

Question	Scheme	Marks	AOs
1(a)	$y \leq 7$	B1	2.5
		(1)	
(b)	$f(1.8) = 7 - 2 \times 1.8^2 = 0.52 \Rightarrow gf(1.8) = g(0.52) = \frac{3 \times 0.52}{5 \times 0.52 - 1} = \dots$	M1	1.1b
	$gf(1.8) = 0.975$ oe e.g. $\frac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \frac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$(g^{-1}(x)) = \frac{x}{5x-3}$	A1	2.2a
		(2)	
(5 marks)			
Notes			
(a) B1: Correct range. Allow $f(x)$ or f for y. Allow e.g. $\{y \in \mathbb{R} : y \leq 7\}$, $-\infty < y \leq 7$, $(-\infty, 7]$ (b) M1: Full method to find $f(1.8)$ and substitutes the result into g to obtain a value. Also allow for an attempt to substitute $x = 1.8$ into an attempt at $gf(x)$. E.g. $gf(x) = \frac{3(7-2x^2)}{5(7-2x^2)-1} = \frac{3(7-2(1.8)^2)}{5(7-2 \times (1.8)^2)-1} = \dots$ A1: Correct value (c) M1: Correct attempt to cross multiply, followed by an attempt to factorise out x from an xy term and an x term. If they swap x and y at the start then it will be for an attempt to cross multiply followed by an attempt to factorise out y from an xy term and a y term. A1: Correct expression. Allow equivalent correct expressions e.g. $\frac{-x}{3-5x}$, $\frac{1}{5} + \frac{3}{25x-15}$ Ignore any domain if given.			

Question	Scheme	Marks	AOs
2(a)	Attempts to solve $\frac{3}{2} = \frac{8x+5}{2x+3} \Rightarrow x = \dots$ Or substitutes $x = \frac{3}{2}$ into $\frac{5-3x}{2x-8}$	M1	3.1a
	$\left(f^{-1}\left(\frac{3}{2}\right) = \right) - \frac{1}{10}$	A1	1.1b
		(2)	
(b)	$\left(\frac{8x+5}{2x+3} = \right) 4 \pm \frac{\dots}{2x+3}$	M1	1.1b
	$\left(\frac{8x+5}{2x+3} = \right) 4 - \frac{7}{2x+3}$	A1	2.1
		(2)	
(c)	$0 \leq g^{-1}(x) \leq 4$	B1	2.2a
		(1)	
(d)	Attempts either boundary $f(0) = \frac{8 \times 0 + 5}{2 \times 0 + 3}$ or $f(4) = \frac{8 \times 4 + 5}{2 \times 4 + 3}$	M1	3.1a
	Attempts both boundaries $f(0) = \frac{8 \times 0 + 5}{2 \times 0 + 3}$ and $f(4) = \frac{8 \times 4 + 5}{2 \times 4 + 3}$	dM1	1.1b
	Range $\frac{5}{3} \leq fg^{-1}(x) \leq \frac{37}{11}$	A1	2.1
		(3)	
	Alternative by attempting $fg^{-1}(x)$		
	$g^{-1}(x) = \sqrt{16-x} \Rightarrow fg^{-1}(x) = \frac{8\sqrt{16-x}+5}{2\sqrt{16-x}+3}$ $fg^{-1}(0) = \frac{8 \times 4 + 5}{2 \times 4 + 3}$ or $fg^{-1}(16) = \frac{8 \times 0 + 5}{2 \times 0 + 3}$	M1	3.1a
	$fg^{-1}(0) = \frac{8 \times 4 + 5}{2 \times 4 + 3}$ and $fg^{-1}(16) = \frac{8 \times 0 + 5}{2 \times 0 + 3}$	dM1	1.1b
	Range $\frac{5}{3} \leq fg^{-1}(x) \leq \frac{37}{11}$	A1	2.1
		(3)	
(8 marks)			
Notes:			

(a)

M1: Attempts to solve $\frac{3}{2} = \frac{8x+5}{2x+3} \Rightarrow x = \dots$ You can condone poor algebra as long as they reach a value for x.

Alternatively attempt to substitute $x = \frac{3}{2}$ into $f^{-1}(x) = \frac{\pm 5 \pm 3x}{\pm 2x \pm 8}$ or equivalent (may be in terms of y). Note that attempts to find e.g. $f'(x)$ or $\frac{1}{f(x)}$ which may be implied by values such as

$\frac{6}{17}, \frac{17}{6}, \frac{7}{18}, \frac{18}{7}$ score M0

A1: Achieves $\left(f^{-1}\left(\frac{3}{2}\right) = \right) - \frac{1}{10}$. Do not be concerned what they call it, just look for the value e.g.

$x = -\frac{1}{10}$ or just $-\frac{1}{10}$ is fine. Correct answer with no (or minimal) working scores both marks.

