

**Questions****Q1.**

$$\frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \equiv A + \frac{B}{(x - 3)} + \frac{C}{(1 - 2x)}$$

(a) Find the values of the constants  $A$ ,  $B$  and  $C$ .

(4)

$$f(x) = \frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \quad x > 3$$

(b) Prove that  $f(x)$  is a decreasing function.

(3)

**(Total for question = 7 marks)**

**Q2.**

$$f(x) = \frac{50x^2 + 38x + 9}{(5x + 2)^2(1 - 2x)} \quad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that  $f(x)$  can be expressed in the form

$$\frac{A}{5x + 2} + \frac{B}{(5x + 2)^2} + \frac{C}{1 - 2x}$$

where  $A$ ,  $B$  and  $C$  are constants(a) (i) find the value of  $B$  and the value of  $C$ (ii) show that  $A = 0$ 

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of  $x$ 

$$f(x) = p + qx + rx^2 + \dots$$

where  $p$ ,  $q$  and  $r$  are simplified fractions to be found.(ii) Find the range of values of  $x$  for which this expansion is valid.

(7)

**(Total for question = 11 marks)**

**Q3.**

(a) Express  $\frac{1}{P(11 - 2P)}$  in partial fractions.

**(3)**

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11 - 2P), \quad t \geq 0, \quad 0 < P < 5.5$$

where  $P$ , in thousands, is the population of meerkats and  $t$  is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

**(6)**

(c) show that

$$P = \frac{A}{B + Ce^{-\frac{1}{2}t}}$$

where  $A$ ,  $B$  and  $C$  are integers to be found.

**(3)****(Total for question = 12 marks)**

**Q4.**

The curve  $C$  with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \quad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where  $p$  and  $q$  are constants, passes through the point  $\left(3, \frac{1}{2}\right)$  and has two vertical asymptotes with equations  $x = 2$  and  $x = -3$

- (a) (i) Explain why you can deduce that  $q = 4$   
 (ii) Show that  $p = 15$

(3)

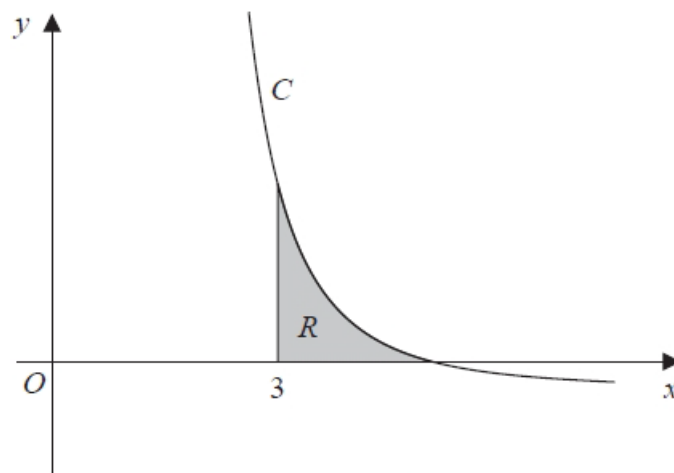
**Figure 4**

Figure 4 shows a sketch of part of the curve  $C$ . The region  $R$ , shown shaded in Figure 4, is bounded by the curve  $C$ , the  $x$ -axis and the line with equation  $x = 3$

- (b) Show that the exact value of the area of  $R$  is  $a \ln 2 + b \ln 3$ , where  $a$  and  $b$  are rational constants to be found.

(8)

**(Total for question = 11 marks)**

**Q5.**

(a) Use the substitution  $x = u^2 + 1$  to show that

$$\int_5^{10} \frac{3 \, dx}{(x-1)(3+2\sqrt{x-1})} = \int_p^q \frac{6 \, du}{u(3+2u)}$$

where  $p$  and  $q$  are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3 \, dx}{(x-1)(3+2\sqrt{x-1})} = \ln a$$

where  $a$  is a rational constant to be found.

