1.

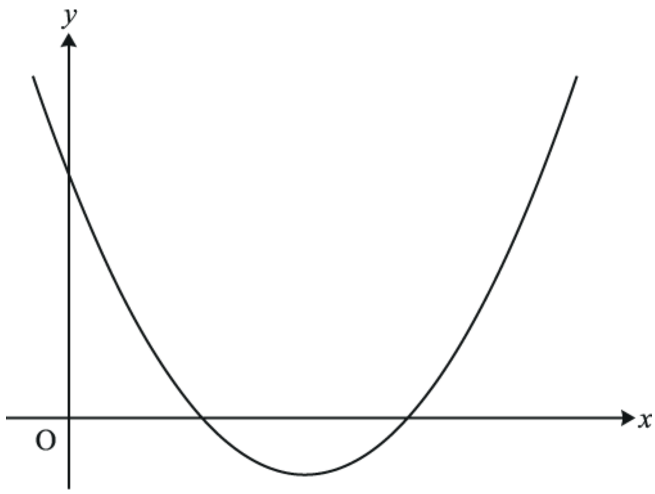

Fig. 11

Fig. 11 shows a sketch of the curve with equation $y = (x - 4)^2 - 3$.

(i) Write down the equation of the line of symmetry of the curve and the coordinates of the minimum point.

[2]

(ii) Find the coordinates of the points of intersection of the curve with the x-axis and the y-axis, using surds where necessary.

[4]

(iii) The curve is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$. Show that the equation of the translated curve may be written as $y = x^2 - 12x + 33$.

[2]

(iv) Show that the line $y = 8 - 2x$ meets the curve $y = x^2 - 12x + 33$ at just one point, and find the coordinates of this point.

[5]

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2. Fig. 9 shows the curves $y = \frac{1}{x+2}$ and $y = x^2 + 7x + 7$.

