1.

Express $\frac{3x}{(2-x)(4+x^2)}$ in partial fractions.

[5]

2.

(i) Express $\frac{x}{(1+x)(1-2x)}$ in partial fractions.

[3]

(ii) Hence use binomial expansions to show that $\frac{x}{(1+x)(1-2x)} = ax + bx^2 + \dots$, where *a* and *b* are constants to be determined.

State the set of values of *x* for which the expansion is valid.

[5]

3.

Solve the equation $\frac{5x}{2x+1} - \frac{3}{x+1} = 1$.

[5]

4.

Express $\frac{14+6x}{(1-x)(3+2x)}$ in partial fractions. [3]

END OF QUESTION PAPER

Question			Answer/Indicative content	Marks	Guidance
1			$\dfrac{3x}{(2-x)(4+x^2)} = \dfrac{A}{2-x} + \dfrac{Bx+C}{4+x^2}$	M1	correct form of partial fractions (condone additional coeffs eg $\dfrac{(Ax+B)}{2-x} + \dfrac{(Cx+D)}{4+x^2}$* for M1 **BUT** $\dfrac{A}{2-x} + \dfrac{B}{4+x^2}$ ** is M0)
			$\Rightarrow 3x = A(4+x^2) + (Bx+C)(2-x)$	M1	Multiplying through oe and substituting values or equating coeffs at **LEAST AS FAR AS FINDING A VALUE** for one of their unknowns (even if incorrect) Can award in cases * and ** above Condone a sign error or single computational error for M1 but not a conceptual error Eg $3x = A(2-x) + (Bx+C)(4+x^2)$ is M0 $3x(2-x)(4+x^2) = A(4+x^2) + (Bx+C)$ $(2-x)$ is M0 Do not condone missing brackets unless it is clear from subsequent work that they were implied. Eg $3x = A(4+x^2) + Bx + C(2-x) = 4A + Ax^2 + Bx + 2C - Cx$ is M0 $= 4A + Ax^2 + 2Bx - Bx^2 + 2C - Cx$ is M1
			$x = 2 \Rightarrow 6 = 8A$, $A = ¾$	A1	oe www [SC B1 $A = 3/4$ from cover up rule can be applied, then the M1 applies to the other coefficients] NB $\dfrac{A}{2-x} + \dfrac{B}{4+x^2} \Rightarrow$ **A=3/4** is A0 ww **(wrong working)**
			x^2 coeffs: $0 = A - B \Rightarrow B = ¾$	A1	oe www

Question			Answer/Indicative content	Marks	Guidance
			constants: $0 = 4A + 2C \Rightarrow C = -1\frac{1}{2}$	A1	oe www [In the case of * above, all 4 constants are needed for the final A1] Ignore subsequent errors when recompiling the final solution provided that the coeffs were all correct. **Examiner's Comments** Most candidates understood the method of expressing the fraction in partial fractions. Many were completely successful and most errors were arithmetic. A few incorrectly used $\frac{A}{(2-x)}+\frac{B}{(4+x^2)}$
			Total	5	

Question			Answer/Indicative content	Marks	Guidance
2		i	$\frac{x}{(1+x)(1-2x)}=\frac{A}{1+x}+\frac{B}{1-2x}$		
		i	$\Rightarrow x = A(1-2x) + B(1+x)$	M1	expressing in partial fractions of correct form (at any stage) and attempting to use cover up, substitution or equating coefficients Condone a single sign error for M1 only.
		i	$x = ½ \Rightarrow ½ = B(1 + ½) \Rightarrow B = 1/3$	A1	www cao
		i	$x = -1 \Rightarrow -1 = 3A \Rightarrow A = -1/3$	A1	www cao (accept A/(1 + x) + B/(1 – 2x), A = –1/3, B = 1/3 as sufficient for full marks without needing to reassemble fractions with numerical numerators)
		i			<u>Examiner's Comments</u> Whilst almost all candidates knew the general method for expressing the given fraction in partial fractions, there were a surprising number of numerical errors.
		ii	$\frac{x}{(1+x)(1-2x)}=\frac{-1/3}{1+x}+\frac{1/3}{1-2x}$		
		ii	$=\frac{1}{3}\left[(1-2x)^{-1}-(1+x)^{-1}\right]$ $=\frac{1}{3}[1+(-1)(-2x)+\frac{(-1)(-2)}{2}(-2x)^2+$ $\ldots-(1+(-1)x+\frac{(-1)(-2)}{2}x^2+\ldots)]$	M1	correct binomial coefficients throughout for first three terms of **either** $(1-2x)^{-1}$ or $(1+x)^{-1}$ oe i.e. 1, (–1), (–1)(–2)/2, not nCr form. Or correct simplified coefficients seen.
		ii	$=\frac{1}{3}[1+2x+4x^2+\ldots-(1-x+x^2+\ldots)]$	A1 A1	1 + 2x + 4x2 $1 - x + x^2$ (or 1/3/ –1/3 of each expression, ft their A/B) If $k(1 - x + x^2)$ (A1) not clearly stated separately, condone absence of inner brackets (i.e. $1 + 2x + 4x^2 - 1 - x + x^2$) **only if** subsequently it is clear that brackets were assumed, otherwise A1A0. [i.e. $-1 - x + x^2$ is A0 unless it is followed by the correct answer] Ignore any subsequent incorrect terms