(b)

M1: Attempts to divide $8x+5$ by $2x+3$ Look for $4 \pm \frac{\dots}{2x+3}$ where ... is a constant or $8x+5 = A(2x+3) + B$ with A or B correct

(which may be in a fraction) or in a long division attempt and obtains a quotient of 4

or attempts to express the numerator in terms of the denominator e.g. $\frac{8x+5}{2x+3} = \frac{4(2x+3) + \dots}{2x+3}$ **A1:** A full and complete method showing $\frac{8x+5}{2x+3} = 4 - \frac{7}{2x+3}$ or $\frac{8x+5}{2x+3} = 4 + \frac{-7}{2x+3}$ Also allow for correct values e.g. $A = 4, B = -7$ Do not isw here e.g. if they obtain $A = 4, B = -7$ and then write $-7 + \frac{4}{2x+3}$ score A0

(c)

B1: Deduces $0 \leq g^{-1}(x) \leq 4$ o.e.E.g. $0 \leq y \leq 4, 0 \leq \text{range} \leq 4, g^{-1}(x) \leq 4$ and $g^{-1}(x) \geq 0, 0 \leq g^{-1} \leq 4, [0, 4]$ but not e.g. $0 \leq x \leq 4, 0 \leq g(x) \leq 4, (0, 4)$

(d)

M1: Attempts either boundary. Look for either $f(0) = \frac{8 \times 0 + 5}{2 \times 0 + 3}$ **or** $f(4) = \frac{8 \times 4 + 5}{2 \times 4 + 3}$ or uses (b) e.g. $f(0) = 4 - \frac{7}{2 \times 0 + 3}$ **or** $f(4) = 4 - \frac{7}{2 \times 4 + 3}$ **dM1:** Attempts both boundaries. Look for $f(0) = \frac{8 \times 0 + 5}{2 \times 0 + 3}$ **and** $f(4) = \frac{8 \times 4 + 5}{2 \times 4 + 3}$ or uses (b) e.g. $f(0) = 4 - \frac{7}{2 \times 0 + 3}$ **and** $f(4) = 4 - \frac{7}{2 \times 4 + 3}$ **A1:** Correct answer written in the correct form.E.g. $\frac{5}{3} \leq fg^{-1}(x) \leq \frac{37}{11}, \frac{5}{3} \leq \text{range} \leq \frac{37}{11}, \frac{5}{3} \leq y \leq \frac{37}{11}, fg^{-1}(x) \leq \frac{37}{11}$ and $fg^{-1}(x) \geq \frac{5}{3}$ $\frac{5}{3} \leq fg^{-1} \leq \frac{37}{11}, fg^{-1}(x) \leq \frac{37}{11} \cap fg^{-1}(x) \geq \frac{5}{3}, \left[\frac{5}{3}, \frac{37}{11} \right]$ but not e.g. $\frac{5}{3} \leq x \leq \frac{37}{11}$ **PTO for an alternative to (d)**

(d) **Alternative:****M1:** Attempts $fg^{-1}(x)$ and either boundary using $x = 0$ or $x = 16$

$$\text{Look for either } fg^{-1}(0) = \frac{8 \times g^{-1}(0) + 5}{2 \times g^{-1}(0) + 3} \text{ or } fg^{-1}(16) = \frac{8 \times g^{-1}(16) + 5}{2 \times g^{-1}(16) + 3}$$

$$\text{Or uses (b) e.g. } fg^{-1}(0) = 4 - \frac{7}{2 \times g^{-1}(0) + 3} \text{ or } fg^{-1}(16) = 4 - \frac{7}{2 \times g^{-1}(16) + 3}$$

The attempt at $fg^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f**dM1:** Attempts both boundaries. Look for $fg^{-1}(0) = \frac{8 \times g^{-1}(0) + 5}{2 \times g^{-1}(0) + 3}$ **and** $fg^{-1}(16) = \frac{8 \times g^{-1}(16) + 5}{2 \times g^{-1}(16) + 3}$

$$\text{Or uses (b) e.g. } fg^{-1}(0) = 4 - \frac{7}{2 \times g^{-1}(0) + 3} \text{ and } fg^{-1}(16) = 4 - \frac{7}{2 \times g^{-1}(16) + 3}$$

The attempt at $fg^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f**A1:** Correct answer written in the correct form with exact values.

$$\text{E.g. } \frac{5}{3} \leq fg^{-1}(x) \leq \frac{37}{11}, \frac{5}{3} \leq \text{range} \leq \frac{37}{11}, \frac{5}{3} \leq y \leq \frac{37}{11}, fg^{-1}(x) \leq \frac{37}{11} \text{ and } fg^{-1}(x) \geq \frac{5}{3}$$

$$\frac{5}{3} \leq fg^{-1} \leq \frac{37}{11}, fg^{-1}(x) \leq \frac{37}{11} \cap fg^{-1}(x) \geq \frac{5}{3}, \left[\frac{5}{3}, \frac{37}{11} \right] \text{ but not e.g. } \frac{5}{3} \leq x \leq \frac{37}{11}$$

Note that the $\frac{37}{11}$ is sometimes obtained fortuitously from incorrect working so check working carefully.

