i. Find a and b . [2]

ii. Find the coordinates of the two points of intersection of the graphs. [4]

4. You are given that $gf(x) = |3x - 1|$, for $x \in \mathbb{R}$.
 (a) Given that $f(x) = 3x - 1$, express $g(x)$ in terms of x . [1]

(b) State the range of $gf(x)$. [1]

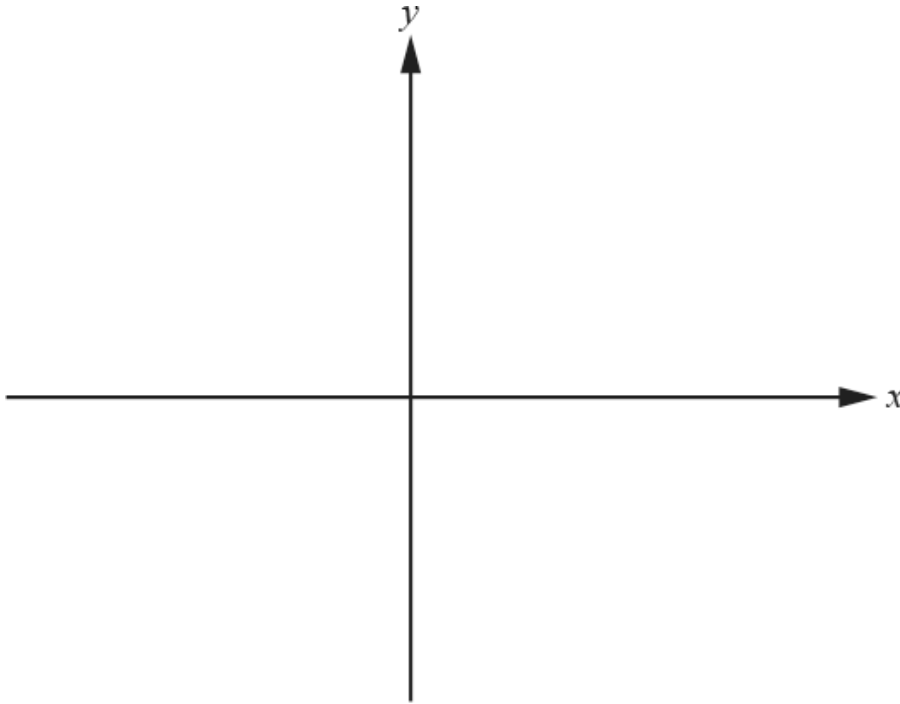
(c) Solve the inequality $|3x - 1| > 1$. [4]

5. Solve the inequality $|2x - 1| \geq 4$. [4]

6. By sketching the graphs of $y = |2x + 1|$ and $y = -x$ on the same axes, show that the equation $|2x + 1| = -x$ has two roots. Find these roots. [4]

7. The function $f(x)$ is defined by $f(x) = |x|$, for $-1 \leq x \leq 1$.
Sketch the graph of $y = g(x)$, where $g(x) = 2 - 2f(x)$. [3]

8. (a) On the same axes, sketch the graphs of $y = x$ and $y = |2x - 1|$. [2]



- (b) In this question you must show detailed reasoning.

Solve the inequality $|2x - 1| > x$. [4]

9. Solve the equation $|2x + 1| < 5$. [3]

10. Solve the equation $|4x - 5| = 3$. [3]

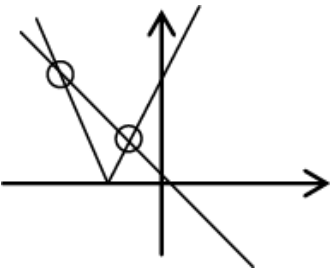
END OF QUESTION paper

Mark scheme

Question	Answer/Indicative content	Marks	Part marks and guidance	
1	$1 < x < 3 \Rightarrow -1 < x - 2 < 1$ $\Rightarrow x - 2 > 1$	B1 B1	oe [or $a = 2$ and $b = 1$] Examiner's Comments The non-standard nature of the question made this one of the harder section A questions. Some candidates were able to write the answer down while others used an algebraic approach.	
Total		2		
2	$ 3 - 2x = 4 x $ $\Rightarrow 3 - 2x = 4x, x = \frac{1}{2}$ or $3 - 2x = -4x, x = -1\frac{1}{2}$ <i>or</i> $(3 - 2x)^2 = 16x^2$ $\Rightarrow 12x^2 + 12x - 9 = 0$ $\Rightarrow x = \frac{1}{2}, -1\frac{1}{2}$	M1A1 M1A1 M1 A1 A1 A1	not $3/(3 - 2)$ squaring both sides correct quadratic o.e. but with single x^2 term Examiner's Comments Although plenty of candidates scored full marks with apparent ease, there were all sorts of errors as well. Some clearly do not understand the modulus function; many duplicate work by solving 4 equations from $\pm(3 - 2x) = \pm 4x$, and in the process produced additional solutions due to poor algebra. A surprisingly common error was to write $3 = 6x \Rightarrow x = 2$! Some even discounted the solution $x = -3/2$ on the grounds that answers to a modulus question need to be positive! Squaring both sides was seen occasionally, and although this method is somewhat long-winded, it does avoid conceptual errors such as $ 3 - 2x = 3 + 2x$.	If 3 or more final answers offered, – 1 for each incorrect additional answer – 1 for final ans written as an inequality $(3 - 2x)^2 = 4x^2$ is MO

