

1.

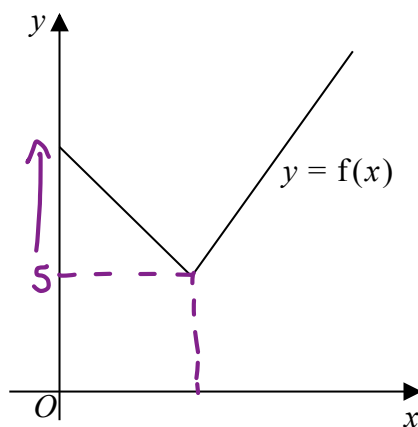


Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geq 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k .

(2)

$$a) \text{ range: } f(x) \geq 5$$

$$b) 2|3-x| + 5 = \frac{1}{2}x + 30$$

$$-2(3-x) + 5 = \frac{1}{2}x + 30 \quad - (1) \quad 2(3-x) + 5 = \frac{1}{2}x + 30$$

$$-6 + 2x + 5 = \frac{1}{2}x + 30$$

$$6 - 2x + 5 = \frac{1}{2}x + 30$$

$$2x - 1 = \frac{1}{2}x + 30$$

$$11 - 2x = \frac{1}{2}x + 30$$

$$\frac{3}{2}x = 31 \quad - (1)$$

$$\frac{5}{2}x = -19$$

$$x = \frac{62}{3}$$

$$x = -\frac{38}{5}$$

$$x = \frac{62}{3} \quad - (1)$$

$$c) \quad k > 5 \quad - \textcircled{1}$$

$$@ \quad x = 0$$

$$2|3-0| + 5 = y$$

$$2|3| + 5 = y$$

$$y = 11$$

$$k \leq 11$$

$$\{k: k \in \mathbb{R}, 5 < k \leq 11\} \quad - \textcircled{1}$$

2. (a) "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

Disprove this statement by means of a counter example.

(2)

(b) (i) Sketch the graph of $y = |x| + 3$

(ii) Explain why $|x| + 3 \geq |x+3|$ for all real values of x .

$$a) \sqrt{x} \times \sqrt{y} = \sqrt{z}$$

$$= a$$

$$\frac{22}{7}, 4 = \frac{4}{1}, 2.5 = \frac{5}{2} \quad (3)$$

$$1, 4, 9, 16$$

$$\pi, e, \sqrt{2}, \sqrt{3} \dots$$

$$\frac{16}{8}, \frac{1}{2}$$

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

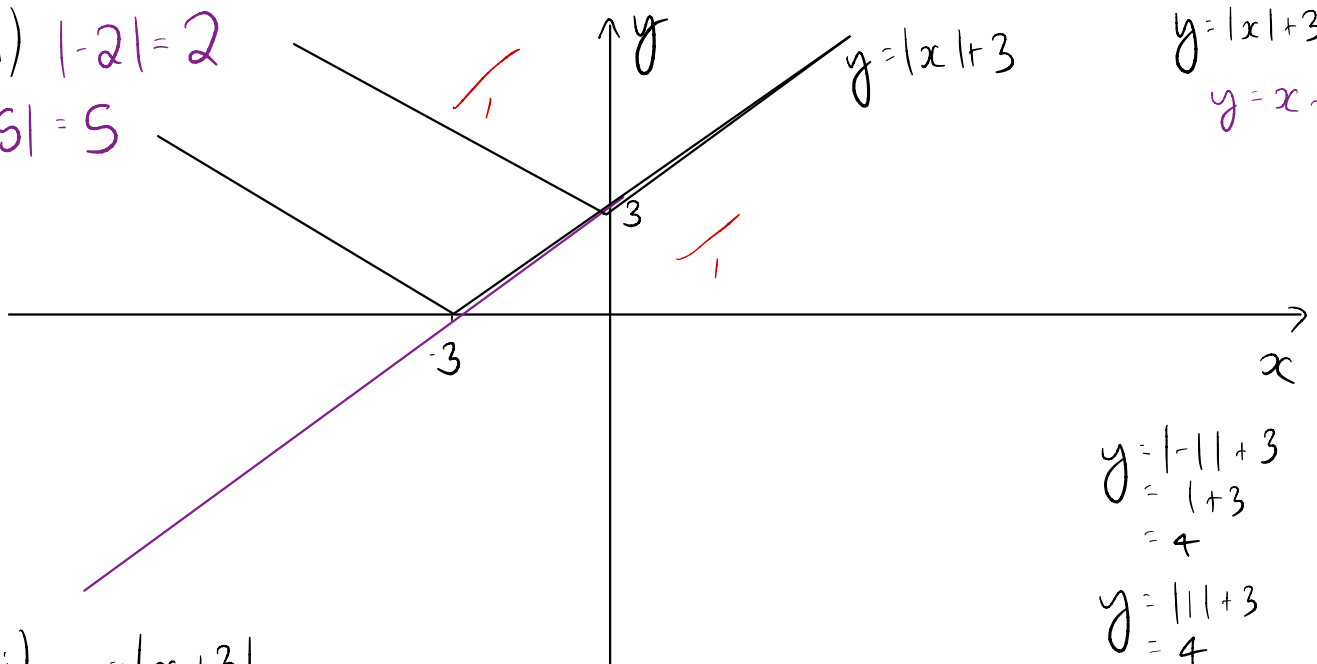
$$\sqrt{8} \times \sqrt{2} = \sqrt{16}$$