Question	Scheme	Marks	AOs
3(a)	$f(x) > 3$	B1	1.1b
		(1)	
(b)	$y = 3 + \sqrt{x-2} \Rightarrow x = \dots$	M1	1.1b
	$f^{-1}(x) = (x-3)^2 + 2$	A1	1.1b
	$x > 3$	B1ft	2.2a
		(3)	
(c)	$f(6) = 3 + \sqrt{6-2} = 5 \Rightarrow g("5") = \frac{15}{"5"-3} = \dots$	M1	1.1b
	$= \frac{15}{2}$	A1	1.1b
		(2)	
(d)	$3 + \sqrt{a^2 + 2} - 2 = \frac{15}{a-3} \Rightarrow "a^2 - 9 = 15"$	M1	1.1b
	$a = 2\sqrt{6}$	A1	2.2a
		(2)	
(8 marks)			
Notes			
(a) B1: $f(x) > 3$ o.e. e.g. $y > 3$, $\text{range} > 3$, $f(x) \in (3, \infty)$, $\{f(x) : f(x) > 3\}$, $f > 3$ but not e.g. $x > 3$, $f(x) \geq 3$, $[3, \infty)$			
(b) M1: Sets $y = 3 + \sqrt{x-2}$ and attempts to make x the subject (or vice versa). Look for the correct order of operations so score for an expression of the form $(x =) (y \pm 3)^2 \pm 2$ or $(y =) (x \pm 3)^2 \pm 2$ A1: $f^{-1}(x) = (x-3)^2 + 2$ Also accept $f^{-1} : x \rightarrow (x-3)^2 + 2$. Condone $f^{-1} = (x-3)^2 + 2$ (or $f^{-1} = y = (x-3)^2 + 2$) but do not allow just $y = \dots$ or $f^{-1} : y =$ Also accept other equivalent expressions such as $f^{-1}(x) = x^2 - 6x + 11$ (simplified or unsimplified) B1ft: $x > 3$ or follow through on their part (a). The omission of $x \in \mathbb{R}$ is condoned. Allow equivalent answers such as $x \in ("3", \infty)$ or $\{x : x > "3"\}$ Note: It is also acceptable to define f^{-1} in any variable e.g. as $f^{-1}(t) = (t-3)^2 + 2$ $t > 3$ as long as the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined e.g. $f^{-1}(t) = \dots$ not just $f^{-1} = \dots$			
(c) M1: Substitutes $x = 6$ into f and substitutes the result into g to find a value for $gf(6)$. Allow an attempt to substitute $x = 6$ into $gf(x) = \frac{15}{\sqrt{x-2}}$ condoning slips. They must proceed to find a value. Condone arithmetical slips and bracket errors/omissions. Condone for M1 attempts where when dealing with $\sqrt{x-2}$ leads to two different answers e.g. $\frac{15}{\sqrt{6-2}} \rightarrow \pm \frac{15}{2}$ A1: $\frac{15}{2}$ only oe isw once a correct answer is seen			

(d)

M1: Attempts to form the equation $3 + \sqrt{a^2 + 2} - 2 = \frac{15}{a-3}$, and proceeds to a quadratic in a (usually $a^2 = k$ or $a^2 - k = 15$ but condone arithmetical, miscopying and sign slips. Condone equations which would lead to complex roots.
May be implied by a correct exact answer.

Alternatively, they attempt to form the equation $a^2 + 2 = f^{-1}g(a) \Rightarrow a^2 + 2 = \left(\frac{15}{a-3} - 3\right)^2 + 2$
 $\Rightarrow (a+3)(a-3) = 15 \Rightarrow a^2 - 9 = 15$ (condone slips)

They should be square rooting both sides so that $\sqrt{a^2 + 2} - 2 \rightarrow a$, before multiplying both sides by $a-3$ and rearranging so that the a^2 term comes from their “ $(a+3)(a-3)$ ”

May be implied by a correct exact answer for their quadratic in a but a correct decimal answer does not imply this mark.

A1: ($a =$) $2\sqrt{6}$ or accept $\sqrt{24}$ (they must reject the negative solution if found as $f(a^2 + 2) \neq g(a)$ when $a = -2\sqrt{6}$) $\sqrt{6} \times \sqrt{4}$ is A0
 isw $\sqrt{24}$ followed by $4\sqrt{6}$ (incorrect manipulation of the surd) but not followed by $\pm\sqrt{24}$ o.e.
 A decimal answer on its own or multiple answers e.g. $\pm\sqrt{24}$ score A0.