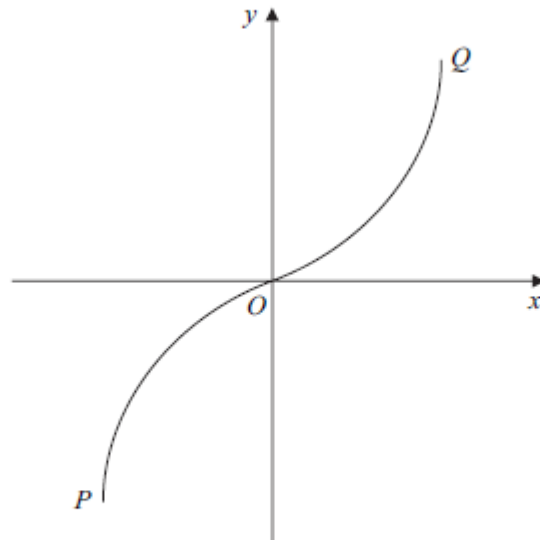


Function Questions

- 7 (a) The sketch shows the graph of $y = \sin^{-1}x$.



Write down the coordinates of the points P and Q , the end-points of the graph.

(2 marks)

- (b) Sketch the graph of $y = -\sin^{-1}(x - 1)$.

(3 marks)

- 8 The functions f and g are defined with their respective domains by

$$f(x) = x^2 \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{x+2} \quad \text{for real values of } x, \quad x \neq -2$$

- (a) State the range of f . (1 mark)
- (b) (i) Find $fg(x)$. (1 mark)
- (ii) Solve the equation $fg(x) = 4$. (4 marks)
- (c) (i) Explain why the function f does **not** have an inverse. (1 mark)
- (ii) The inverse of g is g^{-1} . Find $g^{-1}(x)$. (3 marks)
-

4 (a) Sketch and label on the same set of axes the graphs of:

(i) $y = |x|$; (1 mark)

(ii) $y = |2x - 4|$. (2 marks)

(b) (i) Solve the equation $|x| = |2x - 4|$. (3 marks)

(ii) Hence, or otherwise, solve the inequality $|x| > |2x - 4|$. (2 marks)

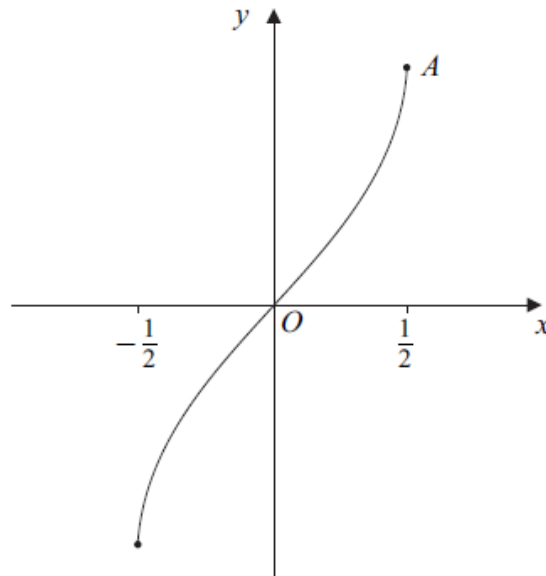
8 A function f is defined by $f(x) = 2e^{3x} - 1$ for all real values of x .

(a) Find the range of f . (2 marks)

(b) Show that $f^{-1}(x) = \frac{1}{3} \ln \left(\frac{x+1}{2} \right)$. (3 marks)

(c) Find the gradient of the curve $y = f^{-1}(x)$ when $x = 0$. (4 marks)

9 The diagram shows the curve with equation $y = \sin^{-1} 2x$, where $-\frac{1}{2} \leq x \leq \frac{1}{2}$.



(a) Find the y -coordinate of the point A , where $x = \frac{1}{2}$. (1 mark)

3 The functions f and g are defined with their respective domains by

$$f(x) = 3 - x^2, \text{ for all real values of } x$$

$$g(x) = \frac{2}{x+1}, \text{ for real values of } x, x \neq -1$$

(a) Find the range of f . *(2 marks)*

(b) The inverse of g is g^{-1} .

(i) Find $g^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1} . *(1 mark)*

(c) The composite function gf is denoted by h .