$$= 4 \quad \checkmark$$

Therefore, the statement has shown to be false by means of a counter example. $\checkmark$

$$b) i) |-2| = 2$$

$$|5| = 5$$



$$ii) y = |x+3|$$

$$y = |-1+3|$$

$$= |2| = 2$$

$$y = |0+3| = 3$$

$$y = |0| + 3 = 3$$

$$\text{When } x < 0, |x+3| < |x| + 3$$

$$\text{When } x \geq 0, |x+3| = |x| + 3$$

$$y = |1+3| = 4$$

$$y = |1| + 3 = 4$$

$$\text{Overall, } |x+3| \leq |x| + 3 \quad \checkmark$$

→ *Proof by exhaustion*

3. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

↳ 1, 2, 3, 4, ...

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

i) For when n is even

let $n = 2k$

$$n^2 + 2 = (2k)^2 + 2$$

$$= 4k^2 + 2$$

↳ *divisible by 4*

①

So when n is even $n^2 + 2$ is not divisible by 4 since it's 2 more than a multiple of 4 and 2 isn't divisible by 4

①

For when n is odd

let $n = 2k + 1$

①

$$n^2 + 2 = (2k + 1)^2 + 2$$

$$= 4k^2 + 4k + 1 + 2$$

$$= 4(k^2 + k) + 3$$

↳ *divisible by 4*

So when n is odd $n^2 + 2$ is not divisible by 4 since it's 3 more than a multiple of 4 and 3 isn't divisible by 4

①

∴ because n is not divisible by 4 when n is odd and when n is even this means that $n^2 + 2$ is not divisible by 4 for $n \in \mathbb{N}$

Proof by exhaustion

- (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

1, 3, 5, 7, ...

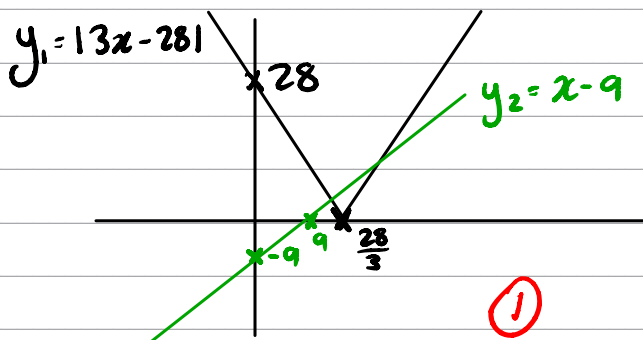
1, 2, 3, 4, ...

- (ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

iii) $|3x - 28| \geq x - 9$



$$y_1 = |3x - 28|$$

$$0 = 3x - 28$$

$$3x = 28$$

$$x = \frac{28}{3}$$

$$y_2 = x - 9$$

$$0 = x - 9$$

$$x = 9$$

Working out x intercepts

$\therefore$ The statement is sometimes true, since there are points where

$|3x - 28| < x - 9$ and points $|3x - 28| > x - 9$

①

4.

