Ch.1 Partial Fractions Questions

1.
$$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of the constants *A*, *B* and *C*.

(4)

(b) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of *x*, as far as the term in x^2. Give each coefficient as a simplified fraction.

(7)
(Total 11 marks)

2.
$$\text{f}(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

(a) Find the values of the constants *A*, *B* and *C*.

(4)

(b) (i) Hence find $\int \text{f}(x)\,\text{d}x$.

(3)

(ii) Find $\int_0^2 \text{f}(x)\,\text{d}x$ in the form 1n *k*, where *k* is a constant.

(3)
(Total 10 marks)

3.
$$\text{f}(x) = \frac{27x^2+32x+16}{(3x+2)^2(1-x)}, |x| < \frac{2}{3}$$

Given that f(*x*) can be expressed in the form

$$\text{f}(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(a) find the values of B and C and show that $A = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^2. Simplify each term.

(6)

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of f (0.2). Give your answer to 2 significant figures.

(4)
(Total 14 marks)

4.

$$\text{f}(x) = \frac{3x-1}{(1-2x)^2} \qquad |x| < \tfrac{1}{2}.$$

Given that, for $x \neq \frac{1}{2}$, $\dfrac{3x-1}{(1-2x)^2} = \dfrac{A}{(1-2x)} + \dfrac{B}{(1-2x)^2}$, where A and B are constants,

(a) find the values of A and B.

(3)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3, simplifying each term.

(6)
(Total 9 marks)

5.

$$\text{f}(x) = \frac{9+4x^2}{9-4x^2}, \qquad x \neq \pm\tfrac{3}{2}.$$

(a) Find the values of the constants A, B and C such that

$$\text{f}(x) = A + \frac{B}{3+2x} + \frac{C}{3-2x}, \qquad x \neq \pm\tfrac{3}{2}.$$

(4)

(b) Hence find the exact value of

$$\int_{-1}^{1} \frac{9+4x^2}{9-4x^2}\,\mathrm{d}x$$

(5)
(Total 9 marks)

6. Given that

$$\frac{3+5x}{(1+3x)(1-x)} \equiv \frac{A}{1+3x} + \frac{B}{1-x},$$

(a) find the values of the constants A and B.

(3)

(b) Hence, or otherwise, find the series expansion in ascending powers of x, up to and including the term in x^2, of

$$\frac{3+5x}{(1+3x)(1-x)}.$$

(5)

(c) State, with a reason, whether your series expansion in part (b) is valid for $x = \frac{1}{2}$.

(2)
(Total 10 marks)

7. (a) Express $\dfrac{13-2x}{(2x-3)(x+1)}$ in partial fractions.

(4)

(b) Given that $y = 4$ at $x = 2$, use your answer to part (a) to find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y(13-2x)}{(2x-3)(x+1)}, \quad x > 1.5$$

Express your answer in the form $y = \mathrm{f}(x)$.

(7)
(Total 11 marks)

8. The function f is given by

$$f(x) = \frac{3(x+1)}{(x+2)(x-1)}, \; x \in \mathbb{R}, x \neq -2, x \neq 1.$$

(a) Express f(x) in partial fractions.

(3)

(b) Hence, or otherwise, prove that $f'(x) < 0$ for all values of x in the domain.

(3)
(Total 6 marks)

9. $f(x) = \dfrac{1+14x}{(1-x)(1+2x)}, \qquad |x| < \dfrac{1}{2}.$

(a) Express f(x) in partial fractions.

(3)

(b) Hence find the exact value of $\int_{\frac{1}{6}}^{\frac{1}{3}} f(x)\, dx$, giving your answer in the form ln p, where p is rational.

(5)

(c) Use the binomial theorem to expand f(x) in ascending powers of x, up to and including the term in x^3, simplifying each term.

