Questions

Q1.

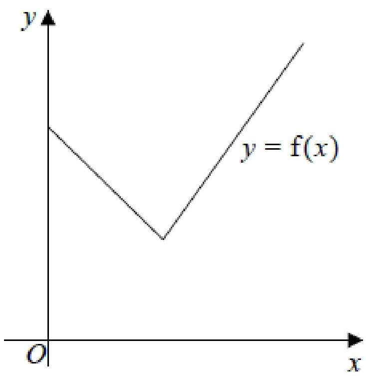


Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geq 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

(Total for question = 6 marks)

Q2.

(a) "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

Disprove this statement by means of a counter example.

(2)

(b) (i) Sketch the graph of $y = |x| + 3$

(ii) Explain why $|x| + 3 \geq |x + 3|$ for all real values of x.

(3)

(Total for question = 5 marks)

Q3.

(i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of (x – 9)."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(Total for question = 6 marks)

Q4.

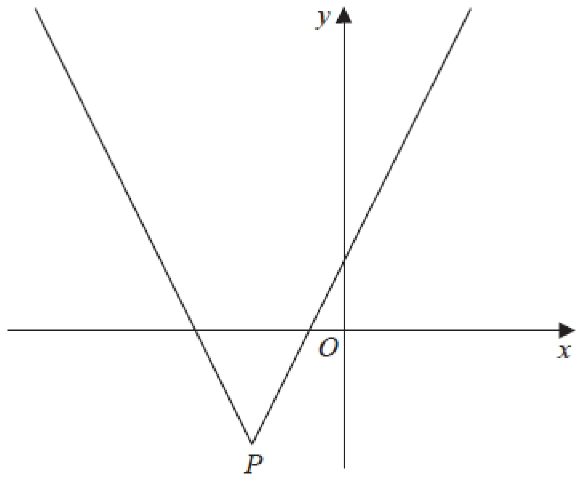


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point *P*, shown in Figure 2.

(a) Find the coordinates of *P*.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line *l* has equation *y* = *ax*, where *a* is a constant.

Given that *l* intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of *a*, writing your answer in set notation.

(3)

(Total for question = 7 marks)

Q5.

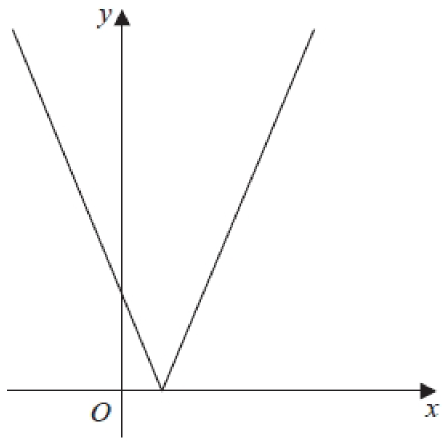


Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = | 2x - 3k |$$

where k is a positive constant.

(a) Sketch the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = k - | 2x - 3k |$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - | 2x - 3k | > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5\mathrm{f}\left(\frac{1}{2}x\right)$$

(2)

(Total for question = 10 marks)

Q6.

$$f(x) = 10e^{-0.25x}\sin x, \quad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

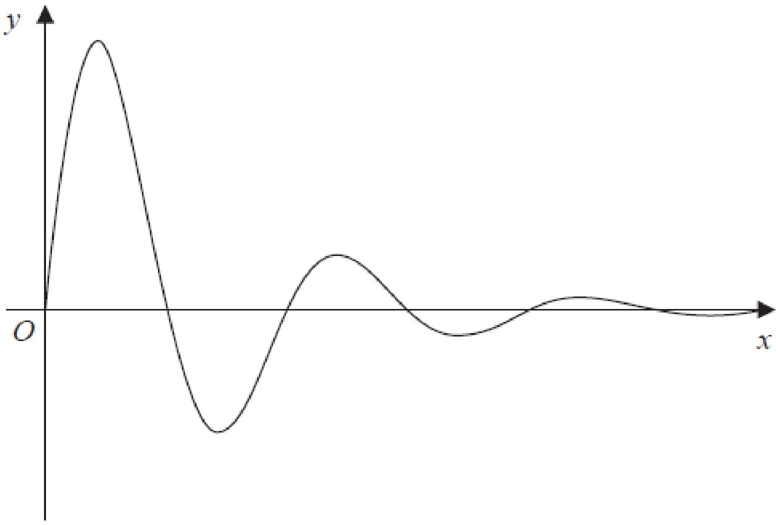


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$$H(t) = \left|10e^{-0.25t}\sin t\right| \quad t \geqslant 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

