

## ALGEBRA

## Answers

1 a  $f(-2) = -8 + 4 + 44 - 40 = 0$

$\therefore (x + 2)$  is a factor of  $f(x)$

b

$$\begin{array}{r} x^2 - x - 20 \\ x + 2 \overline{) x^3 + x^2 - 22x - 40} \\ \underline{x^3 + 2x^2} \phantom{- 40} \\ -x^2 - 22x \phantom{- 40} \\ \underline{-x^2 - 2x} \phantom{- 40} \\ -20x - 40 \\ \underline{-20x - 40} \\ 0 \end{array}$$

$\therefore f(x) \equiv (x + 2)(x^2 - x - 20)$

$\equiv (x + 2)(x + 4)(x - 5)$

c  $f(x) = 0 \Rightarrow (x + 2)(x + 4)(x - 5) = 0$   
 $x = -4, -2$  or  $5$

3 a  $= p(-2) = -16 - 36 + 4 + 11 = -37$

b

$$\begin{array}{r} 2x^2 - x - 6 \\ x - 4 \overline{) 2x^3 - 9x^2 - 2x + 11} \\ \underline{2x^3 - 8x^2} \phantom{- 2x + 11} \\ -x^2 - 2x \phantom{+ 11} \\ \underline{-x^2 + 4x} \phantom{+ 11} \\ -6x + 11 \\ \underline{-6x + 24} \\ -13 \end{array}$$

$\therefore$  quotient  $= 2x^2 - x - 6$

remainder  $= -13$

5 a  $f(1) = 0$

$\therefore 1 - 3 + k + 8 = 0$

$k = -6$

b

$$\begin{array}{r} x^2 - 2x - 8 \\ x - 1 \overline{) x^3 - 3x^2 - 6x + 8} \\ \underline{x^3 - x^2} \phantom{- 6x + 8} \\ -2x^2 - 6x \phantom{+ 8} \\ \underline{-2x^2 + 2x} \phantom{+ 8} \\ -8x + 8 \\ \underline{-8x + 8} \\ 0 \end{array}$$

$\therefore f(x) = (x - 1)(x^2 - 2x - 8)$

$= (x - 1)(x + 2)(x - 4)$

$f(x) = 0 \Rightarrow x = -2, 1, 4$

2 a  $f(2) = f(-3)$

$\therefore 8 - 8 + 2k + 1 = -27 - 18 - 3k + 1$

$k = -9$

b  $= f(-2) = -8 - 8 + 18 + 1 = 3$

4 a  $A$  is  $(0, 12)$

b  $x = 1$  is a root of  $y = 0$

$\therefore (x - 1)$  is a factor of  $y$

$$\begin{array}{r} x^2 - 4x - 12 \\ x - 1 \overline{) x^3 - 5x^2 - 8x + 12} \\ \underline{x^3 - x^2} \phantom{- 8x + 12} \\ -4x^2 - 8x \phantom{+ 12} \\ \underline{-4x^2 + 4x} \phantom{+ 12} \\ -12x + 12 \\ \underline{-12x + 12} \\ 0 \end{array}$$

$\therefore y = (x - 1)(x^2 - 4x - 12)$

$= (x - 1)(x + 2)(x - 6)$

$\therefore y = 0$  when  $x = -2, 1$  or  $6$

$\therefore B$  is  $(-2, 0)$  and  $D$  is  $(6, 0)$

6 let  $f(x) = 2x^3 + x^2 - 13x + 6$

$f(1) = -4, f(2) = 0$

$\therefore (x - 2)$  is a factor of  $f(x)$

$$\begin{array}{r} 2x^2 + 5x - 3 \\ x - 2 \overline{) 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 - 4x^2} \phantom{+ 6} \\ 5x^2 - 13x \phantom{+ 6} \\ \underline{5x^2 - 10x} \phantom{+ 6} \\ -3x + 6 \\ \underline{-3x + 6} \\ 0 \end{array}$$

$\therefore (x - 2)(2x^2 + 5x - 3) = 0$

$(x - 2)(2x - 1)(x + 3) = 0$

$x = -3, \frac{1}{2}, 2$

7 a  $p(-1) = 3$   
 $\therefore -b + a + 10 + b = 3$   
 $a = -7$

b  $p(\frac{1}{3}) = -1$   
 $\therefore \frac{1}{27}b - \frac{7}{9} - \frac{10}{3} + b = -1$   
 $b - 21 - 90 + 27b = -27$   
 $b = 3$

9  $f(\frac{2}{3}) = 6$   
 $\therefore \frac{8}{9} + \frac{4}{9}k - \frac{14}{3} + 2k = 6$   
 $8 + 4k - 42 + 18k = 54$   
 $22k = 88$   
 $k = 4$

