ALGEBRA

1 $f(x) \equiv x^3 + x^2 - 22x - 40.$

a Show that $(x + 2)$ is a factor of $f(x)$. **(2)**

b Express $f(x)$ as the product of three linear factors. **(4)**

c Solve the equation $f(x) = 0$. **(1)**

2 $f(x) \equiv x^3 - 2x^2 + kx + 1.$

Given that the remainder when $f(x)$ is divided by $(x - 2)$ and the remainder when $f(x)$ is divided by $(x + 3)$ are equal,

a find the value of the constant k, **(4)**

b find the remainder when $f(x)$ is divided by $(x + 2)$. **(2)**

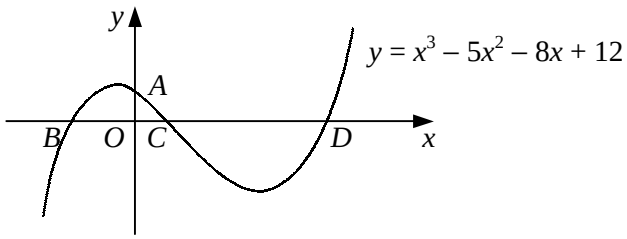
3 The polynomial $p(x)$ is defined by

$$p(x) \equiv 2x^3 - 9x^2 - 2x + 11.$$

a Find the remainder when $p(x)$ is divided by $(x + 2)$. **(2)**

b Find the quotient and remainder when $p(x)$ is divided by $(x - 4)$. **(3)**

4



The diagram shows the curve with the equation $y = x^3 - 5x^2 - 8x + 12$.

a State the coordinates of the point A where the curve crosses the y-axis. **(1)**

The curve crosses the x-axis at the points B, C and D. Given that C has coordinates $(1, 0)$,

b find the coordinates of the points B and D. **(6)**

5 $f(x) \equiv x^3 - 3x^2 + kx + 8.$

Given that $(x - 1)$ is a factor of $f(x)$,

a find the value of k, **(2)**

b solve the equation $f(x) = 0$. **(5)**

6 Solve the equation

$$2x^3 + x^2 - 13x + 6 = 0. \quad \textbf{(7)}$$

7 The polynomial $p(x)$ is defined by

$$p(x) \equiv bx^3 + ax^2 - 10x + b,$$

where a and b are constants.

Given that when $p(x)$ is divided by $(x + 1)$ the remainder is 3,

a find the value of a. **(2)**

Given also that when $p(x)$ is divided by $(3x - 1)$ the remainder is -1,

b find the value of b. **(3)**

8 $f(x) \equiv x^3 - 7x^2 + x + 10.$

a Find the remainder when $f(x)$ is divided by $(x + 1)$. **(2)**

b Hence, or otherwise, solve the equation $f(x) = 1$, giving your answers in exact form. **(6)**

9 $f(x) \equiv 3x^3 + kx^2 - 7x + 2k.$

When $f(x)$ is divided by $(3x - 2)$ the remainder is 6.

Find the value of the constant k. **(3)**

10 $f(x) \equiv 2x^3 - 7x^2 + 4x - 3.$

a Show that $(x - 3)$ is a factor of $f(x)$. **(2)**

b Hence, express $f(x)$ as the product of a linear factor and a quadratic factor. **(3)**

c Show that there is only one real solution to the equation $f(x) = 0$. **(3)**

11 The polynomial $f(x)$ is defined by

$$f(x) \equiv x^3 + px + q,$$

where p and q are constants.

Given that $(x - 2)$ is a factor of $f(x)$,

a find an expression for q in terms of p. **(2)**

Given also that when $f(x)$ is divided by $(x + 1)$ the remainder is -15,

b find the values of p and q. **(4)**

12 $f(x) \equiv x^3 + 4x^2 - 9.$

Given that $x = -3$ is a solution to the equation $f(x) = 0$, find the other two solutions correct to 2 decimal places. **(6)**

13 $f(x) \equiv (x + k)^3 - 8.$

Given that when $f(x)$ is divided by $(x + 2)$ the remainder is -7,

a find the value of the constant k, **(3)**

b show that $(x + 1)$ is a factor of $f(x)$. **(2)**

14 $f(x) \equiv x^3 - 4x^2 - 7x + 8.$

a Find the remainder when $f(x)$ is divided by $(x + 2)$. **(2)**

Given that

$$g(x) \equiv f(x) + c,$$

and that $(x + 2)$ is a factor of $g(x)$,

b state the value of the constant c, **(1)**

c solve the equation $g(x) = 0$. **(4)**

15 $f(x) \equiv x^3 - 4x + 1.$

Given that when $f(x)$ is divded by $(2x - k)$, where k is a constant, the remainder is 4,

a show that $k^3 - 16k - 24 = 0$. **(3)**

Given also that when $f(x)$ is divided by $(x + k)$ the remainder is 1,

b find the value of k. **(3)**