

1. i. The function $f(x)$ is defined by

$$f(x) = \frac{1-x}{1+x}, x \neq -1.$$

Show that $f(f(x)) = x$.

Hence write down $f^{-1}(x)$.

[3]

- ii. The function $g(x)$ is defined for all real x by

$$g(x) = \frac{1-x^2}{1+x^2}.$$

Prove that $g(x)$ is even. Interpret this result in terms of the graph of $y = g(x)$.

[3]

2. The function $f(x)$ is defined by $f(x) = x^3 - 4$ for $-1 \leq x \leq 2$.

For $f^{-1}(x)$, determine

- The domain
- The range.

[5]

3. Functions $f(x)$ and $g(x)$, each defined for $-1 < x < 1$, are given by $f(x) = \ln(1-x)$ and $g(x) = x^2$.

(i) Find $f^{-1}(x)$ and state its domain and range.

[4]

(ii) Show that $f(x) + f(-x) = fg(x)$.

[3]

END OF QUESTION paper

Mark scheme

Question		Answer/Indicative content	Marks	Part marks and guidance	
1	i	$ff(x) = f\left(\frac{1-x}{1+x}\right) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}}$	M1	substituting $(1-x)/(1+x)$ for x in $f(x)$	
	i	$= \frac{1+x-1+x}{1+x+1-x} = \frac{2x}{2} = x^*$	A1	correctly simplified to x NB AG	
	i	$f^{-1}(x) = f(x) = (1-x)/(1+x)$	B1	or just $f^{-1}(x) = f(x)$ Examiner's Comments Most candidates gained a method mark for substituting $(1-x)/(1+x)$ for x in $f(x)$. However, the simplification of the ensuing algebraic fraction proved to be problematic to many candidates, who failed to clear the subsidiary denominators correctly. Concluding that $f^{-1}(x) = f(x)$ should of course be a 'write down' from $ff(x) = x$; however, virtually all candidates found $f^{-1}(x)$ by rearranging the formula for $x = f(y)$, usually correctly. Occasionally we were offered $f^{-1}(x) = 1/f(x) = (1+x)/(1-x)$.	
	ii	$g(-x) = \frac{1 - (-x)^2}{1 + (-x)^2}$	M1	substituting $-x$ for x in $g(x)$ condone use of 'f' for g	if brackets are omitted or misplaced allow M1A0
	ii	$= \frac{1 - x^2}{1 + x^2} = g(x)$	A1	must indicate that $g(-x) = g(x)$ somewhere allow 'reflected', 'reflection' for symmetrical Examiner's Comments This was well answered, with few candidates using particular values of x to 'show' that $g(x)$ was even. We condoned the use of f instead of g . Occasionally the brackets were misplaced in $1 - (-x)^2$ or $1 + (-x)^2$. The geometrical interpretation was well answered: although we would prefer 'symmetrical about the y -axis' to formulations such as 'reflection in the	condone use of 'f' for g
	ii	Graph is symmetrical about the y -axis.	B1		must state axis (y -axis or $x = 0$)

					y-axis', the latter was nevertheless condoned.	
			Total	6		
2			$y = x^3 - 4x \ll y$ $x = y^2 - 4$ $\Rightarrow x + 4 = y^2$ $\Rightarrow y = \sqrt[3]{x+4}$ so $f^{-1}(x) = \sqrt[3]{x+4}$ range of f is $-5 \leq y \leq 4$ so domain of f^{-1} is $-5 \leq x \leq 4$ range is $-1 \leq y \leq 2$	M1(AO1.1) A1(AO1.1) M1(AO1.1) A1(AO1.2) B1(AO1.1) [5]	attempt to invert accept $y = \sqrt[3]{x+4}$ but not $x = \sqrt[3]{y+4}$ May be implied or $[-5, 4]$ or $-1 \leq f^{-1}(x) \leq 2$ or $[-1, 2]$	
			Total	5		
3	i		$y = \ln(1-x) \ll y$ $x = \ln(1-y)$ $\Rightarrow e^x = 1-y$ $\Rightarrow y = 1 - e^x$ [so $f^{-1}(x) = 1 - e^x$] domain $x < \ln 2$ range $-1 < y < 1$	M1 A1 B1 B1 [4]	or $e^y = 1-x$ $y = 1 - e^x$ or $f^{-1}(x) = 1 - e^x$ allow $x < 0.693$ or better, $-\infty < x < \ln 2$ or $-1 < f^{-1}(x) < 1$. Must use x for domain, y or	can interchange x and y at any stage $x \leq \ln 2$ is B0, $\ln 0 < x < \ln 2$ is B0 allow $(-1, 1)$ but not $[-1, 1]$. If not labelled, take inequality with x as domain and with y or $f^{-1}(x)$ as range

				<table border="1"> <tr> <td>$f^{-1}(x)$ for range.</td> <td></td> </tr> </table> <p>Examiner's Comments</p> <p>The first two marks for finding the inverse function were nearly always gained. However, accurate notation for the domain and the range was not often seen. Not many candidates scored full marks, with the domain proving particularly awkward to get right.</p>	$f^{-1}(x)$ for range.				
$f^{-1}(x)$ for range.									
		ii	$f(-x) = \ln(1+x)$ $fg(x) = \ln(1-x^2)$ $\ln(1-x) + \ln(1+x) = \ln(1-x)(1+x)$ <table border="1"> <tr> <td></td> <td>$= \ln(1-x^2)$</td> </tr> </table>		$= \ln(1-x^2)$	<p>B1 B1</p> <p>B1</p> <p>[3]</p> <p>Examiner's Comments</p> <p>Many candidates got all three marks here, though the structure of their 'show' was sometimes weak. Very occasionally we saw $fg(x) = \ln(1-x)^2$.</p> <table border="1"> <tr> <td> soi e.g. from $\ln(1-x) + \ln(1+x) = \dots$ </td> <td> must include brackets must include brackets </td> </tr> </table>	soi e.g. from $\ln(1-x) + \ln(1+x) = \dots$	must include brackets must include brackets	
	$= \ln(1-x^2)$								
soi e.g. from $\ln(1-x) + \ln(1+x) = \dots$	must include brackets must include brackets								
		Total		7					