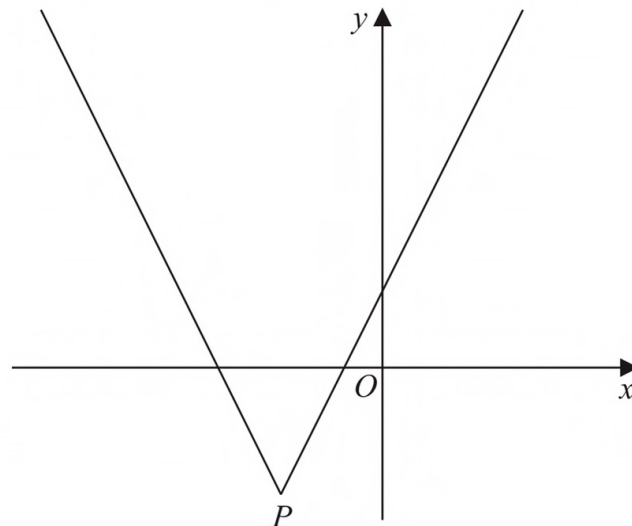


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P , shown in Figure 2.

(a) Find the coordinates of P .

(2)

P is our turning point, so we can read the turning point from the equation.

$$x = -4 \quad \text{and} \quad y = -5 \quad \Rightarrow \quad \underline{P(-4, -5)}$$

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

Option 1: $3x + 40 = 2(x + 4) - 5$

$$3x + 40 = 2x + 8 - 5 \quad \Rightarrow \quad x = -37 \quad \Rightarrow \quad 3(-37) + 40 = 2|-37 + 4| - 5$$

$$-71 \neq 61$$

$$\Rightarrow x = -37 \text{ is not a valid solution.}$$

Option 2: $3x + 40 = -2(x + 4) - 5$

$$3x + 40 = -2x - 8 - 5$$

$$5x = -53$$

$$x = -10.6$$

$$\Rightarrow \text{Check Solution: } \frac{41}{5} = \frac{41}{5} \Rightarrow \text{The solution is } x = \underline{-10.6}.$$

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a , writing your answer in set notation.

(3)

$$y = ax$$

For $a = 2$, there will never be intersection.

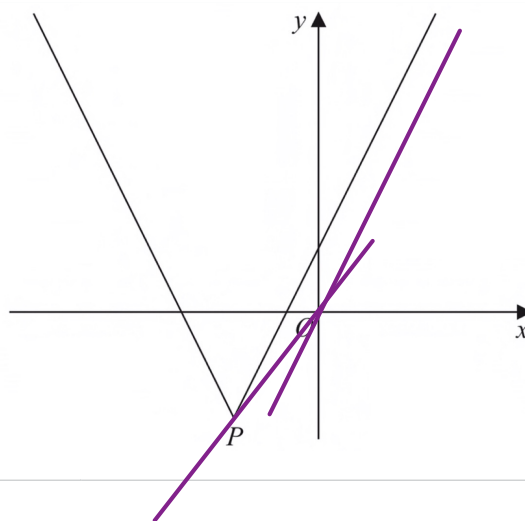
$\Rightarrow$ For $a > 2$, there will always be at least one point of intersection. ①

$$y = ax \text{ with point } P(-4, -5)$$

$$\Rightarrow -5 = -4a \Rightarrow a = 1.25$$

$\Rightarrow$ For $a \leq 1.25$, there will be at least one point of intersection. ①

$\Rightarrow$ Set Notation: $\underline{(-\infty, 1.25] \cup (2, \infty)}$ ①



5.

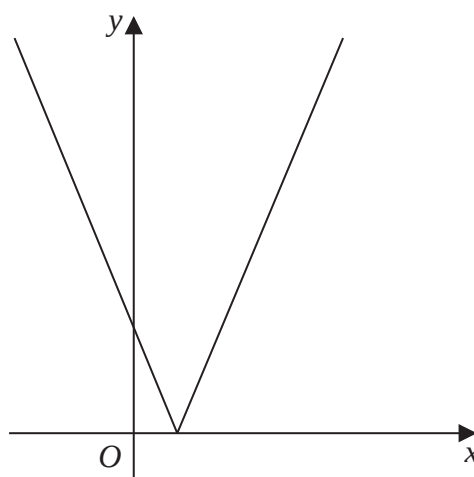


Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = f(x)$ where

$$f(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k , the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k , the coordinates of the minimum point of the graph with equation

$$y = 3 - 5f\left(\frac{1}{2}x\right)$$

(2)