(6)

**(Total for question = 10 marks)**

**Mark Scheme**

Q1.

Question	Scheme	Marks	AOs
	$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{x-3} + \frac{C}{1-2x}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3) \Rightarrow B = \dots, C = \dots$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = \dots$ or $C = \dots$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x) + C(x-3) \Rightarrow B = \dots, C = \dots$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = \dots$ or $C = \dots$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using $-10x+10 \equiv B(1-2x) + C(x-3)$	A1	1.1b
	(4)		
(b)	$f(x) = 3 + \frac{4}{x-3} - \frac{2}{1-2x}$ { = $3 + 4(x-3)^{-1} - 2(1-2x)^{-1}$ }; $x > 3$		
	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2} \left\{ = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2} \right\}$	M1 A1ft	2.1 1.1b
	Correct $f'(x)$ and as $(x-3)^2 > 0$ and $(1-2x)^2 > 0$ , then $f'(x) = -(+ve) - (+ve) < 0$ , so $f(x)$ is a decreasing function	A1	2.4
		(3)	

(7 marks)

**Notes for Question**

(a)	
M1:	Way 1: Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3)$ in a complete method to find values for $B$ and $C$ . Note: Allow one slip in copying $1+11x-6x^2$ Way 2: Uses a correct identity $-10x+10 \equiv B(1-2x) + C(x-3)$ (which has been found from long division) in a complete method to find values for $B$ and $C$
B1:	$A = 3$
M1:	Attempts to find the value of either $B$ or $C$ from their identity This can be achieved by <i>either</i> substituting values into their identity <i>or</i> by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	Way 1: Comparing terms: $x^2: -6 = -2A; x: 11 = 7A - 2B + C; \text{ constant: } 1 = -3A + B - 3C$ Way 1: Substituting: $x = 3: -20 = -5B \Rightarrow B = 4; x = \frac{1}{2}: 5 = -\frac{5}{2}C \Rightarrow C = -2$
Note:	Way 2: Comparing terms: $x: -10 = -2B + C; \text{ constant: } 10 = B - 3C$ Way 2: Substituting: $x = 3: -20 = -5B \Rightarrow B = 4; x = \frac{1}{2}: 5 = -\frac{5}{2}C \Rightarrow C = -2$
Note:	$A = 3, B = 4, C = -2$ from no working scores M1B1M1A1
Note:	The final A1 mark is effectively dependent upon both M marks

Notes for Question Continued	
(a) ctd	
<b>Note:</b>	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4, C=-2$ will get 1 <sup>st</sup> M0, 2 <sup>nd</sup> M1, 1 <sup>st</sup> A0
<b>Note:</b>	Way 1: You can imply a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ from seeing $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
<b>Note:</b>	Way 2: You can imply a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ from seeing $\frac{-10x+10}{(x-3)(1-2x)} \equiv \frac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
(b)	
<b>M1:</b>	Differentiates to give $\{f'(x) = \} \pm \lambda(x-3)^{-2} \pm \mu(1-2x)^{-2}; \lambda, \mu \neq 0$
<b>A1ft:</b>	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ , which can be simplified or un-simplified
<b>Note:</b>	Allow A1ft for $f'(x) = -(their\ B)(x-3)^{-2} + (2)(their\ C)(1-2x)^{-2}$ ; (their B), (their C) $\neq 0$
<b>A1:</b>	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ or $f'(x) = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2}$ and a correct explanation e.g. $f'(x) = -(+ve) - (+ve) < 0$ , so $f(x)$ is a decreasing {function}
<b>Note:</b>	The final A mark can be scored in part (b) from an incorrect $A=...$ or from $A=0$ or no value of $A$ found in part (a)

