

ALGEBRA

1 a Express $(\frac{2}{3})^{-2}$ as an exact fraction in its simplest form. (2)

b Solve the equation

$$x^{\frac{3}{2}} - 27 = 0. \quad (3)$$

2 Solve the simultaneous equations

$$x + 3y = 16$$

$$x^2 - xy + 2y^2 = 46 \quad (7)$$

3 Simplify

a $\sqrt{192} - 2\sqrt{12} + \sqrt{75}$ (4)

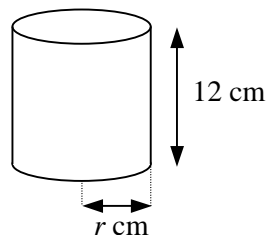
b $(2 + \sqrt{3})(5 - 2\sqrt{3})$ (3)

4 $f(x) \equiv x^2 - 4\sqrt{2}x + 11.$

a Express $f(x)$ in the form $a(x + b)^2 + c$ stating the exact values of the constants a , b and c . (4)

b Sketch the curve $y = f(x)$, showing the coordinates of the turning point of the curve and of any points of intersection of the curve with the coordinate axes. (3)

5



A sealed metal container for food is a cylinder of height 12 cm and base radius r cm.

Given that the surface area of the container must be at most $128\pi \text{ cm}^2$,

a show that $r^2 + 12r - 64 \leq 0$. (3)

b Hence find the maximum value of r . (4)

6 Find the non-zero value of x for which

$$(2\sqrt{x})^3 = 4x. \quad (4)$$

7 a Write down the value of x such that $2^x = 32$. (1)

b Solve the equation

$$32^{y+1} = 4^y. \quad (3)$$

8 a Given that $t = \sqrt{x}$, express $x - 5\sqrt{x}$ in terms of t . (1)

b Hence, or otherwise, solve the equation

$$x - 5\sqrt{x} + 6 = 0. \quad (4)$$

9 Prove, by completing the square, that there is no real value of the constant k for which the equation $x^2 + kx + 3 + k^2 = 0$ has real roots. (6)

- 10 a** Find the value of x such that

$$8^{2x-1} = 32. \quad (3)$$
- b** Find the value of y such that

$$\left(\frac{1}{3}\right)^{y-2} = 81. \quad (3)$$
- 11** Solve the inequality

$$x(2x - 7) < (x - 2)^2. \quad (5)$$
- 12** Express

$$\frac{2}{3\sqrt{2}-4} - \frac{3-\sqrt{2}}{\sqrt{2}+1}$$
in the form $a + b\sqrt{2}$, where a and b are integers. (6)
- 13 a** Solve the equation

$$6y^2 + 25y - 9 = 0. \quad (3)$$
- b** Find the values of the constant k such that the equation

$$x^2 + kx + 16 = 0$$
has equal roots. (3)
- 14 a** Given that $y = 2^x$,
- i** show that $4^x = y^2$,
- ii** express 2^{x-1} in terms of y . (4)
- b** By using your answers to part **a**, or otherwise, find the values of x for which

$$4^x - 9(2^{x-1}) + 2 = 0. \quad (4)$$
- 15** Find the pairs of values (x, y) which satisfy the simultaneous equations

$$x^2 + 2xy + y^2 = 9$$

$$x - 3y = 1 \quad (7)$$
- 16 a** Prove, by completing the square, that the roots of the equation $x^2 + ax + b = 0$ are given by

$$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}. \quad (6)$$
- b** Hence, find an expression for b in terms of a such that the equation $x^2 + ax + b = 0$ has a repeated root. (2)
- 17**
$$f(x) \equiv 2x^2 - 12x + 19.$$
- a** Prove that $f(x) \geq 1$ for all real values of x . (5)
- b** Find the set of values of x for which $f(x) < 9$. (4)
- 18 a** Express $(1 - \sqrt{5})^2$ in the form $a + b\sqrt{5}$. (2)
- b** Hence, or otherwise, solve the equation

$$y^2 = 3 - \sqrt{5},$$
giving your answers in the form $c\sqrt{2} + d\sqrt{10}$, where c and d are exact fractions. (6)