(5)
(Total 13 marks)

Ch.1 Partial Fractions Mark Schemes

1. (a) $A = 2$ B1

$$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$$

$x \to 1$	$-3 = 3B \Rightarrow B = -1$	A1	
$x \to -2$	$-12 = -3C \Rightarrow C = 4$	A1	4

(b) $$\frac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\frac{x}{2}\right)^{-1}$$

$(1-x)^{-1} = 1 + x + x^2 + \ ...$	B1	
$\left(1+\frac{x}{2}\right)^{-1} = 1 - \frac{x}{2} + \frac{x^2}{4} + \ ...$	B1	

$$\frac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\frac{1}{2}\right)x^2 + \ ...$$

$= 5 + \ ...$	ft their $A - B + \frac{1}{2}C$	A1 ft	
$= \ ... + \frac{3}{2}x^2 + \ ...$	$0x$ stated or implied	A1 A1	7

[11]

2. (a) $$\text{f}(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

$4 - 2x = A(x + 1)(x + 3) + B(2x + 1)(x + 3) + C(2x + 1)(x + 1)$

A method for evaluating one constant

$x \to -\frac{1}{2}$, $5 = A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \Rightarrow A = 4$	any one correct constant	A1	
$x \to -1$, $6 = B(-1)(2) \Rightarrow B = -3$			
$x \to -3$ $10 = C(-5)(-2) \Rightarrow C = 1$	all three constants correct	A1	4

(b) (i) $$\int\left(\frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{x+3}\right)\text{d}x$$

$$= \frac{4}{2}\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + C$$

A1 two ln terms correct	A1ft	
All three ln terms correct and "+C" ; ft constants	A1ft	3

(ii) $\left[2\ln(2x+1) - 3\ln(x+1) + \ln(x+3)\right]_0^2$

$= (2\ln 5 - 3\ln 3 + \ln 5) - (2\ln 1 - 3\ln 1 + \ln 3)$

$= 3\ln 5 - 4\ln 3$

$= \ln\left(\dfrac{5^3}{3^4}\right)$

$= \ln\left(\dfrac{125}{81}\right)$ A1 3

[10]

3. (a) $27x^2 + 32x + 16 \equiv A(3x + 2)(1 - x) + B(1 - x) + C(3x + 2)^2$ Forming this identity Substitutes either $x = -\frac{2}{3}$ or

$x = -\frac{2}{3}, 12 - \frac{64}{3} + 16 = \left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3} = \left(\frac{5}{3}\right)B \Rightarrow B = 4$ $x = 1$ into their identity or equates 3 terms or substitutes in values to write down three

$x = 1, 27 + 32 + 16 = 25C \Rightarrow 75 = 25C \Rightarrow C = 3$ simultaneous equations. Both $B = 4$ and $C = 3$ A1

(**Note the A1 is dependent on both method marks in this part.)**

Equate x^2: $27 = -3A + 9C \Rightarrow 27 = -3A + 27 \Rightarrow 0 = -3A$

$\Rightarrow A = 0$ Compares coefficients or substitutes in a third x-value or uses simultaneous

$x = 0, 16 = 2A + B + 4C$ equations to show $A = 0$. B1 4

$\Rightarrow 16 = 2A + 4 + 12 \Rightarrow 0 = 2A \Rightarrow A = 0$

(b) $f(x) = \dfrac{4}{(3x+2)^2} + \dfrac{3}{(1-x)}$

$= 4(3x + 2)^{-2} + 3(1 - x)^{-1}$ Moving powers to top on any one of the two expressions

$= 4\left[2(1 + \frac{3}{2}x)^{-2}\right] + 3(1 - x)^{-1}$

$= \left(1 + \frac{3}{2}x\right)^{-2} + 3(1 - x)^{-1}$

$= 1\left\{\underline{1 + (-2)(\tfrac{3x}{2}); + \dfrac{(-2)(-3)}{2!}(\tfrac{3x}{2})^2 + \ldots}\right\}$ Either $1 \pm (-2)(\frac{3x}{2})$; or $1 \pm (-1)(-x)$ from either first or second expansions respectively

Ignoring 1 and 3, any one

$+ 3\left\{\underline{1 + (-1)(-x); + \dfrac{(-1)(-2)}{2!}(-x)^2 + \ldots}\right\}$ correct $\{\underline{\ldots\ldots\ldots\ldots}\}$ A1 expansion. Both {...........} correct. A1

$= \{1 - 3x + \frac{27}{4}x^2 + ...\} + 3\{1 + x + x^2 + ...\}$

$= 4 + 0x; + \frac{39}{4}x^2$ 4 + (0*x*) ; $\frac{39}{4}x^2$ A1; A1 6

(c) Actual = f (0.2) = $\frac{1.08 + 6.4 + 16}{(6.76)(0.8)}$ Attempt to find the actual value of f(0.2)

$= \frac{23.48}{5.408} = 4.341715976... = \frac{2935}{676}$ or seeing awrt 4.3 and believing it is candidate's actual f(0.2).