Notes for Question Continued - Alternatives	
(a)	
<b>Note:</b>	Be aware of the following alternative solutions, by initially dividing by " $(x-3)$ " or " $(1-2x)$ " <ul style="list-style-type: none"> <li><math>\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{-6x-7}{(1-2x)} - \frac{20}{(x-3)(1-2x)} \equiv 3 - \frac{10}{(1-2x)} - \frac{20}{(x-3)(1-2x)}</math>  <math>\frac{20}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x) + E(x-3) \Rightarrow D = -4, E = -8</math>  <math>\Rightarrow 3 - \frac{10}{(1-2x)} - \left( \frac{-4}{(x-3)} + \frac{-8}{(1-2x)} \right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}; A=3, B=4, C=-2</math></li> <li><math>\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{3x-4}{(x-3)} + \frac{5}{(x-3)(1-2x)} \equiv 3 + \frac{5}{(x-3)} + \frac{5}{(x-3)(1-2x)}</math>  <math>\frac{5}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x) + E(x-3) \Rightarrow D = -1, E = -2</math>  <math>\Rightarrow 3 + \frac{5}{(x-3)} + \left( \frac{-1}{(x-3)} + \frac{-2}{(1-2x)} \right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}; A=3, B=4, C=-2</math></li> </ul>
(b)	
<b>Alternative Method 1:</b>	
$f(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)}, x > 3 \Rightarrow f(x) = \frac{1+11x-6x^2}{-2x^2+7x-3}, \left\{ \begin{array}{l} u=1+11x-6x^2 \quad v=-2x^2+7x-3 \\ u'=11-12x \quad v'=-4x+7 \end{array} \right\}$	
$f'(x) = \frac{(-2x^2+7x-3)(11-12x) - (1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$	Uses quotient rule to find $f'(x)$ Correct differentiation
$f'(x) = \frac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2}$ and a correct explanation, e.g. $f'(x) = -\frac{(+ve)}{(+ve)} < 0$ , so $f(x)$ is a decreasing {function}	M1 A1
<b>Alternative Method 2:</b>	
Allow M1A1A1 for the following solution: Given $f(x) = 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)} = 3 + \frac{4}{(x-3)} + \frac{2}{(2x-1)}$ as $\frac{4}{(x-3)}$ decreases when $x > 3$ and $\frac{2}{(2x-1)}$ decreases when $x > 3$ then $f(x)$ is a decreasing {function}	

**Q2.**

Question	Scheme	Marks	AOs
(a)(i)	$50x^2 + 38x + 9 \equiv A(5x + 2)(1 - 2x) + B(1 - 2x) + C(5x + 2)^2$ $\Rightarrow B = \dots$ or $C = \dots$	M1	1.1b
	$B = 1$ and $C = 2$	A1	1.1b
(a)(ii)	E.g. $x = 0$ $x = 0 \Rightarrow 9 = 2A + B + 4C$ $\Rightarrow 9 = 2A + 1 + 8 \Rightarrow A = \dots$	M1	2.1
	$A = 0^*$	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2} = (5x+2)^{-2} = 2^{-2} \left(1 + \frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2} = 2^{-2} + \dots$	M1	3.1a
	$\left(1 + \frac{5}{2}x\right)^{-2} = 1 - 2\left(\frac{5}{2}x\right) + \frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2 + \dots$	M1	1.1b
	$2^{-2} \left(1 + \frac{5}{2}x\right)^{-2} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots$	A1	1.1b
	$\frac{1}{(1-2x)} = (1-2x)^{-1} = 1 + 2x + \frac{-1(-1-1)}{2!}(2x)^2 + \dots$	M1	1.1b
	$\frac{1}{(5x+2)^2} + \frac{2}{1-2x} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots + 2 + 4x + 8x^2 + \dots$	dM1	2.1
	$= \frac{9}{4} + \frac{11}{4}x + \frac{203}{16}x^2 + \dots$	A1	1.1b
(b)(ii)	$ x  < \frac{2}{5}$	B1	2.2a
		(7)	
<b>(11 marks)</b>			
<b>Notes</b>			

(a)(i)

M1: Uses a correct identity and makes progress using an appropriate strategy (e.g. sub  $x = \frac{1}{2}$ ) to find a value for  $B$  or  $C$ . May be implied by one correct value (cover up rule).