(Total for question = 10 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$f(x) \geqslant 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$	M1	1.1b
	$x=\frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k \leqslant 11$	M1	2.2a
	$\{k : k \in \mathbb{R}, 5<k \leqslant 11\}$	A1	2.5
		(2)	
			(6 marks)

Notes:

(a)

B1: $f(x) \geqslant 5$ Also allow $f(x) \in [5, \infty)$

(b)

M1: Deduces that the solution to $f(x)=\frac{1}{2}x+30$ can be found by solving

$-2(3-x)+5=\frac{1}{2}x+30$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms

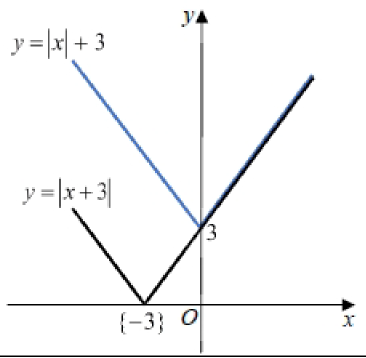
A1: $x=\frac{62}{3}$ only. Do not allow 20.6

(c)

M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k \leqslant 11$

A1: Correct solution only $\{k : k \in \mathbb{R}, 5<k \leqslant 11\}$

Q2.

Question	Scheme	Marks	AOs
	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."		
(a)	E.g. $m = \sqrt{3}$, $n = \sqrt{12}$	M1	1.1b
	$\{mn =\}$ $(\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue or 6 is not irrational or 6 is rational	A1	2.4
		(2)	
(b)(i), (ii) Way 1	 V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret $(0, 3)$ or 3 stated or marked on the positive y-axis	B1	1.1b
	Superimposes the graph of $y = \lvert x+3 \rvert$ on top of the graph of $y = \lvert x \rvert + 3$	M1	3.1a
	the graph of $y = \lvert x \rvert + 3$ is either the same or above the graph of $y = \lvert x+3 \rvert$ {for corresponding values of x} or when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = \lvert x \rvert + 3$ is above the graph of $y = \lvert x+3 \rvert$	A1	2.4
		(3)	
(b)(ii) Way 2	**Reason 1** When $x \geq 0$, $\lvert x \rvert + 3 = \lvert x+3 \rvert$ — Any one of Reason 1 or Reason 2	M1	3.1a
	Reason 2 When $x < 0$, $\lvert x \rvert + 3 > \lvert x+3 \rvert$ — Both Reason 1 and Reason 2	A1	2.4
			(5 marks)