Or

Actual = f (0.2) = $\frac{4}{(3(0.2)+2)^2} + \frac{3}{(1-0.2)}$

Candidates can also attempt to find the actual value by using $\frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)}$

$= \frac{4}{6.76} + 3.75 = 4.341715976... = \frac{2935}{676}$ with their *A*, *B* and *C*.

Estimate = f(0.2) = $4 + \frac{39}{4}(0.2)^2$ Attempt to find an estimate for f(0.2) using their answer to (b) M1ft

= 4 + 0.39 = 4.39

%age error = $\frac{|4.39 - 4.341715976...|}{4.341715976...} \times 100$

$\left|\frac{\text{their estimate - actual}}{\text{actual}}\right| \times 100$

= 1.112095408... = 1.1%(2sf) 1.1% A1 cao 4

[14]

4. (a) 3*x* – 1 ≡ A (1 – 2x) + B

Let *x* = $\frac{1}{2}$; $\frac{3}{2} - 1 =$ B $\Rightarrow$ B $-\frac{1}{2}$

Considers this identity and either substitutes
x = $\frac{1}{2}$ *, equates coefficients or solves simultaneous equations*

Equate *x* terms; 3 = –2A ⇒ A = $-\frac{3}{2}$ A1;A1 3

$A = -\frac{3}{2}; B = \frac{1}{2}$

***(No working seen,** but A and B correctly stated $\Rightarrow$ award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.)*

(b) $\text{f}(x) = -\frac{3}{2}(1-2x)^{-1} + \frac{1}{2}(1-2x)^{-2}$

Moving powers to top on any one of the two expressions

$= -\frac{3}{2}\left\{\underline{1 + (-1)(-2x); + \frac{(-1)(-2)}{2!}(-2x)^2 + \frac{(-1)(-2)(-3)}{3!}(-2x)^3 + \ldots}\right\}$ dM1

Either 1 ± 2x or 1 ± 4x from either first or second expansions respectively

$+ \frac{1}{2}\left\{\underline{1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots}\right\}$

Ignoring $-\frac{3}{2}$ and $\frac{1}{2}$,
any one correct
{…………} expansion. A1
Both {………..} correct. A1

$= \frac{3}{2}\{1 + 2x + 4x^2 + 8x^3 + \ldots\} + \frac{1}{2}\{1 + 4x + 12x^2 + 32x^3 + \ldots\}$

$= 1 - x; + 0x^2 + 4x^3$ A1; A1 6

$-1 - x; (0x^2) + 4x^3$

[9]

Aliter Way 2

(b) $\text{f}(x) = (3x - 1)(1 - 2x)^{-2}$

Moving power to top

$= (3x-1) \times \left(1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots\right)$ dM1;

1 ± 4x;
Ignoring (3x – 1), correct
(…………..) expansion A1

$= (3x - 1)(1 + 4x + 12x^2 + 32x^8 + \ldots)$

$-3x + 12x^2 + 36x^3 - 1 - 4x - 12x^2 - 32x^3 + \ldots$ Correct expansion A1

$= -1 - x; + 0x^2 + 4x^3$ $-1 - x; (0x^2) + 4x^3$ A1; A1 6

Aliter Way 3

(b) Maclaurin expansion

$\mathrm{f}(x) = -\frac{3}{2}(1-2x)^{-1} + \frac{1}{2}(1-2x)^{-2}$

Bringing (b) both powers to top

$\mathrm{f}'(x) = -3(1-2x)^{-2} + 2(1-2x)^{-3}$

Differentiates to give

$a(1 - 2x)^{-2} \pm b(1 - 2x)^{-3}$;