Question			Answer/Indicative content	Marks	Guidance
		ii	$= \frac{1}{3}(3x + 3x^2 + ...) = x + x^2 + ...$ so $a = 1$ and $b = 1$	A1	or from expansion of $x(1 - 2x)^{-1}(1 + x)^{-1}$ www cao
		ii	OR $x(1 - x - 2x^2) = x(1 - (x + 2x^2))$ $= x(1 + x + 2x^2 + (-1)(-2)(x + 2x^2)^2/2 + \dots\dots\dots)$	M1	correct binomial coefficients throughout for $(1 - (x + 2x^2))$ oe (i.e. 1, –1), at least as far as necessary terms $(1 + x)$ (NB third term of expansion unnecessary and can be ignored)
		ii	$= x(1 + x + 2x^2 + x^2 \dots\dots\dots)$	A2	$x(1 + x)$ www
		ii	$= x + x^2 \dots$ so $a = 1$ and $b = 1$	A1	www cao
		ii	Valid for $-\frac{1}{2} < x < \frac{1}{2}$ or $\lvert x \rvert < \frac{1}{2}$	B1	independent of expansion. Must combine as one overall range. condone $\leq$ s (although incorrect) or a combination. Condone also, say $-\frac{1}{2} < \lvert x \rvert < \frac{1}{2}$ but not $x < \frac{1}{2}$ or $-1 < 2x < 1$ or $-\frac{1}{2} > x > \frac{1}{2}$
		ii			<u>Examiner's Comments</u> Most candidates were able to use the binomial expansion correctly although there were sign errors - often from using (-2x) as (2x). The most common error-which was very common- was using $\frac{1}{3(1+x)} = 3(1+x)^{-1}$ $= 3(1 - x + x^2 \dots) = 3 - 3x + 3x^2$ and similarly for $\frac{1}{3(1-2x)}$. The other frequent error was in the validity. Some candidates omitted this completely but many others failed to combine the validities from the two expansions, or failed to choose the more restrictive option.
			Total	8	

Question			Answer/Indicative content	Marks	Guidance
3			$\Rightarrow 5x(x + 1) - 3(2x + 1) = (2x + 1)(x + 1)$	M1*	Multiplying throughout by $(2x + 1)(x + 1)$ or combining fractions and multiplying up oe (eg can retain denominator throughout) Condone a single numerical error, sign error or slip provided that there is no conceptual error in the process involved Do not condone omission of brackets unless it is clear from subsequent work thatthey were assumed eg $5x(x + 1) - 3(2x + 1) = (2x + 1)(x - 1)$ gets M1 $5x(x + 1) - 3(2x + 1) = 1$ gets M0 $5x(x + 1)(2x + 1) - 3(2x + 1)(x + 1) = (x + 1)(2x + 1)$ gets M0 $5x(x + 1) - 3(2x + 1) = (2x + 1)$ gets M1, just, for slip in omission of $(x + 1)$
				M1dep*	Multiplying out, collecting like terms and forming quadratic (= 0). Follow through from their equation provided the algebra is not significantly eased and it isa quadratic. Condone a further sign or numerical error or a minor slip when rearranging
			$\Rightarrow 3x^2 - 4x - 4 = 0$	A1	oe www (not fortuitously obtained – check for double errors)
			$\Rightarrow (3x + 2)(x - 2) = 0$	M1	Solving their three term quadratic (= 0) provided $b^2 - 4ac \geq$ Use of correct quadratic equation formula (if formula isquoted correctly then only one sign slip is permitted, if the formula is quoted incorrectly M0, if not quoted at all substitution must be completely correct to earn the M1) or factorising (giving their x^2 term and one other term when factors multiplied out) or comp. the square (must get to the square root stage involving ± and arithmetical errors may be condoned provided their $3(x - 2/3)^2$ seen or implied)

Question			Answer/Indicative content	Marks	Guidance
			⇒ $x = -2/3$ or 2	A1	cao for both obtained www (condone – 0.667 or better) (If no factorisation (oe) seen B1 for each answer stated following correct quadratic) **Examiner's Comments** Common errors included: • $5(x + 1) - 3(2x + 1) = (2x + 1)(x + 1)$ (and not the correct $5x(x + 1)$ –…) • Expanding $-3(2x+1)$ as either $-6x+3$ or –{6x –1 or –{6x +1 • There were some candidates who did not multiply up on the right-hand side and so obtained $5x(x+1) - 3(2x1) = 1$ • Some lost the final two marks for not applying the quadratic formula correctly However, this question was generally done well with most candidates scoring ful marks and demonstrating sound basic algebraic manipulation skills. It was commonto see the use of the quadratic formula as much as factorising to solve the final quadratic equation. Very few completed the square but those that did were mainly successful.
			Total	5	
4			$\frac{A}{1-x}+\frac{B}{3+2x}$ seen $14 + 6x = A(3 + 2x) + B(1 - x)$ $A = 4$ and $B = 2$	B1(AO 1.1a) M1(AO 1.1) A1(AO 1.1) [3]	allow 1 slip eg sign error
			Total	3	