Notes for Question			
(a)			
M1:	States or uses any pair of ***different*** numbers that will disprove the statement. E.g. $\sqrt{3},\ \sqrt{12}$; $\sqrt{2},\ \sqrt{8}$; $\sqrt{5},\ -\sqrt{5}$; $\frac{1}{\pi},\ 2\pi$; $3\mathrm{e},\ \frac{4}{5\mathrm{e}}$;		
A1:	Uses correct reasoning to disprove the given statement, with a correct conclusion		
Note:	Writing $(3\mathrm{e})\left(\frac{4}{5\mathrm{e}}\right)=\frac{12}{5} \Rightarrow$ untrue is sufficient for M1A1		
(b)(i)			
B1:	See scheme		
(b)(ii)			
M1:	For constructing a method of comparing $\lvert x\rvert+3$ with $\lvert x+3\rvert$. See scheme.		
A1:	Explains fully why $\lvert x\rvert+3 \geq \lvert x+3\rvert$. See scheme.		
Note:	Do not allow either $x>0,\ \lvert x\rvert+3\geq\lvert x+3\rvert$ or $x\geq 0,\ \lvert x\rvert+3\geq\lvert x+3\rvert$ as a valid reason		
Note	$x=0$ (or where necessary $x=-3$) need to be considered in their solutions for A1		
Note:	Do not allow an incorrect statement such as $x\leq 0,\ \lvert x\rvert+3>\lvert x+3\rvert$ for A1		
(b)(ii)			
Note:	Allow M1A1 for $x>0,\ \lvert x\rvert+3=\lvert x+3\rvert$ and for $x\leq 0,\ \lvert x\rvert+3\geq\lvert x+3\rvert \geqslant$		
Note:	Allow M1 for any of • x is positive, $\lvert x\rvert+3=\lvert x+3\rvert$ • x is negative, $\lvert x\rvert+3>\lvert x+3\rvert$ • $x>0,\ \lvert x\rvert+3=\lvert x+3\rvert$ • $x\leq 0,\ \lvert x\rvert+3\geq\lvert x+3\rvert$ • $x>0,\ \lvert x\rvert+3$ and $\lvert x+3\rvert$ are equal • $x\geq 0,\ \lvert x\rvert+3$ and $\lvert x+3\rvert$ are equal • when $x\geq 0$, both graphs are equal • for positive values $\lvert x\rvert+3$ and $\lvert x+3\rvert$ are the same Condone for M1 • $x\leq 0,\ \lvert x\rvert+3>\lvert x+3\rvert$ • $x<0,\ \lvert x\rvert+3\geq\lvert x+3\rvert$		
(b)(ii) Way 3	• For $x>0,\ \lvert x\rvert+3=\lvert x+3\rvert$ • For $-3<x<0$, as $\lvert x\rvert+3>3$ and $\{0<\}\ \lvert x+3\rvert<3$, then $\lvert x\rvert+3>\lvert x+3\rvert$	M1	3.1a
	• For $x\leq -3$, as $\lvert x\rvert+3=-x+3$ and $\lvert x+3\rvert=-x-3$, then $\lvert x\rvert+3>\lvert x+3\rvert$	A1	2.4

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	For even numbers $n = 2m$, $n^2 + 2 = 4m^2 + 2$	M1	This mark is given for showing the case for all even numbers
	This is a multiple of 4 with 2 added, so cannot be divisible by 4	A1	This mark is given for a correct conclusion with a reason why $n^2 + 2$ is not divisible by 4 for all even numbers
	For odd numbers $n = 2m + 1$, $n^2 + 2 = (2m + 1)^2 + 2 = 4m^2 + 4m + 3$ $= 4(m^2 + m) + 3$	M1	This mark is given for showing the case for all odd numbers
	This is a multiple of 4 with 3 added, so cannot be divisible by 4 Hence, for all $n \in \mathbb{N}$, $n + 2$ is not divisible by 4	A1	This mark is given for a correct conclusion with a reason why $n^2 + 2$ is not divisible by 4 for all odd numbers and a full concluding statement that for all $n \in \mathbb{N}$, $n + 2$ is not divisible by 4
(ii)	For example, for $x = 9.4$ $\lvert 3x - 28 \rvert = 0.2$ and $(x - 9) = 0.4$	M1	This mark is given for showing that the statement is not true for $9.25 < x < 9.5$
	The statement is sometimes true; For example, for $x = 12$ $\lvert 3x - 28 \rvert = 8$ and $(x - 9) = 3$	A1	This mark is given for a correct statement and an example where the statement is true

Q4.

Question	Scheme	Marks	AOs
(a)	$x=-4$ **or** $y=-5$	B1	1.1b
	$P(-4,-5)$	B1	2.2a
		(2)	
(b)	$3x+40=-2(x+4)-5 \Rightarrow x=...$	M1	1.1b
	$x=-10.6$	A1	2.1
		(2)	
(c)	$a>2$	B1	2.2a
	$y=ax \Rightarrow -5=-4a \Rightarrow a=\frac{5}{4}$	M1	3.1a
	$\{a:a\leqslant 1.25\}\cup\{a:a>2\}$	A1	2.5
		(3)	
			(7 marks)

Notes:

(a)

B1: One correct coordinate. Either $x=-4$ or $y=-5$ or $(-4, \ldots)$ or $(\ldots,-5)$ seen.