Total			4	
3	i	$a = \frac{1}{2}$	B1	or 0.5
	i	$b = 1$	B1	
	ii	$\frac{1}{2} x + 1 = x $ $\Rightarrow \frac{1}{2} (x + 1) = x,$	M1	o.e. ft their $a (\neq 0), b$ (but allow recovery to correct values) or verified by subst $x = 1, y = 1$ into $y = \frac{1}{2} x + 1 $ and $y = x $
	ii	$\Rightarrow x = 1, y = 1$	A1	unsupported answers M0A0
	ii	or $\frac{1}{2}(x + 1) = -x,$	M1	o.e., ft their a, b ; or verified by subst $(-1/3, 1/3)$ into $y = \frac{1}{2} x + 1 $ and $y = x $
	ii	$\Rightarrow x = -1/3, y = 1/3$	A1	or 0.33, -0.33 or better unsupported answers M0A0
	ii	or $\frac{1}{4}(x + 1)^2 = x^2$	M1	ft their a and b
	ii	$\Rightarrow 3x^2 - 2x - 1 = 0$	M1ft	obtaining a quadratic = 0, ft their previous line, but must have an x^2 term
	ii	$\Rightarrow x = -1/3$ or 1	A1	SC3 for $(1, 1) (-1/3, 1/3)$ and one or more additional points
	ii	$y = 1/3$ or 1	A1	
				<u>Examiner's Comments</u> Most candidates, who knew what they were doing here either used $\frac{1}{2} (x + 1) = \pm x$ or squared both sides to find a quadratic in x . In the latter approach, some forgot to square the $\frac{1}{2}$ and got the wrong quadratic. Examiners followed through their values for a and b . Some candidates omitted the y -coordinates. Candidates who found $(1, 1)$ without showing a valid method got no marks, and there was evidence of the usual mistakes in using modulus, such as $ x + 1 = x + 1$, etc.
Total			6	

4	a	$[g(x) =] x $	B1(AO 1.1) [1]	<table border="1"> <tr> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> </tr> </table>							
	b	All non-negative real numbers	B1(AO 1.1) [1]	Allow any reasonable notation, e.g. $y \geq 0$							
	c	<p>Either</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="width: 40px;">$3x - 1 > 1$</td> <td>$\Rightarrow 3x - 1 > 1$</td> </tr> <tr> <td></td> <td>or $3x - 1 < -1$</td> </tr> </table> <p>Or</p> $(3x - 1)^2 > 1$ $9x^2 - 6x > 0 \Rightarrow 3x(3x - 2) > 0$ $x > \frac{2}{3}, x < 0$ <p>Hence</p> $\{x : x < 0\} \cup \{x : x > \frac{2}{3}\}$	$ 3x - 1 > 1$	$\Rightarrow 3x - 1 > 1$		or $3x - 1 < -1$	M1(AO 1.1) M1(AO 1.1) M1 M1 A1(AO 1.1) A1(AO 2.5) [4]	Allow for sketch of $y = (3x - 1)^2$ and $y = 1$ Allow for critical values $x = 0, \frac{2}{3}$ For either of these inequalities Final answer must be in this set notation form or else stated as ' $x < 0$ or $x > \frac{2}{3}$ '			
$ 3x - 1 > 1$	$\Rightarrow 3x - 1 > 1$										
	or $3x - 1 < -1$										
		Total	6								
5		<p>Either</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="width: 40px;">$2x - 1 \geq 4$</td> <td>\Rightarrow</td> <td>$2x - 1 \geq 4$</td> </tr> <tr> <td></td> <td>or</td> <td>$2x - 1 \leq -4$</td> </tr> </table> <p>Or</p> $(2x - 1)^2 \geq 16$ $4x^2 - 4x - 15 \geq 0$	$ 2x - 1 \geq 4$	\Rightarrow	$2x - 1 \geq 4$		or	$2x - 1 \leq -4$	M1(AO1.1) M1(AO1.1) M1(AO1.1) M1(AO1.1) A1(AO1.1) A1(AO2.5)	M1 for sketch graph of $y = (2x - 1)^2$	
$ 2x - 1 \geq 4$	\Rightarrow	$2x - 1 \geq 4$									
	or	$2x - 1 \leq -4$									