(i) Find $h(x)$, simplifying your answer. *(2 marks)*

(ii) State the greatest possible domain of h . *(1 mark)*

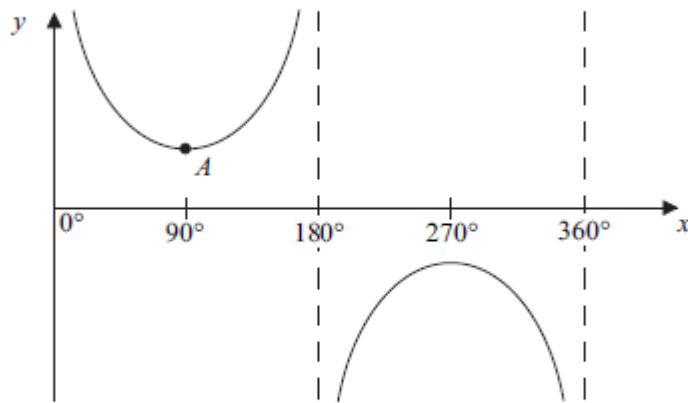
7 (a) Sketch the graph of $y = |2x|$. *(1 mark)*

(b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(3 marks)*

(c) Solve $4 - |2x| = x$. *(3 marks)*

(d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. *(2 marks)*

- 3 (a) Solve the equation $\operatorname{cosec} x = 2$, giving all values of x in the interval $0^\circ < x < 360^\circ$.
(2 marks)
- (b) The diagram shows the graph of $y = \operatorname{cosec} x$ for $0^\circ < x < 360^\circ$.



- (i) The point A on the curve is where $x = 90^\circ$. State the y -coordinate of A .
(1 mark)
- (ii) Sketch the graph of $y = |\operatorname{cosec} x|$ for $0^\circ < x < 360^\circ$.
(2 marks)
- (c) Solve the equation $|\operatorname{cosec} x| = 2$, giving all values of x in the interval $0^\circ < x < 360^\circ$.
(2 marks)
-

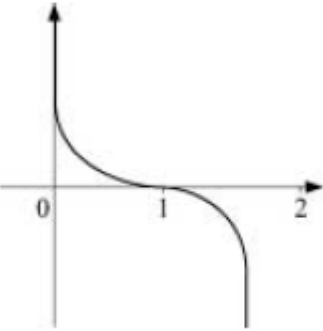
- 5 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{x-2} \quad \text{for } x \geq 2$$

$$g(x) = \frac{1}{x} \quad \text{for real values of } x, \quad x \neq 0$$

- (a) State the range of f .
(2 marks)
- (b) (i) Find $fg(x)$.
(1 mark)
- (ii) Solve the equation $fg(x) = 1$.
(3 marks)
- (c) The inverse of f is f^{-1} . Find $f^{-1}(x)$.
(3 marks)
-

Functions Answers

7(a)	$\left(1, \frac{\pi}{2}\right)$ OE in decimals $\left(-1, -\frac{\pi}{2}\right)$	B1		Or for -1 and 1
		B1	2	
(b)		M1		Translation in +ve x direction
		M1		Correct shape
		A1	3	Correct Graph Through $(1,0)$ touching y -axis
8(a)	$(\text{Range of } f) \geq 0$	B1	1	
(b)(i)	$fg(x) = \frac{1}{(x+2)^2}$	B1	1	OE Maybe in part (ii)
(ii)	$\frac{1}{(x+2)^2} = 4$			
	$(x+2)^2 = \frac{1}{4}$	M1		Or $4(x+2)^2 = 1$
	$x+2 = (\pm)\frac{1}{2}$	M1		$(2x+5)(2x+3) = 0$
	$x = -\frac{5}{2}, -\frac{3}{2}$	A1 A1	4	
(c)(i)	Not one to one	E1	1	OE
(ii)	$x = \frac{1}{y+2}$	M1		$x \Leftrightarrow y$
	$y+2 = \frac{1}{x}$	M1		Attempt to isolate
	$y = \frac{1}{x} - 2 \quad \left(\frac{1-2x}{x}\right)$	A1	3	
Total			10	

4(a)(i)		B1	1	$y = x $
(ii)		M1		2 branches mod graph $x > 0$ for $y = 0$
		A1	2	for 2, 4
(b)(i)	$x = 2x - 4, x = 4$ $-x = 2x - 4$ $x = \frac{4}{3}$	B1		
	Alternative: $x^2 = (2x - 4)^2$ $x = 4, \frac{4}{3}$	M1	3	OE one value only
		A1A1		
(ii)	$\frac{4}{3} < x < 4$	M1		$\frac{4}{3}, 4$ (ft) identified as extremes
		A1	2	CAO
Total			8	

