Algebra & Functions Questions

1 (a) The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 3x^3 + 2x^2 - 7x + 2$.

(i) Find $\mathrm{f}(1)$. *(1 mark)*

(ii) Show that $\mathrm{f}(-2) = 0$. *(1 mark)*

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax+b}$$

where a and b are integers. *(3 marks)*

(b) The polynomial $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = 3x^3 + 2x^2 - 7x + d$.

When $\mathrm{g}(x)$ is divided by $(3x-1)$, the remainder is 2. Find the value of d. *(3 marks)*

(c) Given that $\dfrac{2x^2 - 3}{(3-2x)(1-x)^2}$ can be written in the form $\dfrac{A}{(3-2x)} + \dfrac{B}{(1-x)} + \dfrac{C}{(1-x)^2}$,

find the values of A, B and C. *(5 marks)*

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1 (a) The polynomial $\mathrm{p}(x)$ is defined by $\mathrm{p}(x) = 6x^3 - 19x^2 + 9x + 10$.

(i) Find $\mathrm{p}(2)$. *(1 mark)*

(ii) Use the Factor Theorem to show that $(2x+1)$ is a factor of $\mathrm{p}(x)$. *(3 marks)*

(iii) Write $\mathrm{p}(x)$ as the product of three linear factors. *(2 marks)*

(b) Hence simplify $\dfrac{3x^2 - 6x}{6x^3 - 19x^2 + 9x + 10}$. *(2 marks)*

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3 (a) Given that $\dfrac{9x^2 - 6x + 5}{(3x-1)(x-1)}$ can be written in the form $3 + \dfrac{A}{3x-1} + \dfrac{B}{x-1}$, where A and B are integers, find the values of A and B. *(4 marks)*

(b) Hence, or otherwise, find $\displaystyle\int \frac{9x^2 - 6x + 5}{(3x-1)(x-1)}\,\mathrm{d}x$. *(4 marks)*

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2 The polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 13$.

(a) Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(2x - 3)$. *(2 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + 13 + d$, where d is a constant.

Given that $(2x - 3)$ is a factor of $g(x)$, show that $d = -4$. *(2 marks)*

(c) Express $g(x)$ in the form $(2x - 3)(x^2 + ax + b)$. *(2 marks)*

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4 (a) (i) Express $\dfrac{3x-5}{x-3}$ in the form $A + \dfrac{B}{x-3}$, where A and B are integers. *(2 marks)*

(ii) Hence find $\displaystyle\int \frac{3x-5}{x-3}\,dx$. *(2 marks)*

(b) (i) Express $\dfrac{6x-5}{4x^2-25}$ in the form $\dfrac{P}{2x+5} + \dfrac{Q}{2x-5}$, where P and Q are integers. *(3 marks)*

(ii) Hence find $\displaystyle\int \frac{6x-5}{4x^2-25}\,dx$. *(3 marks)*

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1 (a) Find the remainder when $2x^2 + x - 3$ is divided by $2x + 1$. *(2 marks)*

(b) Simplify the algebraic fraction $\dfrac{2x^2 + x - 3}{x^2 - 1}$. *(3 marks)*

(b) Express $\dfrac{1+4x}{(1+x)(1+3x)}$ in partial fractions. *(3 marks)*

Algebra & Functions Answers

1(a)(i)	$f(1)=0$	B1	1	
(ii)	$f(-2)=-24+8+14+2=0$	B1	1	
(iii)	$\frac{(x-1)(x+2)}{3x^3+2x^2-7x+2}=\frac{(x-1)(x+2)}{(x-1)(x+2)(ax+b)}$	B1		Recognising $(x-1),(x+2)$ as factors PI
	$ax^3=3x^3$ $\quad -2b=2$ $a=3$ $\quad b=-1$	B1 B1	3	a b **Or** By division M1 attempt started M1 complete division A1 Correct answers
(b)	Use $\frac{1}{3}$	B1		
	$3\left(\frac{1}{3}\right)^3+2\left(\frac{1}{3}\right)^2-7\times\frac{1}{3}+d=2$	M1		Remainder Th$^{\underline{M}}$ with $\pm\frac{1}{3,}\pm 3$
	$d=4$	A1F	3	Ft on $-\frac{1}{3}\left(\text{answer}-\frac{4}{9}\right)$ **Or** by division M1 M1 A1 as above
	Total		**8**	

5(c)	$2x^2-3=$ $A(1-x)^2+B(3-2x)(1-x)+C(3-2x)$	M1		**Or** by equating coefficients
	$x=1\quad -1=C\times 1\quad x=\frac{3}{2}\quad \frac{3}{2}=A\times\frac{1}{4}$	M1		M1 same A1 collect terms M1 equate coefficients A1 2 correct A1 3 correct
	$C=-1$ $\quad A=6$	A1		Follow on A and C
	$x=0\quad (-3=6+3B-3)$ or other value $\Rightarrow$ equation in A, B, C	m1		
	$B=-2$	A1	5	

1 (a)(i)	$p(2)=0$	B1	1	
(ii)	See $-\frac{1}{2}$	B1		
	$p\left(-\frac{1}{2}\right)=6\times\left(-\frac{1}{8}\right)-19\times\frac{1}{4}+9\left(-\frac{1}{2}\right)+10$	M1		Use $\pm\frac{1}{2}$
	$=0$	A1	3	Arithmetic to show = 0 and conclusion. Long division : 0/3
(iii)	$p(x)=(2x+1)(x-2)(3x-5)$	B1 B1	2	$x-2$ Complete expression
(b)	$\frac{3x(x-2)}{(2x+1)(x-2)(3x-5)}$	M1		For $\frac{3x(x-2)}{\text{their (a)(iii)}}$
	$=\frac{3x}{(2x+1)(3x-5)}$	A1	2	Or $\frac{3x}{6x^2-7x-5}$ No ISW on A1
	Total		**8**	