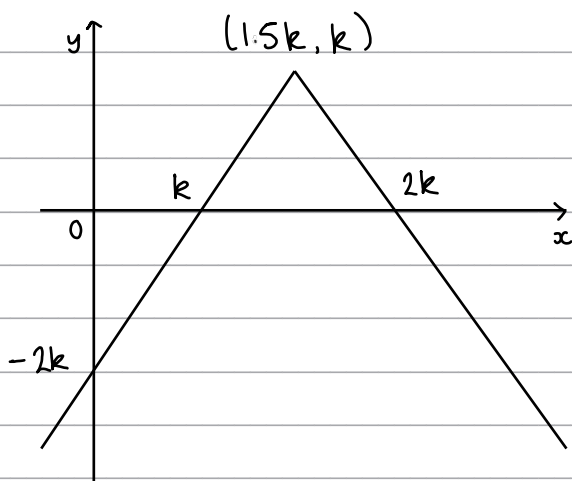
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a) $y = k - |2x - 3k|$ where $k > 0$



find max point:

$$y = 0$$

$$0 = |2x - 3k|$$

$$3k = 2x$$

$$x = \frac{3}{2}k$$

$$y = 2\left(\frac{3}{2}k - 3k\right)$$

$$y = k$$

$$\text{max. point } \left(\frac{3}{2}k, k\right)$$

find the x-intercept.

$$f(x) = 0$$

$$k - |2k - 3k| = 0$$

$$k = |2k - 3k|$$

$$k = \pm(2k - 3k)$$

↓

$$k = 2x - 3k$$

$$4k = 2x$$

$$x = 2k$$

$$\therefore (2k, 0)$$

↓

$$k = -(2x - 3k)$$

$$k = -2x + 3k$$

$$2x = 2k$$

$$x = k$$

$$\therefore (k, 0)$$

find the y-intercept:

$$x = 0$$

$$f(x) = k - |2(0) - 3k|$$

$$= k - |-3k|$$

$$= k - 3k$$

$$= -2k \rightarrow (0, -2k)$$



$$b) \quad k - |2x - 3k| = x - k \quad (1)$$

$$k + k - x = |2x - 3k|$$

$$2k - x = |2x - 3k|$$

$$2k - x = \pm (2x - 3k)$$

↓

$$2k - x = 2x - 3k$$

$$5k = 3x$$

$$x = \frac{5}{3}k$$

↓

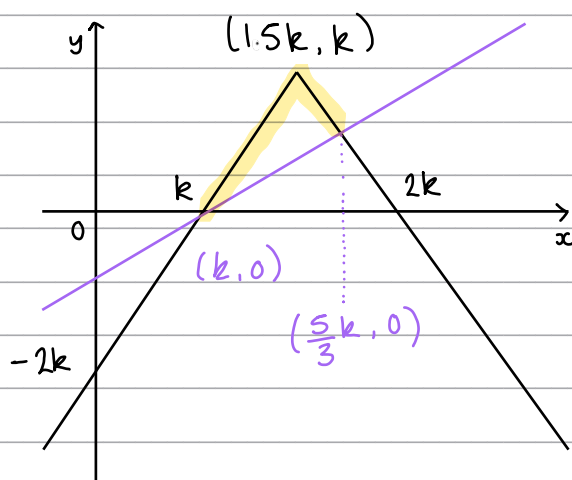
$$2k - x = -(2x - 3k)$$

$$2k - x = -2x + 3k$$

$$x = k$$

$$\left\{ x : x < \frac{5k}{3} \right\} \cap \left\{ x : x > k \right\} \quad (1) \text{ must be in set notation}$$

To picture it on the graph:



The set of values where $k - |2x - 3k| > x - k$ are highlighted.

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c) This question uses knowledge of graph transformations

$f(x) \rightarrow f(\frac{1}{2}x)$ is a stretch $2\times$ parallel to the y -axis

$$x_2 = 2 \times x_1 = 2 \times 1.5k = 3k \quad \textcircled{i} \quad \text{use old maximum } (1.5k, k)$$

$$y_2 = 3 - 5(k) = 3 - 5k \quad \textcircled{i}$$

this is a translation 3 units up and a stretch scale factor 5 parallel to the x -axis.