8(a)	$f(x) = 2e^{3x} - 1$			
	Range: $f(x) > -1$ (or $y > -1$ or $f > -1$)	M1		for -1 only
		A1	2	exactly correct
(b)	$y = 2e^{3x} - 1$ $x = 2e^{3y} - 1$ $2e^{3y} = x + 1$ $e^{3y} = \frac{x+1}{2}$ $y = \frac{1}{3} \ln\left(\frac{x+1}{2}\right)$	M1		$x \leftrightarrow y$
		M1		attempt to isolate
		A1	3	all correct with no error AG (be convinced)
(c)	$f^{-1}(x) = \frac{1}{3} \left(\frac{2}{x+1} \right) \times \frac{1}{2}$ OE	M1		for differentiation of $\ln; \frac{k}{\text{their}(x \pm 1)}$
		A1		for $\frac{1}{2}$
		A1		all correct
	$x = 0$			
	$f^{-1}(x) = \frac{1}{3}$	A1	4	CSO
	Alternative			
	$f^{-1}(x) = \frac{1}{3} \ln(x+1) - \frac{1}{3} \ln 2$	M1A1		
	$f^{-1}(x) = \frac{1}{3(x+1)}$	A1		
	$f^{-1}(0) = \frac{1}{3}$	A1		CSO
Total			9	

9(a)	$x = \frac{1}{2} \quad y = \frac{\pi}{2}$ (or 1.57, $\sin^{-1}1$)	B1	1	ignore 90°
3(a)	$f(x) \leq 3$	M1A1	2	M1 for $f < 3, x \leq 3$ Condone y, f, range
(b)(i)	$y = \frac{2}{x+1}$ $x+1 = \frac{2}{y}$ $x = \frac{2}{y} - 1$ $y/g^{-1}(x) = \frac{2}{x} - 1 = \frac{2-x}{x}$	M1 M1 A1	3	Attempt to obtain x as a function of y or y as a function of x $x \leftrightarrow y$ at any stage Any correct form
(ii)	$(g^{-1}(x)) \neq -1$	B1	1	
(c)(i)	$h(x) = \frac{2}{3-x^2+1}$ $= \frac{2}{4-x^2} = \frac{2}{(2-x)(2+x)}$	M1 A1	2	
(ii)	$(x \in \mathbb{R}), x \neq +2, x \neq -2$	B1	1	Condone omit 'x is real' Allow $x^2 \neq 4$
Total			9	

7(a)		B1	1	
(b)		M1 A1 A1	3	Shape inverted V in all four quadrants Symmetrical about y axis Coordinates
(c)	$4 - 2x = x$ $4 - 2x = x \quad x = \frac{4}{3}$ $4 + 2x = x \quad x = -4$	M1 A1 A1	3	Attempt to solve And no others
(d)	$-4 < x < \frac{4}{3}$	M1 A1	2	Either correct Other solution and no extras SC $-4 \leq x \leq \frac{4}{3}$ B1

3(a)	$\operatorname{cosec} x = 2$ $\Rightarrow \sin x = \frac{1}{2}$ $x = 30, 150$	M1		30° scores M1 implied
		A1	2	and no extras in range
(b)(i)	1	B1	1	
(ii)		M1		all positive, 2 U shapes
		A1	2	minima consistent > 0, not intersecting with each other or y-axis
(c)	$x = 30, 150, 210, 330$	B1F		3 correct values from their (a), which must be $\theta, 180 - \theta$
		B1	2	all correct and no extras in range

5(a)	$f(x) \geq 0$	allow $y \geq 0$	M1		> 0 or $f \geq 0$ or ≥ 0
			A1	2	
(b)(i)	$\sqrt{\frac{1}{x} - 2}$		B1	1	
(ii)	$\frac{1}{x} - 2 = 1$		M1		squaring their (b)(i) in an equation
	$\frac{1}{x} = 3$	OE	A1		
	$x = \frac{1}{3}$		A1	3	CSO
(c)	$y = \sqrt{x-2}$		M1		attempt to isolate; condone 1 slip
	$y^2 = x-2$		M1		reverse $x \Leftrightarrow y$
	$x^2 = y-2$		A1	3	
	$y = x^2 + 2$				
Total				9	