B1: Deduces that $P(-4,-5)$ Accept written separately e.g. $x=-4,\ y=-5$

(b)

M1: Attempts to solve $3x+40=-2(x+4)-5 \Rightarrow x=...$ Must reach a value for x.

You may see the attempt crossed out but you can still take this as an attempt to solve the required equation.

A1: $x=-10.6$ oe e.g. $-\frac{53}{5}$ only. If other values are given, e.g. $x=-37$ they must be rejected or the $-\frac{53}{5}$ clearly chosen as their answer. Ignore any attempts to find y.

Alternative by squaring:

$$3x+40=2|x+4|-5 \Rightarrow 3x+45=2|x+4| \Rightarrow 9x^2+270x+2025=4\left(x^2+8x+16\right)$$

$$\Rightarrow 5x^2+238x+1961=0 \Rightarrow x=-37,-\frac{53}{5}$$

M1 for isolating the $|x+4|$, squaring both sides and solving the resulting quadratic

A1 for selecting the $-\frac{53}{5}$

Correct answer with no working scores both marks.

(c)

B1: Deduces that $a>2$

M1: Attempts to find a value for a using their $P(-4,-5)$

Alternatively attempts to solve $ax=2(x+4)-5$ and $ax=2(x+4)-5$ to obtain a value for a.

A1: Correct range in acceptable set notation.

Examples:

$\{a:a\leqslant 1.25\}\cup\{a:a>2\}$

$\{a:a\leqslant 1.25\},\ \{a:a>2\}$

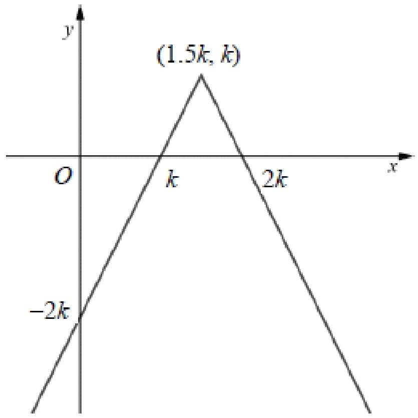
$\{a:a\leqslant 1.25 \text{ or } a>2\}$

$\{a:a\leqslant 1.25,\ a>2\}$

$(-\infty,1.25]\cup(2,\infty)$

$(-\infty,1.25],\ (2,\infty)$

Q5.

Question	Scheme	Marks	AOs
(a)			
	$\wedge$ shape in any position	B1	1.1b
	Correct x-intercepts or coordinates	B1	1.1b
	Correct y-intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a $\wedge$ shape	B1	1.1b
		(4)	
(b)	$x = k$	B1	2.2a
	$k-(2x-3k)=x-k \Rightarrow x = ...$	M1	3.1a
	$x=\frac{5k}{3}$	A1	1.1b
	Set notation is required here for this mark $\left\{x : x<\frac{5k}{3}\right\} \cap \{x : x>k\}$	A1	2.5
		(4)	
(c)	$x = 3k$ **or** $y = 3 - 5k$	B1ft	2.2a
	$x = 3k$ **and** $y = 3 - 5k$	B1ft	2.2a
		(2)	
			(10 marks)