A1: Both values correct

(a)(ii)

M1: Uses an appropriate method to establish an equation connecting  $A$  with  $B$  and/or  $C$  and uses their values of  $B$  and/or  $C$  to find a suitable equation in  $A$ .

Amongst many different methods are:

Compare terms in  $x^2 \Rightarrow 50 = -10A + 25C$  which would be implied by  $50 = -10A + 25 \times 2$

Compare constant terms or substitute  $x = 0 \Rightarrow 9 = 2A + B + 4C$  implied by  $9 = 2A + 1 + 4 \times 2$

A1\*: Fully correct proof with no errors.

Note: The second part is a proof so it is important that a suitable proof/show that is seen.

Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0



(b)(i)

M1: Applies the key steps of writing  $\frac{1}{(5x+2)^2}$  as  $(5x+2)^{-2}$  and takes out a factor of  $2^{-2}$  to form an

expression of the form  $(5x+2)^{-2} = 2^{-2}(1+*x)^{-2}$  where \* is not 1 or 5

Alternatively uses direct expansion to obtain  $2^{-2} + \dots$

M1: Correct attempt at the binomial expansion of  $(1+*x)^{-2}$  up to the term in  $x^2$

Look for  $1+(-2)*x + \frac{(-2)(-3)}{2}*x^2$  where \* is not 5 or 1.

Condone sign slips and lack of  $*^2$  on term 3. ....

Alt Look for correct structure for 2<sup>nd</sup> and 3<sup>rd</sup> terms by direct expansion. See below

A1: For a fully correct expansion of  $(2+5x)^{-2}$  which may be unsimplified. This may have been combined with their 'B'

A direct expansion would look like  $(2+5x)^{-2} = 2^{-2} + (-2)2^{-3} \times 5x + \frac{(-2)(-3)}{2}2^{-4} \times (5x)^2$

M1: Correct attempt at the binomial expansion of  $(1-2x)^{-1}$

Look for  $1+(-1)*x + \frac{(-1)(-2)}{2}*x^2$  where \* is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.

There must be some attempt to use their values of *B* and *C*

A1: Correct expression or correct values for *p*, *q* and *r*.

(b)(ii)

B1: Correct range. Allow also other forms, for example  $-\frac{2}{5} < x < \frac{2}{5}$  or  $x \in \left(-\frac{2}{5}, \frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered

Q3.

Question	Scheme	Marks	AOs
(a)	Sets $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\frac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A$ or $B$	M1	1.1b
	$\frac{1}{P(11-2P)} = \frac{1/11}{P} + \frac{2/11}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\int \frac{22}{P(11-2P)} dP = \int 1 dt$	B1	3.1a
	Uses (a) and attempts to integrate $\int \frac{2}{P} + \frac{4}{(11-2P)} dP = t + c$	M1	1.1b
	$2 \ln P - 2 \ln(11-2P) = t + c$	A1	1.1b
	Substitutes $t=0, P=1 \Rightarrow t=0, P=1 \Rightarrow c = (-2 \ln 9)$	M1	3.1a
	Substitutes $P=2 \Rightarrow t = 2 \ln 2 + 2 \ln 9 - 2 \ln 7$	M1	3.1a
	Time = 1.89 years	A1	3.2a
		(6)	
(c)	Uses ln laws $2 \ln P - 2 \ln(11-2P) = t - 2 \ln 9$ $\Rightarrow \ln\left(\frac{9P}{11-2P}\right) = \frac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow \left(\frac{9P}{11-2P}\right) = e^{\frac{1}{2}t}$ $\Rightarrow 9P = (11-2P)e^{\frac{1}{2}t}$ $\Rightarrow P = f\left(e^{\frac{1}{2}t}\right)$ or $\Rightarrow P = f\left(e^{\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P = \frac{11}{2+9e^{\frac{1}{2}t}} \Rightarrow A=11, B=2, C=9$	A1	1.1b
		(3)	
<b>(12 marks)</b>			