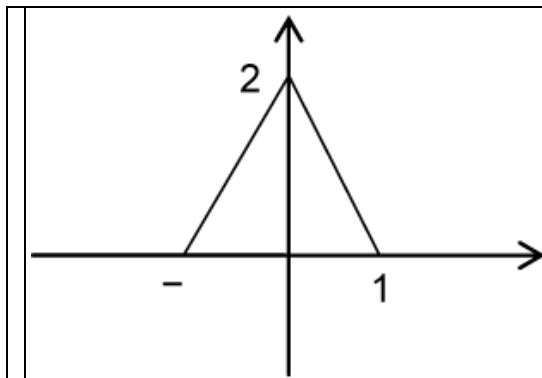
		$(2x - 5)(2x + 3) \geq 0$ $\Rightarrow x \geq 2\frac{1}{2}$ <div style="border: 1px solid black; display: inline-block; padding: 2px;">$\Rightarrow x \leq -1\frac{1}{2}$</div> $\{x : x \leq -1\frac{1}{2}\} \cup \{x : x \geq 2\frac{1}{2}\}$	[4]	and $y = 16$ M1 for $x = 2\frac{1}{2}, -1\frac{1}{2}$ OR $x \geq 2\frac{1}{2}$ or $x \leq -1\frac{1}{2}$ If final ans not in one of these forms then withhold final A1	
	Total		4		
6		 $x = -1$ $x = -1/3$	M1 Sketch of $y = 2x + 1 $ A1 $y = -x$ and two intersections indicated B1 not from ww, condone $(-1, 1)$ B1 not from ww, condone $(-1/3, 1/3)$	condone no intercept labels, but must be a 'V' shape with vertex on $-ve$ x axis squaring: $(2x + 1)^2 =$ $x^2 \Rightarrow 3x^2 + 4x + 1 =$ 0 $\Rightarrow (3x + 1)(x + 1) =$ $0, x = -1, -1/3$ Examiner's Comments Sketches of the modulus function with $y = -x$ were generally well done, though quite a few lost a mark for neither clearly indicating the intercepts nor making a clear	

statement that there were two of them. The roots were then usually found correctly, with less evidence of faulty modulus algebra than in recent years.

Total

4

7



M1
A1

inverted 'v'
shape
through
(-1, 0),
(1, 0) and
(0, 2)
correct
domain
(-1 ≤ x ≤ 1)

A1

Examiner's Comments

[3]

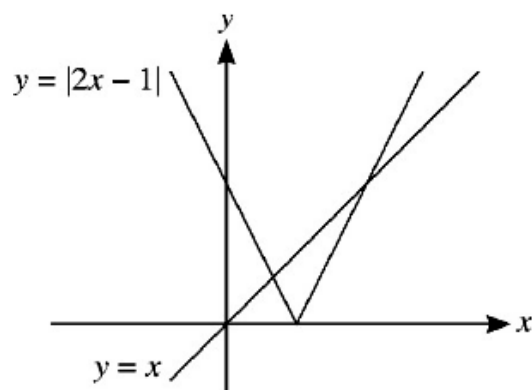
Only a third of candidates scored all three marks here. The final mark required the domain of the graph to be correct – often the 'v' shape extended beyond $x = -1$ to 1. Other attempts bore no relation to the correct answer.

Total

3

8

a



B1(AO1.1)

$y = x$
straight line
with
positive
gradient
through
origin

B1(AO1.1)

[2]

$y = |2x - 1|$
steeper
than $y = x$
and
crossing $y = x$ and $y =$

				axis as shown			
		<p>DR</p> $2x - 1 > x$		Allow $2x - 1 = x$			
		$-(2x - 1) > x$		Allow $-(2x - 1) = x$			
	b	<table border="1"> <tr> <td>Hence</td> <td>$x > 1, \frac{1}{3} > x$</td> </tr> </table>	Hence	$x > 1, \frac{1}{3} > x$	<p>M1(AO1.1)</p> <p>M1(AO1.1a)</p> <p>A1(AO1.1)</p> <p>A1(AO2.5)</p>	For either of these inequalities	
Hence	$x > 1, \frac{1}{3} > x$						
		$\{x : x < \frac{1}{3}\} \cup \{x : x > 1\}$	[4]	Must be in this set notation form or else stated as ' $x < \frac{1}{3}$ or $x > 1$ '			
		Total	9				
9		$-5 < 2x + 1 < 5$ $-6 < 2x < 4$ $-3 < x < 2$	<p>M1 (AO 2.1)</p> <p>A1 (AO 1.1)</p> <p>A1 (AO 1.1)</p>	$-(2x + 1) < 5$ or $2(2x + 1)^2 < 25$ oe and $2x + 1 < 5$ -3 and 2 identified if M0 allow B1 for either condition identified $-3 < x < 2$ allow $x > -3$ and $x < 2$			
		Total	3				
10		<p>Either $4x - 5 = 3$ giving $x = 2$</p> <p>Or $4x - 5 = -3$</p> <p>$4x = 2$ giving $x = 0.5$</p>	<p>B1 (AO 1.1)</p> <p>M1 (AO 1.1a)</p> <p>A1 (AO 1.1)</p>	<p>For answer $x = 2$ www</p> <p>May be implied</p>			
			[3]				

			Total	3	
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