Notes
(a) **Note that the sketch may be seen on Figure 4** B1: See scheme B1: Correct x-intercepts. Allow as shown or written as $(k, 0)$ and $(2k, 0)$ and condone coordinates written as $(0, k)$ and $(0, 2k)$ as long as they are in the correct places. B1: Correct y-intercept. Allow as shown or written as $(0, -2k)$ or $(-2k, 0)$ as long as it is in the correct place. Condone $k - 3k$ for $-2k$. B1: Correct coordinates as shown **Note that the marks for the intercepts and the maximum can be seen away from the sketch but the coordinates must be the right way round or e.g. as $y = 0, x = k$ etc. These marks can be awarded without a sketch but if there is a sketch, such points must not contradict the sketch.** (b) B1: Deduces the correct critical value of $x = k$. May be implied by e.g. $x > k$ or $x < k$ M1: Attempts to solve $k - (2x - 3k) = x - k$ or an equivalent equation/inequality to find the other critical value. Allow this mark for reaching $k = \ldots$ or $x = \ldots$ as long as they are solving the required equation. A1: Correct value A1: Correct answer using the correct set notation. Allow e.g. $\left\{x : x \in \mathbb{R}, k < x < \frac{5k}{3}\right\}$, $\left\{x :\ k < x < \frac{5k}{3}\right\}$, $x \in \left(k, \frac{5k}{3}\right)$ and allow "|" for " :" But $\left\{x : x < \frac{5k}{3}\right\} \cup \{x : x > k\}$ scores A0 $\quad \left\{x :\ k < x,\ x < \frac{5k}{3}\right\}$ scores A0 (c) B1ft: Deduces one correct coordinate. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" or $y = 3 - 5\times$"k" but must be in terms of k. Allow as coordinates or $x = ..,\ y = \ldots$ B1ft: Deduces both correct coordinates. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" and $y = 3 - 5\times$"k" but must be in terms of k. Allow as coordinates or $x = ..,\ y = \ldots$ If coordinates are given the wrong way round and not seen correctly as $x = \ldots, y = \ldots$ e.g. $(3 - 5k, 3k)$ this is B0B0

Alternative to part (b) by squaring:

$$k - |2x - 3k| = x - k \Rightarrow |2x - 3k| = 2k - x$$

$$4x^2 - 12kx + 9k^2 = 4k^2 - 4kx + x^2 \Rightarrow 3x^2 - 8kx + 5k^2 = 0$$

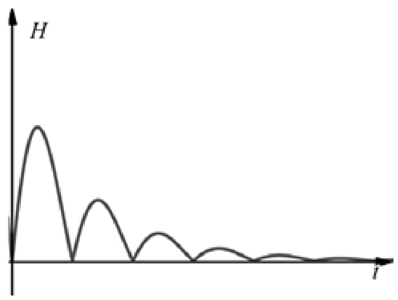
$$(3x - 5k)(x - k) = 0 \Rightarrow x = \frac{5k}{3},\ k$$

Score M1 for isolating the $|2x - 3k|$, squaring both sides to obtain 3 appropriate terms for each side, collects terms to obtain $Ax^2 + Bkx + Ck^2 = 0$ and solves for x

A1 for $x = \frac{5k}{3}$ and B1 for $x = k$

Then A1 as in the scheme.

Q6.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$f'(x) = -2.5e^{-0.25x}\sin x + 10e^{-0.25x}\cos x$	M1	This mark is given for a method to differentiate to find an expression for $f'(x)$
		A1	This mark is given for correctly differentiating to find an expression for $f'(x)$
	$f'(x) = 0$ $\Rightarrow e^{-0.25x}(-2.5\sin x + 10\cos x) = 0$ $\Rightarrow (-2.5\sin x + 10\cos x) = 0$	M1	This mark is given for setting $f'(x) = 0$ and finding as method to solve for $\tan x$
	$\frac{\sin x}{\cos x} = \frac{10}{2.5}$ $\tan x = 4$	A1	This mark is given for showing that $\tan x = 4$ as required.
(b)		M1	This mark is given for a graph with a correct shape
		A1	This mark is given for a graph with heights > 0
(c)	$\tan x = 4$, $x = 1.326$ $t = \pi + 1.326 = 4.47$	M1	This method is given for finding a value for t between the first and second bounce
	$H(4.47) = \lvert 10e^{-0.25\times 4.47}\sin 4.47\rvert$	M1	This mark is given for substituting the value of $t = \pi + \arctan 4$ into $H(t)$
	$= \lvert 3.27 \times -0.97\rvert$ $= 3.17$ metres	A1	This mark is given for finding the maximum height of the ball
(d)	The time between each bounce should not stay the same when the heights of each bounce are getting smaller	B1	This mark is given for a valid explanation of why the model should not be used the predict the time of each bounce