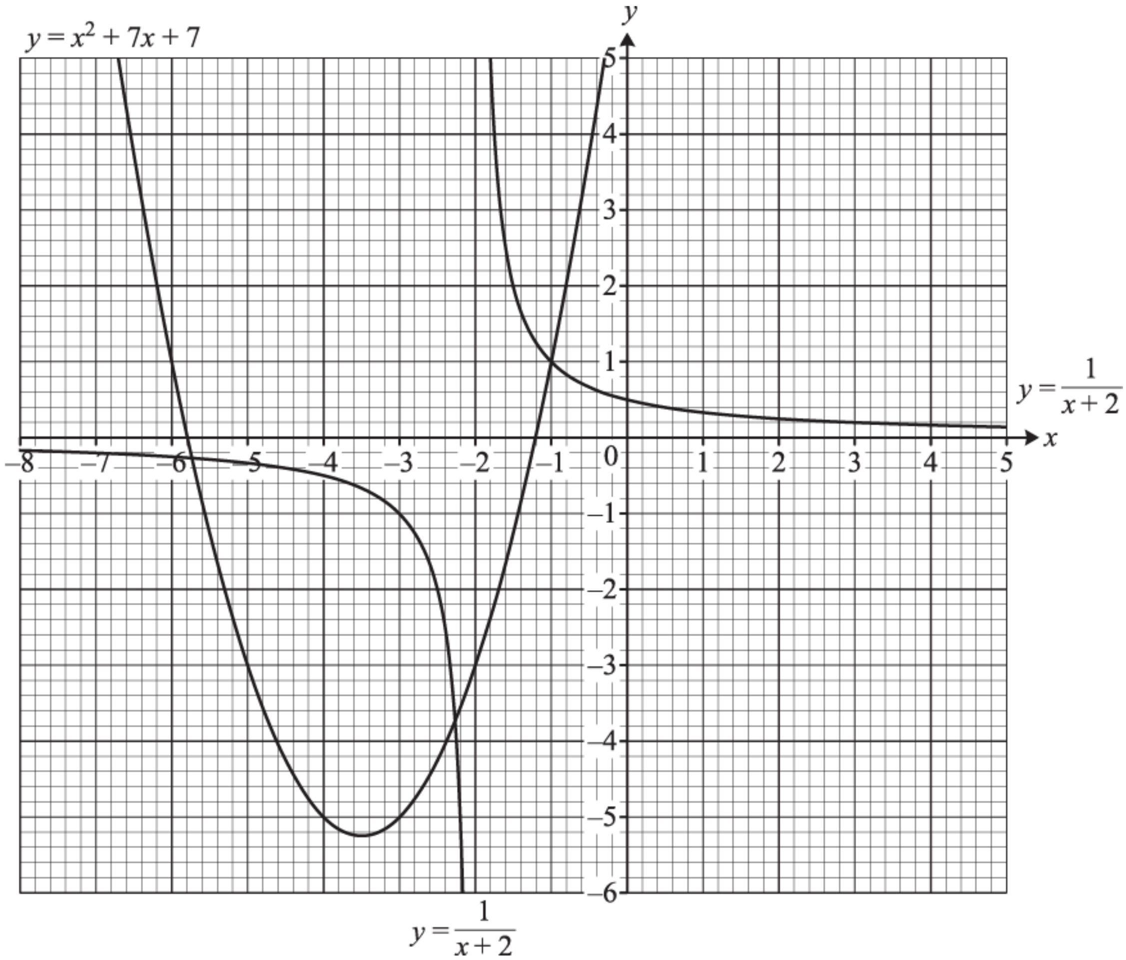

Fig.9

(i) Use Fig. 9 to estimate graphically the roots of the equation $\frac{1}{x+2} = x^2 + 7x + 7$.

[2]

(ii) Show that the equation in part **(i)** may be simplified to $x^3 + 9x^2 + 21x + 13 = 0$. Find algebraically the exact roots of this equation.

[7]

(iii) The curve $y = x^2 + 7x + 7$ is translated by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

(*A*) Show graphically that the translated curve intersects the curve $y = \dfrac{1}{x+2}$ at only one point. Estimate the coordinates of this point.

[2]

(*B*) Find the equation of the translated curve, simplifying your answer.

[2]

END OF QUESTION PAPER

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Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$x = 4$	B1		
		i	(4, –3)	B1	or $x = 4$, $y = –3$ **Examiner's Comments** The minimum point was generally well found, although some just gave the *y* coordinate. The question said, “Write down ...”, which should suggest to candidates that differentiating and putting the differential equal to zero was not needed. The line of symmetry was also usually well done, but some gave $x = –4$ or $y = 4$.	condone 4, –3
		ii	(0, 13) isw	1	or [when $x = 0$], $y = 13$ isw 0 for just (13, 0) or (k, 13) where $k \neq 0$	annotate this question if partially correct
		ii	[when $y = 0$,] $(x – 4)^2 = 3$	M1	or $x^2 – 8x + 13$ [= 0]	may be implied by correct value(s) for *x* found allow M1 for $y = x^2 – 8x + 13$ only if they go on to find values for *x* as if *y* were 0
		ii	$[x =]4 \pm \sqrt{3}$ or $\frac{8 \pm \sqrt{12}}{2}$ isw	A2	need not go on to give coordinate form A1 for one root correct **Examiner's Comments** The *y* intercept was usually correct. For the *x* intercept, many went the long way round: expanding brackets and then using the quadratic formula rather than using the completing the square method.	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	replacement of x in their eqn by $(x-2)$	M1	may be simplified; eg [y =] $(x-6)^2-3$ or allow M1 for $(x-6-\sqrt{3})(x-6+\sqrt{3})$ [=0 or y]	condone omission of 'y =' for M1, but must be present in final line for A1
		iii	completion to given answer $y = x^2 - 12x + 33$, showing at least one correct interim step	A1	cao; condone using f$(x-2)$ in place of y **Examiner's Comments** Some candidates lost a mark as they forgot that an equation has 2 sides and omitted the 'y=', only giving an expression. Most candidates realised that they should replace x with $(x-2)$. A minority expanded brackets before replacing x with $(x-2)$ which was a less efficient method.	
		iv	$x^2 - 12x + 33 = 8 - 2x$ or $(x-6)^2 - 3 = 8 - 2x$	M1	for equating curve and line; correct eqns only; or for attempt to subst $(8-y)/2$ for x in $y = x^2 - 12x + 33$	annotate this question if partially correct
		iv	$x^2 - 10x + 25 = 0$	M1	for rearrangement to zero, condoning one error such as omission of '= 0'	
		iv	$(x-5)^2$ [= 0]	A1	or showing $b^2 = 4ac$	allow $\frac{10 \pm \sqrt{0}}{2}$ oe if $b^2 - 4ac = 0$ is not used explicitly A0 for $(x-5)^2 = y$
		iv	$x = 5$ www [so just one point of contact]	A1	may be part of coordinates (5, k)	allow recovery from $(x-5)^2 = y$
		iv	point of contact at (5, –2)	A1	dependent on previous A1 earned; allow for $y = -2$ found	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iv				examiners: use one mark scheme or the other, to the benefit of the candidate if both methods attempted, but do not use a mixture of the schemes
		iv	for curve, $y' = 2x - 12$	M1		
		iv	$2x - 12 = -2$	M1	for equating their y' to –2	
		iv	$x = 5$, and y shown to be –2 using eqn to curve	A1		
		iv	tgt is $y + 2 = -2(x - 5)$	A1		
		iv	deriving $y = 8 - 2x$	A1	**Examiner's Comments** Since both equations were given, those were the ones which had to be used, and most candidates did so successfully. Some candidates did not realise that obtaining $(x - 5)^2 = 0$ led to sufficient evidence of a repeated root and also showed that the discriminant was zero. Some, of course, did not attempt to factorise anyway but opted for using the formula. The main error was in the very last mark, where some candidates substituted their x value back into the quadratic that they had just solved to find $y = 0$, rather than using the line or the curve to give $y = -2$.	condone no further interim step if all working in this part is correct so far
			Total	13		

