# GRAPHS OF FUNCTIONS — *Answers*

**1** **a** $4x^2 - 9x + 5 = 3x - 4$

$4x^2 - 12x + 9 = 0$

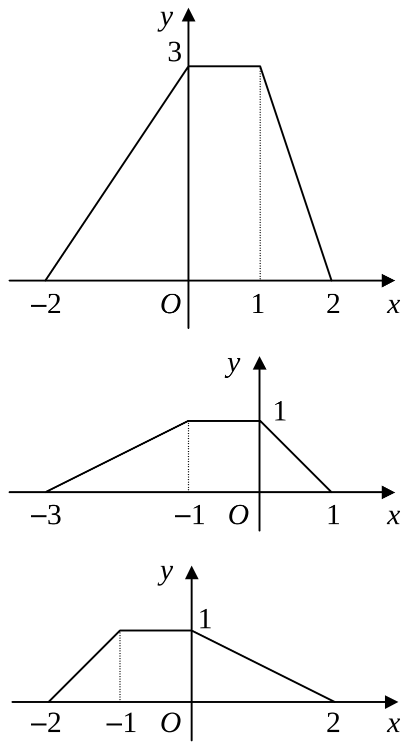
$(2x - 3)^2 = 0$

$x = \frac{3}{2}$

$\therefore\ x = \frac{3}{2},\ y = \frac{1}{2}$

**b** $y = 3x - 4$ is a tangent to the curve $y = 4x^2 - 9x + 5$ at the point $(\frac{3}{2}, \frac{1}{2})$

**2** **a**




**3** **a** $x^2 + 5x + 2 = 4x + 1$

$x^2 + x + 1 = 0$

$b^2 - 4ac = 1 - 4 = -3$

$b^2 - 4ac < 0\ \therefore$ no real roots

$\therefore$ does not intersect

**b** $x^2 + 5x + 2 = mx + 1$

$x^2 + (5 - m)x + 1 = 0$

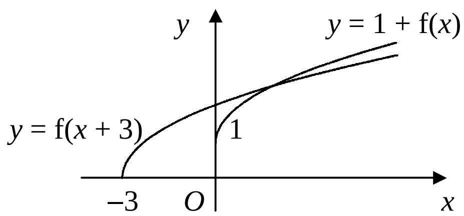
only one root $\therefore\ b^2 - 4ac = 0$

$(5 - m)^2 - 4 = 0$

$5 - m = \pm 2$

$m = 3$ or $7$

**4** **a**




**b** $1 + \sqrt{x} = \sqrt{x+3}$

$(1 + \sqrt{x})^2 = x + 3$

$1 + 2\sqrt{x} + x = x + 3$

$\sqrt{x} = 1$

$x = 1\ \therefore\ (1, 2)$

**5** $x^2 + kx - 3 = k - x$

$x^2 + (k + 1)x - (k + 3) = 0$

$b^2 - 4ac = (k + 1)^2 + 4(k + 3)$

$= k^2 + 6k + 13$

$= (k + 3)^2 - 9 + 13$

$= (k + 3)^2 + 4$

real $k \Rightarrow (k + 3)^2 \geq 0$

$\Rightarrow (k + 3)^2 + 4 \geq 4$

$\therefore\ b^2 - 4ac > 0$

$\Rightarrow$ real and distinct roots

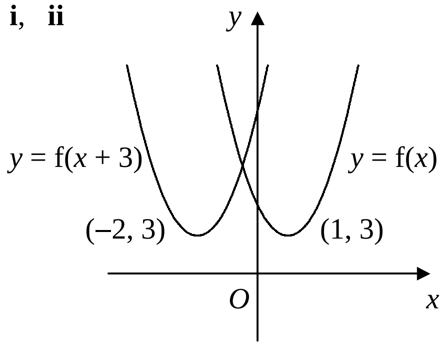
$\therefore\ l$ intersects $C$ at exactly two points

**6** **a** $f(x) = 2[x^2 - 2x] + 5$

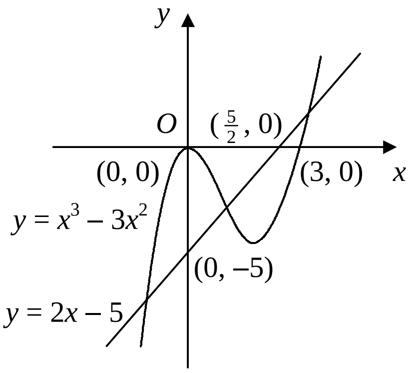
$= 2[(x - 1)^2 - 1] + 5$

$= 2(x - 1)^2 + 3$

**b** **i**, **ii**



**7** **a** $y = x^3 - 3x^2 = x^2(x - 3)$




**b** 3 real roots

$x^3 - 3x^2 - 2x + 5 = 0 \Rightarrow x^3 - 3x^2 = 2x - 5$

the graphs of $y = x^3 - 3x^2$ and $y = 2x - 5$ intersect at three points

**8** touches $x$-axis at (2, 0)

$\therefore\; y = k(x - 2)^2$

crosses $y$-axis at (0, –6)

$\therefore\; -6 = 4k$

$k = -\frac{3}{2}$

$\therefore\; y = -\frac{3}{2}(x - 2)^2$

$y = -\frac{3}{2}x^2 + 6x - 6$

$\therefore\; a = -\frac{3}{2}$, $b = 6$ and $c = -6$

**9** **a** LHS $= (1 - x)(2 + x)^2$

$= (1 - x)(4 + 4x + x^2)$

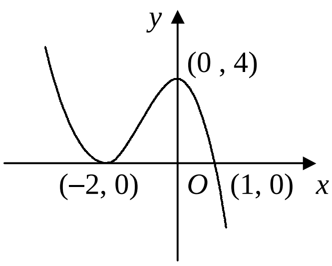
$= (4 + 4x + x^2) - x(4 + 4x + x^2)$

$= 4 + 4x + x^2 - 4x - 4x^2 - x^3$

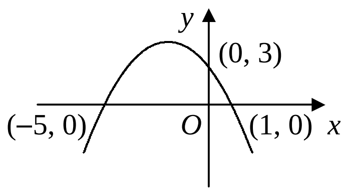
$= 4 - 3x^2 - x^3$

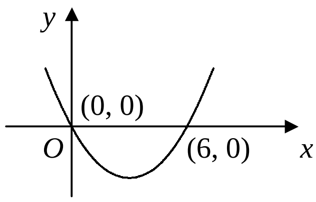
= RHS

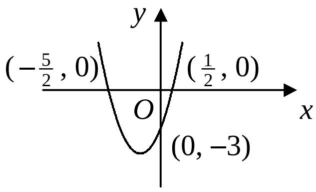
**b**




**10** **a**

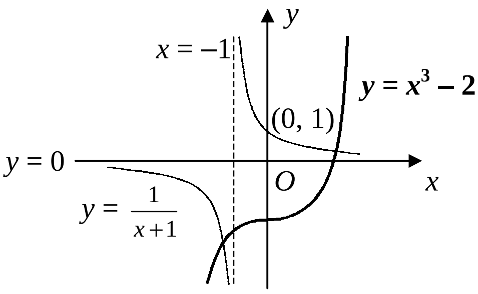



**11** **a** translation by 1 unit in the negative $x$-direction

**b**




**c** $x^3 - \frac{1}{x+1} = 2 \Rightarrow x^3 - 2 = \frac{1}{x+1}$

the graphs $y = x^3 - 2$ and $y = \frac{1}{x+1}$ intersect

at one point for $x > 0$ and at one point for $x < 0$

$\therefore$ one positive and one negative real root