**Notes:**

(a)

**B1:** Sets  $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$

**M1:** Substitutes  $P=0$  or  $P=\frac{11}{2}$  into  $1 = A(11-2P) + BP \Rightarrow A$  or  $B$

Alternatively compares terms to set up and solve two simultaneous equations in  $A$  and  $B$

**A1:**  $\frac{1}{P(11-2P)} = \frac{1/11}{P} + \frac{2/11}{(11-2P)}$  or equivalent  $\frac{1}{P(11-2P)} = \frac{1}{11P} + \frac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

(b)

**B1:** Separates the variables to reach  $\int \frac{22}{P(11-2P)} dP = \int 1 dt$  or equivalent

**M1:** Uses part (a) and  $\int \frac{A}{P} + \frac{B}{(11-2P)} dP = A \ln P \pm C \ln(11-2P)$

**A1:** Integrates both sides to form a correct equation including a 'c' Eg  
 $2 \ln P - 2 \ln(11-2P) = t + c$

**M1:** Substitutes  $t=0$  and  $P=1$  to find  $c$

**M1:** Substitutes  $P=2$  to find  $t$ . This is dependent upon having scored both previous M's

**A1:** Time = 1.89 years

(c)

**M1:** Uses correct log laws to move from  $2 \ln P - 2 \ln(11-2P) = t + c$  to  $\ln\left(\frac{P}{11-2P}\right) = \frac{1}{2}t + d$  for their numerical 'c'

**M1:** Uses a correct method to get  $P$  in terms of  $e^{\frac{1}{2}t}$

This can be achieved from  $\ln\left(\frac{P}{11-2P}\right) = \frac{1}{2}t + d \Rightarrow \frac{P}{11-2P} = e^{\frac{1}{2}t+d}$  followed by cross multiplication and collection of terms in  $P$  (See scheme)

Alternatively uses a correct method to get  $P$  in terms of  $e^{-\frac{1}{2}t}$  For example

$\frac{P}{11-2P} = e^{\frac{1}{2}t+d} \Rightarrow \frac{11-2P}{P} = e^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \frac{11}{P} - 2 = e^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \frac{11}{P} = 2 + e^{-\left(\frac{1}{2}t+d\right)}$  followed by division

**A1:** Achieves the correct answer in the form required.  $P = \frac{11}{2 + 9e^{-\frac{1}{2}t}} \Rightarrow A=11, B=2, C=9$  oe

**Q4.**

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The asymptote is found where $2x - q = 0$ Hence $q = 4$	B1	This mark is given for explaining that the asymptote at $x = 2$ is a solution of $2x - q = 0$
	$y = \frac{p-3x}{(2x-4)(x+3)}$ $\frac{1}{2} = \frac{p-9}{(6-4)(3+3)}$	M1	This mark is given for substituting $x = 3$ , $y = \frac{1}{2}$ (and $q = 4$ )

(b)	$\frac{15-3x}{(2x-4)(x+3)} = \frac{A}{(2x-4)} + \frac{B}{(x+3)}$	M1	This mark is given for a method to use partial fractions
	$= \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)}$	M1	This mark is given for finding values for $A$ and $B$
	$= \frac{0.9}{(x-2)} - \frac{2.4}{(x+3)}$	A1	This mark is given for a fully simplified expression
	$I = \int \frac{15-3x}{(2x-4)(x+3)} dx$ $= m \ln(2x-4) + n \ln(x+3)$	M1	This mark is given for a method to integrate to find the area of $R$
	$= 0.9 \ln(2x-4) + 2.4 \ln(x+3)$	A1	This mark is given for a correct expression for the area of $R$
	$\text{Area } R = \left[ 0.9 \ln(2x-4) - 2.4 \ln(x+3) \right]_3^5$	M1	This mark is given for deducing an expression for the area of $R$ ( $y = 0$ when $x = 5$ )
	$= [0.9 \ln 6 - 2.4 \ln 8] - [0.9 \ln 2 - 2.4 \ln 6]$ $= [0.9 \ln 6 + 2.4 \ln 6] - [7.2 \ln 2 + 0.9 \ln 2]$ $= 3.3 \ln 6 - 8.1 \ln 2$ $= 3.3 \ln 3 + 3.3 \ln 2 - 8.1 \ln 2$	M1	This mark is given for a method to find the exact area of $R$
	$= 3.3 \ln 3 - 4.8 \ln 2$	A1	This mark is given for a correct value of the area of $R$ with $a = 3.3$ and $b = 4.8$