Question			Answer/Indicative content	Marks	Part marks and guidance	
2		i	–5.7 to –5.8, –2.2 to –2.3, –1 isw	2	**B1** for 2 correct or for all 3 only stated in coordinate form, ignoring y coordinates	**Examiner's Comments** About the same number of candidates gave the coordinates of intersection of the two graphs as gave the requested roots of the given equation in x. A few misread from the graph and/or struggled with the scale.
		ii	$1=(x+2)(x^2+7x+7)$	M1	condone missing brackets if expanded correctly; or **M1** for correct expansion of $(x+2)(x^2+7x+7)$	
		ii	correct completion with at least one interim stage of working to given answer: $x^3+9x^2+21x+13=0$	A1		
		ii	[$x=-1$ is root so] $(x+1)$ is factor soi	M1	implied by division of cubic by $x+1$	condone some confusion of root/factor for this mark if division of cubic by $x+1$ seen
		ii	correctly finding other factor as $x^2+8x+13$	M2	**M1** for correct division of cubic by $(x+1)$ as far as obtaining x^2+8x (may be in grid) or for two correct terms of $x^2+8x+13$ obtained by inspection	allow seen in grid without + signs
		ii	$\frac{-8\pm\sqrt{8^2-4\times13}}{2}$ oe	M1	for use of formula, condoning one error, for $x^2+8x+13=0$	or **M1** for $(x+4)^2=4^2-13$ oe or further stage, condoning one error

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$\frac{-8 \pm \sqrt{12}}{2}$ isw or $-4 \pm \sqrt{3}$ isw and $x = -1$	A1	$x = -1$ may be stated earlier	**Examiner's Comments** isw wrong simplification or giving as coordinates **Examiner's Comments** In the main this part was completed well, with almost all candidates gaining the first two marks for multiplying by (x+2) and expanding to prove the stated equality. A significant number of candidates were unable to progress further, unsure of how to solve a cubic equation. Stronger candidates produced a well-organised solution, leading directly to the fully factorised expression (sometimes in only a few lines of working, having used the root of x =–1 from the graph and/or part (i) to obtain the first factor). The majority were able to find the correct quadratic following division by (x + 1), with a few using synthetic division and a sizeable minority finding the solution by inspection. At this stage many found the correct final solution, but a significant number failed to include x =–1 in their final solution to this question, or stated incorrectly that (x + 1) was a root.
		iii	(A) drawing the translated quadratic	B1	must be a reasonable translation of given quadratic, only intersecting given curve once; intersections with x axis –3 to –2.5 and 1.5 to 2; ignore above y = 1	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	or showing that the horizontal gap between the relevant parts of the curve is always less than 3			
		iii	estimated coordinates of the point of intersection (1.8 to 2, 0.2 to 0.3)	B1		**Examiner's Comments** Many candidates were able to translate correctly although there were issues with the intersections on the *x*-axis for some. Quite a few candidates pointed out the intersection but did not write down the coordinates as requested.
		iii	$y = x^2 + x - 5$ or $y = \left(x + \frac{1}{2}\right)^2 - \frac{21}{4}$ (B)	2	**M1** for [*y* =] $(x-3)^2 + 7(x-3) + 7$ oe or for simplified equation with '*y* =' omitted or for $y = (x-a)(x-b)$ where *a* and *b* are the values $3 + \frac{-7 \pm \sqrt{21}}{2}$ oe (may have been wrongly simplified)	M0 for use of estimated roots in (*A*) **Examiner's Comments** Many candidates failed to recognise that they should substitute $(x-3)$ for *x* in the original equation, with a variety of different methods attempted. Substituting $(x+3)$ was quite a common error, as was adding 3 to the original equation, or changing the constant term to –5. Some used estimated roots. Many failed to gain full marks because they omitted '*y* =' from their final answer.
			Total	13		