3(a)	$9x^2-6x+5$ $=3(3x-1)(x-1)+A(x-1)+B(3x-1)$	B1		Or $3+\frac{6x+2}{(3x-1)(x-1)}$
	$x=1$ $x=\frac{1}{3}$	M1		Substitute $x=1$ or $x=\frac{1}{3}$
	$B=4$ $A=-6$	A1A1	4	Or equivalent method (equating coefficients, simultaneous equations)
(b)	$\int=\int 3-\frac{6}{3x-1}+\frac{4}{x-1}\,dx$	M1		Attempt to use partial fractions
	$=3x\ldots$	B1		
	$-2\ln(3x-1)+4\ln(x-1)(+c)$	M1		$p\ln(3x-1)+q\ln(x-1)$ Condone missing brackets
		A1F	4	Follow through on A and B; brackets needed.
	Total		**8**	

Q	Solution	Marks	Total	Comments
2(a)	$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 7\left(\frac{3}{2}\right)^2 + 13$	M1		Substitute $\pm\frac{3}{2}$ in f(x)
	$= 4$	A1	2	
(b)	$g\left(\frac{3}{2}\right) = 0 \Rightarrow d + 4 = 0 \Rightarrow d = -4$	M1A1	2	AG (convincingly obtained) SC Written explanation with $g\left(\frac{3}{2}\right) = 0$ not seen/clear E2,1,0
(c)	$a = -2$, $b = -3$	B1, B1	2	Inspection expected By division: M1 – complete method A1 CAO Multiply out and compare coefficients: M1 – evidence of use A1 – both a and b correct
	Total		**6**	

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{3x-5}{x-3} = 3 + \frac{4}{x-3}$	B1, B1	2	By division: B1 for 3, B1 for $\frac{4}{x-3}$ or $B = 4$ By partial fractions: M1 multiply by $x - 3$ and using 2 values of x, A1 both correct
(ii)	$\int 3 + \frac{4}{x-3}\,dx = 3x + 4\ln(x-3)(+c)$	M1A1F	2	M1 $\int 3 + \frac{4}{x-3}\,dx$ and attempt at integrals ft on A and B; condone omission of brackets around $x - 3$
	Alternative: By substitution $u = x - 3$			
	$\int \frac{3x-5}{x-3}\,dx = \int \frac{3u+4}{u}\,du$	(M1)		Integral in terms of u
	$= 3(x-3) + 4\ln(x-3)$	(A1)		Correct, in x
(b)(i)	$6x - 5 = P(2x-5) + Q(2x+5)$	M1		Clear evidence of use of cover-up rule M2
	$x = \frac{5}{2}$ $\quad x = -\frac{5}{2}$	m1		
	$10 = 10Q$ $\quad -20 = -10P$			
	$Q = 1$ $\quad P = 2$	A1	3	
(ii)	$\int \frac{2}{2x+5} + \frac{1}{2x-5}\,dx$	M1		
	$\ln(2x+5) + \frac{1}{2}\ln(2x-5)(+c)$	M1 A1F	3	ft on P and Q; must have brackets
	Total		**10**	

1(a)	$2\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)-3=-3$	M1A1	2	use of $\pm\frac{1}{2}$ SC NMS –3 1/2 No ISW, so subsequent answer "3" AO
	Alt algebraic division: $2x+1\overline{)2x^2+x-3}$ quotient x $2x^2+x$ -3	(M1)		complete division with integer remainder
		(A1)	(2)	remainder = –3 stated, or –3 highlighted
	Alt $\frac{x(2x+1)-3}{2x+1}$	(M1)		attempt to rearrange numerator with $(2x+1)$ as a factor
		(A1)	(2)	remainder = –3 stated, or –3 highlighted
(b)	$\frac{(2x+3)(x-1)}{(x+1)(x-1)}$	B1 B1		numerator, denominator } not necessarily in fraction
	$=\frac{2x+3}{x+1}$	B1	3	CAO in this form. Not $\frac{2x+3}{x+1}\frac{\cancel{x-1}}{\cancel{x-1}}$
(b)	**Alternative** $\frac{2x^2-2+x-1}{x^2-1}$			
	$=2+\frac{x-1}{x^2-1}$	(M1)		
	$=2+\frac{x-1}{(x-1)(x+1)}$	(B1)		
	$=2+\frac{1}{x+1}$	(A1)	(3)	
	Total		**5**	

(b)	$\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$ $1+4x=A(1+3x)+B(1+x)$	M1		correct partial fractions form, and multiplication by denominator
	$x=-1,\ x=-\frac{1}{3}$	m1		Use (any) two values of x to find A and B
	$A=\frac{3}{2},\ B=-\frac{1}{2}$	A1	3	A and B both correct
	Alt: $\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$ $1+4x=A(1+3x)+B(1+x)$	(M1)		correct partial fractions form, and multiplication by denominator
	$A+B=1,\ 3A+B=4$	(m1)		Set up and solve
	$A=\frac{3}{2},\ B=-\frac{1}{2}$	(A1)	(3)	A and B both correct