11 a  $f(2) = 0$   
 $\therefore 8 + 2p + q = 0$   
 $q = -2p - 8$   
 b  $f(-1) = -15$   
 $\therefore -1 - p + q = -15$   
 $q = p - 14$   
 $\therefore p - 14 = -2p - 8$   
 $p = 2, q = -12$

8 a  $= f(-1) = -1 - 7 - 1 + 10 = 1$   
 b  $x^3 - 7x^2 + x + 10 = 1$   
 $x^3 - 7x^2 + x + 9 = 0$   
 $x = -1$  is solution  $\therefore (x + 1)$  is factor

$$\begin{array}{r} x^2 - 8x + 9 \\ x+1 \overline{) x^3 - 7x^2 + x + 9} \\ \underline{x^3 + x^2} \phantom{+ 9} \\ -8x^2 + x \phantom{+ 9} \\ \underline{-8x^2 - 8x} \phantom{+ 9} \\ 9x + 9 \\ \underline{9x + 9} \\ 0 \end{array}$$

$\therefore (x + 1)(x^2 - 8x + 9) = 0$   
 $x = -1, \frac{8 \pm \sqrt{64 - 36}}{2} = -1, 4 \pm \sqrt{7}$

10 a  $f(3) = 54 - 63 + 12 - 3 = 0$   
 $\therefore (x - 3)$  is a factor of  $f(x)$

b

$$\begin{array}{r} 2x^2 - x + 1 \\ x-3 \overline{) 2x^3 - 7x^2 + 4x - 3} \\ \underline{2x^3 - 6x^2} \phantom{+ 4x - 3} \\ -x^2 + 4x \phantom{- 3} \\ \underline{-x^2 + 3x} \phantom{- 3} \\ x - 3 \\ \underline{x - 3} \\ 0 \end{array}$$

$\therefore f(x) \equiv (x - 3)(2x^2 - x + 1)$   
 c  $f(x) = 0 \Rightarrow (x - 3)(2x^2 - x + 1) = 0$   
 $x = 3$  or  $2x^2 - x + 1 = 0$   
 for  $2x^2 - x + 1 = 0$ ,  $b^2 - 4ac = -7$   
 $b^2 - 4ac < 0 \Rightarrow$  no real roots  
 $\therefore$  only one real solution

12  $f(-3) = 0 \therefore (x + 3)$  is a factor of  $f(x)$

$$\begin{array}{r} x^2 + x - 3 \\ x+3 \overline{) x^3 + 4x^2 + 0x - 9} \\ \underline{x^3 + 3x^2} \phantom{+ 0x - 9} \\ x^2 + 0x \phantom{- 9} \\ \underline{x^2 + 3x} \phantom{- 9} \\ -3x - 9 \\ \underline{-3x - 9} \\ 0 \end{array}$$

$\therefore f(x) = (x + 3)(x^2 + x - 3)$   
 other solutions given by  $x^2 + x - 3 = 0$   
 $x = \frac{-1 \pm \sqrt{1 + 12}}{2} = \frac{-1 \pm \sqrt{13}}{2}$   
 $x = -2.30$  or  $1.30$

**13 a**  $f(-2) = -7$   
 $\therefore (-2 + k)^3 - 8 = -7$   
 $(k - 2)^3 = 1$   
 $k = 3$

**b**  $f(x) \equiv (x + 3)^3 - 8$   
 $\therefore f(-1) = 2^3 - 8 = 0$   
 $\therefore (x + 1)$  is a factor

**14 a**  $= f(-2) = -8 - 16 + 14 + 8 = -2$

**b**  $c = 2$

**c**  $g(x) \equiv x^3 - 4x^2 - 7x + 10$

$$\begin{array}{r} x^2 - 6x + 5 \\ x + 2 \overline{) x^3 - 4x^2 - 7x + 10} \\ \underline{x^3 + 2x^2} \phantom{+ 10} \\ -6x^2 - 7x \phantom{+ 10} \\ \underline{-6x^2 - 12x} \phantom{+ 10} \\ 5x + 10 \\ \underline{5x + 10} \\ 0 \end{array}$$

$\therefore g(x) = (x + 2)(x^2 - 6x + 5)$

$= (x + 2)(x - 1)(x - 5)$

$g(x) = 0 \Rightarrow x = -2, 1, 5$

**15 a**  $f(\frac{1}{2}k) = 4$   
 $\therefore \frac{1}{8}k^3 - 2k + 1 = 4$   
 $k^3 - 16k + 8 = 32$   
 $k^3 - 16k - 24 = 0 \quad (1)$

**b**  $f(-k) = 1$   
 $\therefore -k^3 + 4k + 1 = 1$   
 $k^3 = 4k$

sub (1)  $\Rightarrow 4k - 16k - 24 = 0$   
 $12k = -24$   
 $k = -2$