$-3(1 - 2x)^{-2} + 2(1 - 2x)^{-3}$ A1 oe

$\mathrm{f}''(x) = -12(1-2x)^{-3} + 12(1-2x)^{-4}$

$\mathrm{f}'''(x) = -72(1-2x)^{-4} + 96(1-2x)^{-5}$ A1

Correct $f''(x)$ and $f'''(x)$

$\therefore \mathrm{f}(0) = -1$, $\mathrm{f}'(0) = -1$, $\mathrm{f}''(0) = 0$ and $\mathrm{f}'''(0) = 24$

gives $\mathrm{f}(x) = -1 - x; + 0x^2 + 4x^3 + \ldots$ A1; A1 6

$-1 - x$; $(0x^2) + 4x^3$

Aliter Way 4

(b) $\mathrm{f}(x) = -3(2-4x)^{-1} + \frac{1}{2}(1-2x)^{-2}$

Moving powers to top on any one of the two expressions

$$= -3\left\{\underline{(2)^{-1} + (-1)(2)^{-2}(-4x); + \frac{(-1)(-2)}{2!}(2)^{-3}(-4x)^2 + \frac{(-1)(-2)(-3)}{3!}(2)^{-4}(-4x)^3 + \ldots}\right\}$$ dM1;

Either $\frac{1}{2} \pm x$ or $1 \pm 4x$ from either first or second expansions respectively

$$+\frac{1}{2}\left\{\underline{1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots}\right\}$$

Ignoring -3 and $\frac{1}{2}$,

any one correct

{$\underline{\ldots\ldots\ldots}$} expansion. A1

Both {$\underline{\ldots\ldots\ldots}$} correct. A1

$$= -3\left\{\tfrac{1}{2} + x + 2x^2 + 4x + \ldots\right\} + \tfrac{1}{2}\left\{1 + 4x + 12x^2 + 32x^3 + \ldots\right\}$$

$= 1 - x; +0x^2 + 4x^3$ $-1 - x$; $(0x^2) + 4x^3$ A1; A1 6

5. (a) $\dfrac{9+4x^2}{9-4x^2} = -1 + \dfrac{18}{(3+2x)(3-2x)}$, so $A = -1$ B1

Uses $18 = B(3 - 2x) + C(3 + 2x)$ and attempts to find B and C

$B = 3$ and $C = 3$ A1 A1 4

Or

Uses $9 + 4x^2 = A(9 - 4x^2) + B(3 - 2x) + C(3 + 2x)$ and attempts to find A, B and C

$A = -1$, $B = 3$ and $C = 3$ A1, A1, A1 4

(b) Obtains $Ax + \dfrac{B}{2}\ln(3 + 2x) - \dfrac{C}{2}\ln(3 - 2x)$ A1

Substitutes limits and subtracts to give $2A + \dfrac{B}{2}\ln(5) - \dfrac{C}{2}\ln(\tfrac{1}{5})$ A1ft

$= -2 + 3\ln 5$ or $-2 + \ln 125$ A1 5

[9]

6. (a) $3 + 5x \equiv A(1 - x) + B(1 + 3x)$ Method for A or B

$(x = 1) \Rightarrow 8 = 4B$ $B = 2$ A1

$(x = -\tfrac{1}{3}) \Rightarrow \tfrac{4}{3} = \tfrac{4}{3}A$ $A = 1$ A1 3

(b) $2(1 - x)^{-1} = 2[1 + x + x^2 + \dots]$ [A1]

Use of binomial with n = –1 scores ×2)

$(1 + 3x)^{-1} = [1 - 3x + \tfrac{(-1)(-2)}{2!}(3x)^2 + \dots]$ [A1]

$\therefore \dfrac{3+5x}{(1-x)(1+3x)} = 2 + 2x + 2x^2 + 1 - 3x + 9x^2 = \underline{3 - x + 11x^2}$ A1 5

(c) $(1 + 3x)^{-1}$ requires $|x| < \tfrac{1}{3}$, so expansion is *not* valid. A1 2

[10]

7. (a) Uses $\dfrac{A}{(2x-3)} + \dfrac{B}{(x+1)}$

Considers $-2x + 13 = \text{A}(x + 1) + \text{B}(2x - 3)$ and substitutes $x = -1$ **or** $x = 1.5$, or compares coefficients and solves simultaneous equations

To obtain A = 4 and B = –3. A1, A1 4

(b) Separates variables $\int \frac{1}{y}\,dy = \int \frac{4}{2x-3} - \frac{3}{x+1}\,dx$

$\ln y = 2\ln(2x-3) - 3\ln(x+1) + C$ A1, B1 ft

Substitutes to give $\ln 4 = 2\ln 1 - 3\ln 3 + C$ and finds C (ln 108)