$$f(x) = 10e^{-0.25x} \sin x, \quad x \geq 0$$

6. (a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

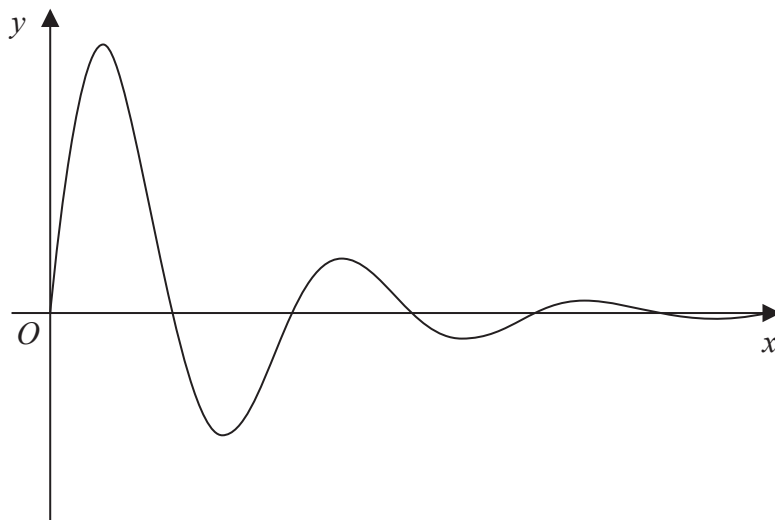


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

- (b) Sketch the graph of H against t where

$\nearrow f(x)$

$$H(t) = |10e^{-0.25t} \sin t| \quad t \geq 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

- (c) the maximum height of the ball above the ground between the first and second bounce.

(3)

- (d) Explain why this model should not be used to predict the time of each bounce.

(1)

$$a) p(x) = 10e^{-0.25x} \sin x$$

$$p'(x) = -0.25(10e^{-0.25x}) \sin x + \cos x (10e^{-0.25x})$$

$$= -2.5 \sin x e^{-0.25x} + 10 \cos x e^{-0.25x} \quad (2)$$

$$0 = -2.5 \sin x e^{-0.25x} + 10 \cos x e^{-0.25x}$$

$$= e^{-0.25x} (-2.5 \sin x + 10 \cos x) \quad (1)$$

$$\therefore e^{-0.25x} = 0 \quad \text{or} \quad -2.5 \sin x + 10 \cos x = 0$$

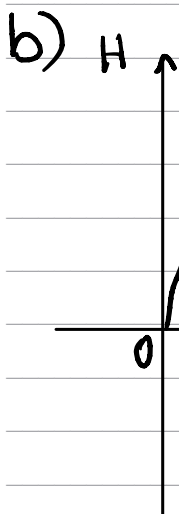
↑
reject because
no solution

$$10 \cos x = 2.5 \sin x$$

$$\frac{10}{2.5} = \frac{\sin x}{\cos x} \quad (1)$$

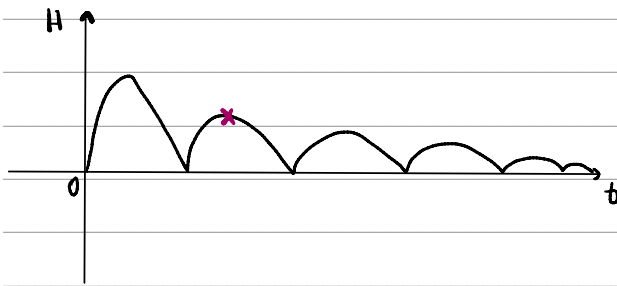
$$\frac{\sin x}{\cos x} = \tan x$$

$$\tan x = 4 \quad \text{as needed}$$



① correct shape
① decreasing heights

c)



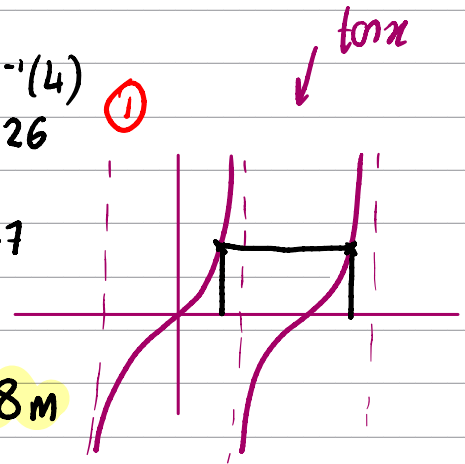
$$\tan x = 4$$

$$x = \tan^{-1}(4)$$

$$= 1.326$$

 $+ \pi$

$$= 4.47$$



$$H(4.47) = |10e^{-0.25(4.47)} \times \sin(4.47)| = 3.18 \text{ m}$$

d)

The times between each bounce should not stay the same when the heights of each bounce are getting smaller ①