**Q5.**

Question	Scheme	Marks	AOs
(a)	$x = u^2 + 1 \Rightarrow dx = 2u du$ oe	B1	1.1b
	Full substitution $\int \frac{3dx}{(x-1)(3+2\sqrt{x-1})} = \int \frac{3 \times 2u du}{(u^2+1-1)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p = 2, q = 3$	B1	1.1b
	$= \int \frac{3 \times 2u du}{u^2(3+2u)} = \int \frac{6 du}{u(3+2u)}$ *	A1*	2.1
	(4)		
(b)	$\frac{6}{u(3+2u)} = \frac{A}{u} + \frac{B}{3+2u} \Rightarrow A = \dots, B = \dots$	M1	1.1b
	Correct PF. $\frac{6}{u(3+2u)} = \frac{2}{u} - \frac{4}{3+2u}$	A1	1.1b
	$\int \frac{6 du}{u(3+2u)} = 2 \ln u - 2 \ln(3+2u) \quad (+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u = "3", u = "2"$ with some correct $\ln$ work leading to $k \ln b$ E.g. $(2 \ln 3 - 2 \ln 9) - (2 \ln 2 - 2 \ln 7) = 2 \ln \frac{7}{6}$	M1	1.1b
	$\ln \frac{49}{36}$	A1	2.1
	(6)		
<b>(10 marks)</b>			
<b>Notes: Mark (a) and (b) together as one complete question</b>			

(a)

**B1:**  $dx = 2u du$  or exact equivalent. E.g.  $\frac{dx}{du} = 2u, \frac{du}{dx} = \frac{1}{2}(x-1)^{\frac{1}{2}}$

**M1:** Attempts a full substitution of  $x = u^2 + 1$ , including  $dx \rightarrow \dots du$  to form an integrand in terms of  $u$ .

Condone slips but there should be an attempt to use the correct substitution on the denominator.

**B1:** Finds correct limits either states  $p = 2, q = 3$  or sight of embedded values as limits to the integral

**A1\*:** Clear reasoning including one fully correct intermediate line, including the integral signs, leading to the given expression ignoring limits. So B1, M1, B0, A1 is possible if the limits are incorrect, omitted or left as 5 and 10.

(b)

**M1:** Uses correct form of PF leading to values of  $A$  and  $B$ .

**A1:** Correct PF  $\frac{6}{u(3+2u)} = \frac{2}{u} - \frac{4}{3+2u}$  (Not scored for just the correct values of  $A$  and  $B$ )

**dM1:** This is an overall problem solving mark. It is for using the correct PF form and integrating using  $\ln$ s.

Look for  $P \ln u + Q \ln(3+2u)$

**A1ft:** Correct integration for their  $\frac{A}{u} + \frac{B}{3+2u} \rightarrow A \ln u + \frac{B}{2} \ln(3+2u)$  with or without modulus signs

**M1:** Uses their 2 and 3 as limits, with at least one correct application of the addition law or subtraction law leading to the form  $k \ln b$  or  $\ln a$ . PF's must have been attempted. Condone bracketing slips. Alternatively changing the  $u$ 's back to  $x$ 's and use limits of 5 and 10.

**A1:** Proceeds to  $\ln \frac{49}{36}$ . Answers without working please send to review.