$\ln y = \ln(2x-3)^2 - \ln(x+1)^3(+\ln 108)$

$= \ln\frac{C(2x-3)^2}{(x+1)}$ A1

$\therefore y = \frac{108(2x-3)^2}{(x+1)^3}$ A1 cso 7

Or $y = e^{2\ln(2x-3) - 3\ln(x+1) + \ln 108}$ special case A2

[11]

8. (a) $\frac{3(x+1)}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1}$, and correct method for finding A or B

$A = 1$, $B = 2$ A1, A1 3

(b) $f'(x) = -\frac{1}{(x+2)^2} - \frac{2}{(x-1)^2}$ A1

Argument for negative, including statement that square terms are positive for all values of x. (f.t. on wrong values of A and B) A1 ft 3

[6]

9. (a) $\frac{1+14x}{(1-x)(1+2x)} \equiv \frac{A}{1-x} + \frac{B}{1+2x}$ and attempt A and or B

$A = 5$, $B = -4$ A1, A1 3

(b) $\int \frac{5}{1-x} - \frac{4}{1+2x}\,dx = [-5\ln|1-x| - 2\ln|1+2x|]$ A1

$= (-5\ln\frac{2}{3} - 2\ln\frac{5}{3}) - (-5\ln\frac{5}{6} - 2\ln\frac{4}{3})$

$= 5\ln\frac{5}{4} + 2\ln\frac{4}{5}$

$= 3\ln\frac{5}{4} = \ln\frac{125}{64}$ A1 5

(c)	$5(1 - x)^{-1} - 4(1 + 2x)^{-1}$	B1 ft	
	$= 5(1 + x + x^2 + x^3) - 4$ $\left(1 - 2x + \frac{(-1)(-2)(2x)^2}{2} + \frac{(-1)(-2)(-3)(2x)^3}{6} + \ldots\right)$	A1	
	$= 1 + 13x - 11x^2 + 37x^3 \ldots$	A1	5

[13]

Ch.1 Partial Fractions Examiner Reports

1. The first part of question 5 was generally well done. Those who had difficulty generally tried to solve sets of relatively complicated simultaneous equations or did long division obtaining an incorrect remainder. A few candidates found *B* and *C* correctly but either overlooked finding *A* or did not know how to find it. Part (b) proved very testing. Nearly all were able to make the connection between the parts but there were many errors in expanding both $(x - 1)^{-1}$ and $(2 + x)^{-1}$. Few were able to write $(x - 1)^{-1}$ as $-(1 - x)^{-1}$ and the resulting expansions were incorrect in the majority of cases, both $1 + x - x^2$ and $1 - x - x^2$ being common.

$(2 + x)^{-1}$ was handled better but the constant $\frac{1}{2}$ in $\frac{1}{2}\left(1+\frac{x}{2}\right)^{-1}$ was frequently incorrect. Most recognised that they should collect together the terms of the two expansions but a few omitted their value of *A* when collecting the terms.

2. Part (a) was well done with the majority choosing to substitute values of x into an appropriate identity and obtaining the values of *A*, *B* and *C* correctly. The only error commonly seen was failing to solve $5 = \frac{5}{4}A$ for A correctly. Those who formed simultaneous equations in three unknowns tended to be less successful. Any incorrect constants obtained in part (a) were followed through for full marks in part (b)(i). Most candidates obtained logs in part (b)(i). The commonest error was, predictably, giving $\int \frac{4}{2x+1}dx = 4\ln(2x+1)$, although this error was seen less frequently than in some previous examinations. In indefinite integrals, candidates are expected to give a constant of integration but its omission is not penalised repeatedly throughout the paper. In part (b)(ii) most applied the limits correctly although a minority just ignored the lower limit 0. The application of log rules in simplifying the answer was less successful. Many otherwise completely correct solutions gave 3 ln 3 as ln 9 and some "simplified" 3 ln 5 – 4 ln 3 to $\frac{3}{4}\ln\left(\frac{5}{3}\right)$.

3. Part (a) was tackled well by many candidates. The majority of candidates were able to write down the correct identity. The most popular strategy at this stage (and the best!) was for candidates to substitute $x = 1$ and $x = -\frac{2}{3}$ into their identity to find the values of the constants *B* and *C*. The substitution of $x = -\frac{2}{3}$ caused problems for a few candidates which led them to find an incorrect value for *B*. Many candidates demonstrated that constant *A* was zero by use of a further value of *x* or by comparing coefficients in their identity. A significant minority of candidates manipulated their original identity and then compared coefficients to produce three equations in order to solve them simultaneously.

In part (b), most candidates were able to rewrite their partial fractions with negative powers and apply the two binomial expansions correctly, usually leading to the correct answer. A significant minority of candidates found the process of manipulating $4(3x + 2)^{-2}$ to $\left(1+\frac{3}{2}x\right)^{-2}$ challenging.

A significant number of candidates were unsure of what to do in part (c). Some candidates found the actual value only. Other candidates found the estimated value only. Of those who progressed further, the most common error was to find the difference between these values and then divide by their estimate rather than the actual value. Some candidates did not follow the instruction to give their final answer correct to 2 significant figures and thus lost the final accuracy mark.

4. In part (a) candidates needed to start with the correct identity; although correct solutions were seen from a good proportion of the candidature, a significant number of candidates started with the wrong identity and thus gained no marks. The most common wrong starting point was to use $3x - \equiv A(1-2x)^2 + B(1-2x)$, but $3x-1 \equiv A(1-2x)^2+B$ and $3x-1 \equiv A(1-2x)^2+Bx$ also occasionally appeared. Candidates using the first identity often produced answers A= $\frac{1}{2}$, B= $-\frac{3}{2}$; the same values but for the wrong constants. Candidates using the second identity could produce the 'correct' answers B= $\frac{1}{2}$,A= $-\frac{3}{2}$ (eg by setting $x = 0$ and $x = \frac{1}{2}$) but this is fortuitous and clearly gains no marks.

Generally candidates showed a good understanding of the work on expanding series in part (b) and most were able to gain some credit. The mark scheme allowed four marks to be gained for the correct unsimplified expansions, as far as the term in x^3, of $(1-2x)^{-1}$ and $(1-2x)^{-2}$ This helped some candidates who went on to make numerical or sign errors when simplifying their expansions and errors in part (a) only affected the final two accuracy marks.

Candidates who multiplied $(3x-1)$ by the expansion of $(1-2x)^{-2}$ gave solutions that were not dependent on their answers in part (a) and it was not uncommon to see a score of zero marks in part (a) followed by a score of six marks in part (b).

5. Many correct answers were seen to part (a). Candidates who used long division were generally less successful often leading to the misuse of $1+\frac{8x^2}{9-4x^2}$ to attempt partial fractions. The few candidates who ignored the 'hence' in part (b) made no progress in their integration, but would not have gained any marks anyway. Although candidates knew $\int \frac{1}{ax+b}\,dx = k\ln(ax+b)$, they were less accurate with the value of k. This question provided another challenge for candidates who were careless in their use of brackets whose answers often led to giving –[–(–1)] as +1. A few candidates ignored the requirement for an exact value.

6. Part (a) was answered very well by almost all the candidates and there were few numerical errors seen. The binomial expansion was used confidently in part (b) but the usual sign slips and lost brackets led to a number of errors. Most used their values of A and B from part (a) and addition to obtain the final expansion but occasionally a candidate tried to multiply the series together. Some weaker candidates thought that $\frac{2}{(1-x)} = (1-x)^{-2}$. Part (c) was not answered well. Some knew the conditions and explained thoroughly that the expansion was only valid for $|x| < \frac{1}{3}$ and therefore $x = \frac{1}{2}$ was not valid. Others tried substituting $x = \frac{1}{2}$ and declared it was invalid when the series gave a different answer to the original expression, and a few thought that the only condition was that $1+3x \neq 0$ and $1-x \neq 0$.

7. The partial fractions were found easily by most of the candidates, with very few errors. Some had difficulty separating the variables, but it was still possible to continue with the answer and to obtain some credit. The log integration was performed well this time and the majority of difficulties were finding the correct constant and using it correctly in conjunction with combining the logs.

8. Most answered part (a) correctly, and errors were usually because candidates had miscopied a sign, or written $3x + 1$ instead of $3(x + 1)$.

The differentiation was usually correct, though a minority integrated and some misunderstood the notation and found the inverse function instead of the derived function. Those who returned to the original expression and differentiated gave themselves more work. Many found it difficult to answer the final part of this question. The examiners were looking for a statement that square terms are always positive

9. No Report available for this question.